NONINTERACTION IN THE PRIMITIVE EQUATION SYSTEM

$$\frac{\partial \bar{u}}{\partial t} = \left(f - \frac{\partial \bar{u}}{\partial y}\right)\bar{v} - \bar{\omega}\frac{\partial \bar{u}}{\partial p} - \left(\frac{\partial}{\partial y}\overline{u'v'} + \frac{\partial}{\partial p}\overline{u'\omega'}\right) - g\frac{\partial}{\partial p}\bar{\tau}_x$$

$$\frac{\partial \bar{\theta}}{\partial t} = -\bar{v}\frac{\partial \bar{\theta}}{\partial y} - \bar{\omega}\frac{\partial \bar{\theta}}{\partial p} - \left(\frac{\partial}{\partial y}\overline{v'\theta'} + \frac{\partial}{\partial p}\overline{\omega'\theta'}\right) + \frac{1}{c_p}\left(\frac{p_0}{p}\right)^{\kappa}\bar{Q}$$

new

generalized

For neutral non-disspative linear waves, Arakawa (1957 a,b) showed

$$\frac{\partial}{\partial y}\overline{u'v'} + \frac{\partial}{\partial p}\overline{u'\omega'} = \left(f - \frac{\partial \bar{u}}{\partial y}\right)\frac{\partial}{\partial p}\left(\frac{\overline{v'\theta'}}{\partial \bar{\theta}/\partial p}\right) + \frac{\partial \bar{u}}{\partial p}\frac{\partial}{\partial y}\left(\frac{\overline{v'\theta'}}{\partial \bar{\theta}/\partial p}\right)$$

A geralization of Eliassen & Palm (1960)

$$\overline{\omega'\theta'} = -\frac{\partial \bar{\theta}}{\partial y}\frac{\overline{v'\theta'}}{\partial \bar{\theta}/\partial p}$$

Then,

$$\frac{\partial \bar{u}}{\partial t} = \hat{v}\left(f - \frac{\partial \bar{u}}{\partial y}\right) - \hat{\omega}\frac{\partial \bar{u}}{\partial p} - g\frac{\partial}{\partial p}\bar{\tau}$$

$$\frac{\partial \bar{\theta}}{\partial t} = -\left(\hat{v}\frac{\partial \bar{\theta}}{\partial y} + \hat{\omega}\frac{\partial \bar{\theta}}{\partial p}\right) + \frac{1}{c_p}\left(\frac{p_0}{p}\right)^{\kappa}\bar{Q}$$

where

$$\hat{v} \equiv \bar{v} - \frac{\partial}{\partial p}\left(\frac{\overline{v'\theta'}}{\partial \bar{\theta}/\partial p}\right)$$

$$\hat{\omega} \equiv \bar{\omega} + \frac{\partial}{\partial y}\left(\frac{\overline{v'\theta'}}{\partial \bar{\theta}/\partial p}\right)$$

Noninteraction theorem still holds, but now 3D advection is involved, instead of advection by horizontal geostrophic velocity.