STABILITY OF 2D MULTI-POINT FORWARD-BACKWARD SCHEME

Let $h_j^n = \hat{h}\lambda^n e^{i(ki\Delta x + \ell j\Delta y)}$, $u_{i+1/2}^n = \hat{u}\lambda^n e^{i(k(i+1/2)\Delta x + \ell j\Delta y)}$ and $v_{i+1/2}^n = \hat{v}\lambda^n e^{i(kii\Delta x + \ell(j+1/2)\Delta y)}$. Then

$$\lambda = (1-2C) \pm \sqrt{4C(-1+C)}$$

where $C = \mu^2\left[\sin^2(k\Delta x/2) + \sin^2(l\,\Delta y/2)\right]\left[a_0 + 4\sum_{i'=1}^{J}\sum_{j=0}^{J} a_{i',j'}\left(\cos(ki'\Delta x)\cos(l\,j'\Delta y)\right)\right]$

$C < 1$ is required for the wave to be progressive. Then

$$|\lambda|^2 = (1-2C)^2 + 4C(1-C) = 1$$ i.e., neutral.

Equivalent trapezoidal scheme (ETZ):

$$F(\xi,\eta) \equiv \frac{1}{2\sqrt{1+\mu^2\left((\sin\xi/2)^2 + (\sin\eta/2)^2\right)}}$$

$$= \frac{a_0}{4} + \sum_{i'=1}^{J}\sum_{j'=0}^{J} a_{i',j'}\cos(i'\xi)\cos(j'\eta)$$

Thus the coefficients “a” for ETZ are the double Fourier coefficients of the function $F(\xi,\eta)$.