STABILITY OF MULTI-POINT 2-LEVEL EXPLICIT SCHEMES

Let $h_j^n = \hat{h}\lambda^n e^{ikj\Delta x}$ and $u_{i-1/2}^n = \hat{u}\lambda^n e^{ik(i-1/2)\Delta x}$.

$$\lambda = (1-2C\alpha) \pm \sqrt{4C\left(-1+C\alpha^2\right)}$$

where $C \equiv \mu^2 \sin^2 \frac{k\Delta x}{2} \left(a_0 + 2\sum_{j'=1}^{J} a_{j'} \cos(kj'\Delta x) \right)^2$

$C\alpha^2 < 1$ is required for the wave to be progressive. Then $|\lambda|^2 = 1 + 4C(1-\alpha)$.

Regardless of C,

$\alpha > 1$: damping

$\alpha = 1$: neutral

$\alpha < 1$: amplifying

When $\alpha = 1$ (backward scheme for the momentum equation), the requirement becomes

$$C < 1$$