NONINTERACTION THEOREM VIEWED FROM THE MOMENTUM EQUATION

$$\frac{\partial \bar{\theta}}{\partial t} = -\frac{\partial}{\partial y}\overline{v'\theta'} + \sigma\bar{\omega} + \frac{1}{c_p}\left(\frac{p_0}{p}\right)^{\kappa}\bar{Q}$$

$$\frac{\partial \bar{u}}{\partial t} = -\frac{\partial}{\partial y}\overline{u'v'} + f\bar{v} - g\frac{\partial}{\partial p}\bar{\tau}_x$$

$$\frac{\partial \bar{v}}{\partial y} + \frac{\partial \bar{\omega}}{\partial p} = 0$$

Define "residual" mean meridional circulation by $\hat{\omega} \equiv \bar{\omega} - \frac{\partial}{\partial y}\left(\frac{1}{\sigma}\overline{v'\theta'}\right)$.

Then

$$\frac{\partial \bar{\theta}}{\partial t} = \sigma\hat{\omega} + \frac{1}{c_p}\left(\frac{p_0}{p}\right)^{\kappa}\bar{Q}$$

When stationary, $\hat{\omega}$ is "thermally forced".

Require $\frac{\partial \hat{v}}{\partial y} + \frac{\partial \hat{\omega}}{\partial p} = 0$. Then $\hat{v} = \bar{v} + \frac{\partial}{\partial p}\left(\frac{1}{\sigma}\overline{v'\theta'}\right)$.

$$\frac{\partial \bar{u}}{\partial t} = f_0\hat{v} - \underbrace{\left[\frac{\partial}{\partial y}\left(\overline{u'v'}\right) + \frac{\partial}{\partial p}\left(\frac{f_0}{\sigma}\overline{v'\theta'}\right)\right]}_{\overline{v'q'}} - g\frac{\partial}{\partial p}\bar{\tau}_x$$

$\overline{v'q'} \rightarrow 0$ for neutral waves without potential enstrophy cascade