J1.18 HOW WELL CAN ENSEMBLE PERTURBATIONS EXPLAIN FORECAST ERRORS?

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1. INTRODUCTION

In this paper the performances of global ensemble forecasts generated by NCEP and ECMWF are evaluated. Current products from NCEP consist of 10 ensemble forecasts both at 00 UTC and 12 UTC each day, while ECMWF produces a 50-member ensemble. We will study how much the forecast errors can be explained by the ensemble perturbations. For this purpose, we compare the structures of forecast errors and the first 10 individual ensemble perturbations which are the differences between the perturbed forecasts and the unperturbed control forecast. For different forecast lead times, the correlations between the forecast errors and the individual ensemble perturbations are computed. In addition, the optimally combined perturbations from the first 10 ensemble forecasts are computed and also compared with the forecast errors. The dependence of correlations on the number of ensembles will also be evaluated for different lead times by using data from both centers.

2. DEFINITIONS

NCEP ensemble perturbations (NPs) are defined as the differences between the perturbed forecasts and the control forecasts (Toth and Kalnay, 1997; Szunyogh et al. , 1997)

$$\mathbf{NP}_i(t) = \mathbf{X}_i^{NCEP}(t) - \mathbf{X}_{control}^{NCEP}(t), \tag{1}$$

Note that at 24-hour lead time these are, by definition, the bred vectors used after rescaling as initial ensemble perturbations. ECMWF ensemble perturbations (EPs) are similarly defined to NPs,

$$\mathbf{EP}_i(t) = \mathbf{X}_i^{ECMWF}(t) - \mathbf{X}_{control}^{ECMWF}(t) \tag{2}$$

$i = 1, 2, \ldots, N$, $N = 10$ (20) for NCEP and 50 for ECMWF. The ECMWF perturbations are bond in both initial and evolved singular vectors (Buizza and Palmer,1995; Molteni et al. , 1996; Barkmeijer et al. , 1999). i.e.

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In both systems, forecast errors $F(t)$ are defined as a difference between a control forecast and verifying analysis from the same center,

$$\mathbf{F}(t) = \mathbf{X}_{control}(t) - \mathbf{X}_{analysis}(t). \tag{3}$$

The optimal combination of n NPs or EPs is obtained by solving a least-square problem

$$Min|\mathbf{F} - \sum_{i=1}^{n} \alpha_i \mathbf{V}_i|_{L2} \tag{4}$$

where $\mathbf{V}_i$ can be either $\mathbf{EP}_i(t)$ or $\mathbf{NP}_i(t)$. Having obtained α_i from the above equation, the optimally combined vector is defined as

$$\mathbf{V}_{optimal} = \sum_{i=1}^{n} \alpha_i \mathbf{V}_i. \tag{5}$$

The pattern anomaly correlation between any vectors $\mathbf{X}$ and $\mathbf{Y}$ is defined by

$$A_c(\mathbf{X}, \mathbf{Y}) = \frac{\{\mathbf{X}, \mathbf{Y}\}}{\{\mathbf{X}, \mathbf{X}\}^{\frac{1}{2}} \{\mathbf{Y}, \mathbf{Y}\}^{\frac{1}{2}}}. \tag{6}$$

The inner product of two vectors are defined for different regions. In addition to the global domain, results are computed for the tropics, Southern and Northern Hemisphere extra-tropics, North America and Europe. We will compute the correlations between the forecast errors $\mathbf{F}(t)$ and individual NPs and EPs, and present the correlations averaged over the 10 individual perturbations studied. Besides, the correlations between $\mathbf{V}_{optimal}$ and $\mathbf{F}(t)$ will also be computed for different regions and different forecast lead times. To save space in this paper, we show the results for only the global, North American (140W-50W, 20N-60N) and European (20W-40E, 77.5N-30N) domains.

3. EXPLAINED ERROR VARIANCE

In Fig. 1, we show the correlations between the forecast errors from NCEP and its perturbations (NP), and errors from ECMWF and its ensemble perturbations (EPs) for 500hPa geopotential height over the global domain. The correlation values are the averages over 30 days starting from 12 UTC April

1, 2001. The thick lines are the correlations between the forecast errors and the optimally combined vectors from the first 10 EPs or NPs for different forecast lead times. The corresponding thin lines are the averaged correlations between the ensemble forecast errors and the first 10 individual NPs or EPs respectively.

The correlation between the NCEP forecast errors and the optimally combined vector from the first 10 NPs from NCEP is shown in as a thick solid line. The average correlation between the NCEP forecast errors and the individual top 10 NPs from NCEP is displayed as a thin solid line. Clearly the correlation of the forecast error with the optimally combined bred vector is much higher than with the individual NPs. A linearly optimally combined vector from the first 10 NPs can explain much more forecast error than individual NPs at any forecast lead time.

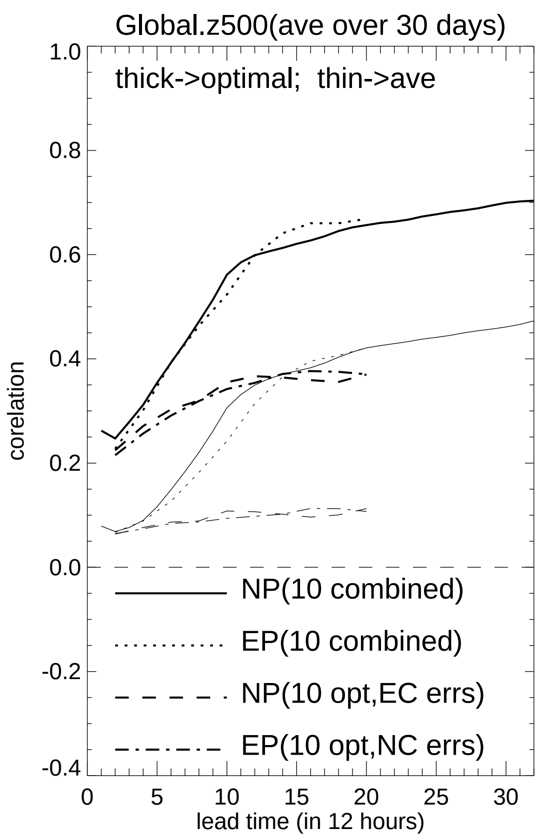

Figure 1: The correlations between the forecast errors from both NCEP and ECMWF and corresponding ensemble perturbations (EPs and NPs) in 500hPa geopotential height in global domain, averaged over 30 days period started at 12UTC from April 01, 2001

The thick dotted line shows the correlation between the ECMWF forecast error and the optimally combined vector from the top 10 EPs from ECMWF. The average correlation between the ECMWF forecast error and the first 10 EPs from ECMWF is shown as a thin dotted line. As for the NCEP results, an optimally combined vector from the first 10 EPs explains much more forecast error than the individual EPs at any forecast lead time.

At longer lead time, the average correlations of the first 10 individual NPs and 10 EPs with their respective forecast errors are converging. For shorter lead times, however the first 10 individual NPs can better explain NCEP forecast errors than the first 10 individual EPs can explain the ECMWF forecast errors.

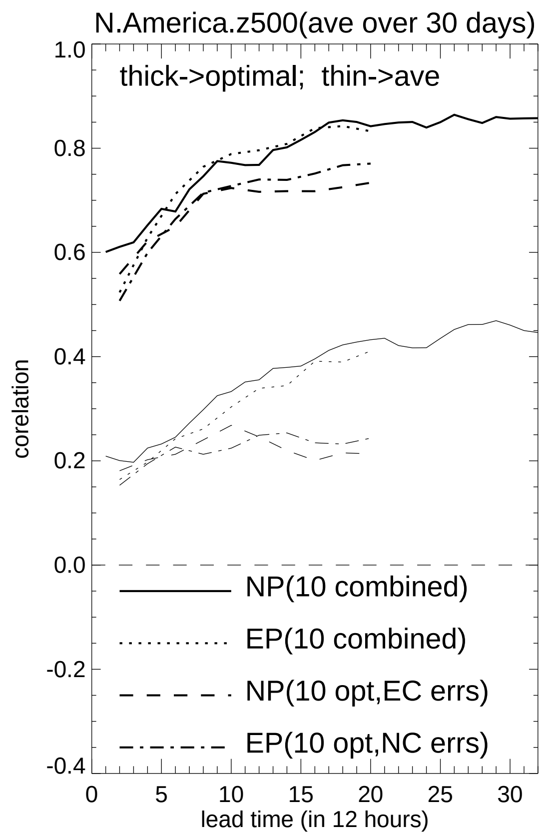

Figure 2: As in Fig.1, but for North American region.

Let us now compare the optimal linear combinations of the first 10 NPs and 10 EPs from NCEP and ECMWF. For the 0 up to 6 days lead time range, the optimally combined vector from the first 10 NPs has a slightly higher correlation with the NCEP forecast error than that of the optimally combined vector from the first 10 EPs with the ECMWF forecast error. For lead times of 6 to 10 days, it is the optimally combined vector from the first 10 EPs that can better explain the respective forecast errors. It is expected that the initial perturbations play a more important role at short lead times (0-6 days), while model errors may become relevant at larger lead times (6 to 10 days).

Next, we use the first 10 NCEP perturbations to explain the ECMWF forecast errors and use the first 10 ECMWF perturbations to explain the NCEP forecast errors. The results are shown in Fig.1 as dashed and dash-dotted lines. The thick dashed line shows the correlation between the optimally combined first 10 NPs and the ECMWF forecast error. The correlation between the optimally combined vector of first 10 EPs and the NCEP forecast error is shown as a thick dash-dotted line. The results show that for lead times up to 7 days, the optimally combined NPs can explain the ECMWF forecast errors a little better than the optimally combined EPs can explain the NCEP forecast errors. After 7 days lead time, the optimally combined EPs have a slight advantage.

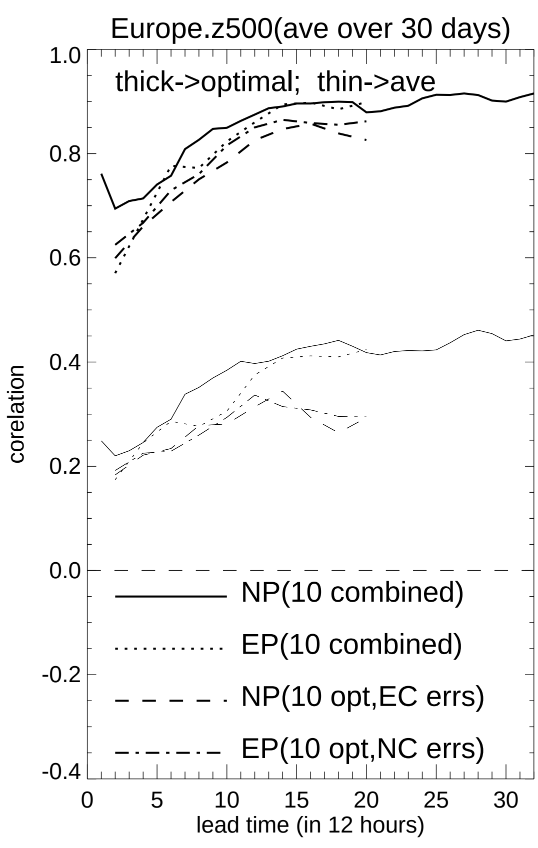

Figure 3: As in Fig.1, but for European region.

The corresponding results of average correlations between one center's forecast error and another center's first 10 individual dynamical vectors are displayed as thin dashed and dash-dotted lines. Again, in comparison with the corresponding results indicated by the thick lines, the optimally combined first 10 NPs or EPs can explain much more forecast errors than the individual NPs or EPs. In terms of the performance of individual perturbation vectors, neither ensemble systems show an ability to explain the other center's forecast errors.

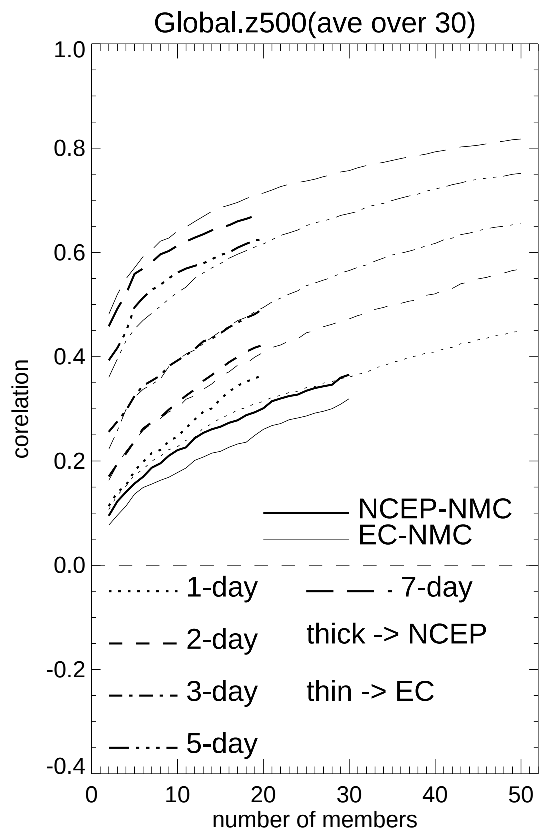

Figure 4: Correlations as a function of number of ensemble members

Similar computations were carried out for smaller regions. The results for North America are shown in Fig. 2. The obvious difference between this figure and Fig. 1 is that the ensemble perturbations from both NCEP and ECMWF, either individually or optimally combined, can explain more forecast errors in the North American region than over the global domain. This is due to the fewer degree of freedom in the error fields over a limited domain. In comparison with the global results, the NPs and EPs can explain relatively more forecast errors from the other center, though still somewhat less than what they can in their own forecast errors. Fig.2 also shows that the NPs slightly outperforms EPs in terms of explaining their own forecast errors for all lead times over North America during the time period studied (thin solid and dotted lines).

Fig. 3 shows the results for the European region. The overall results for Europe are very similar to those over North America displayed in Fig. 2. The optimally combined dominant NPs and EPs in this region can explain a larger part of the forecast error variance over North America. Again, NCEP perturbations are slightly more successful in explaining their own forecast errors than ECMWF perturbations.

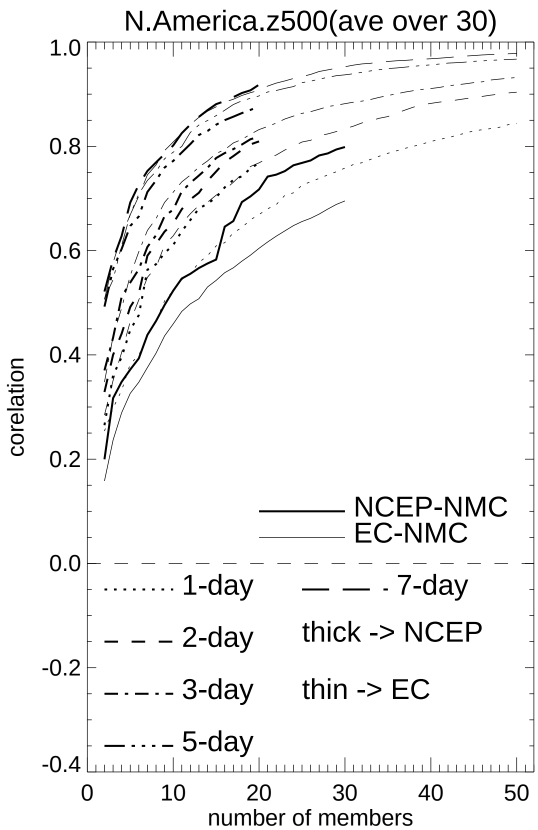

Figure 5: As in Fig.4, but for North American region.

4. EFFECT OF ENSEMBLE SIZE

Here we study the dependence of the correlations between the dominant NPs or EPs and the respective forecast errors on ensemble size. The results shown in Fig. 4 are the correlations between various number of optimally combined NPs and EPs and the respective NCEP and ECMWF forecast errors over the global domain. The results are displayed for various lead times (1, 2, 3, 5 and 7 days). The results from NCEP are indicated in thick lines and the thin lines show the results from ECMWF.

For 1 and 2 days lead times, any available number of the optimally combined NPs can explain more forecast error than the same number of optimally combined EPs can explain the ECMWF forecast error (thick and thin dotted lines respectively). While the ECMWF ensemble has 50 members initiated at 12 UTC, NCEP has only 10. To study the effect of a larger ensemble for the NCEP system, here we combine the 10 ensemble at 12 UTC and the subsequent 00 UTC NCEP ensembles. We consequently choose from the subsequent set of ensemble since as seen from Fig. 1, the use of longer lead time ensemble initiated 12 hour earlier would have led to larger correlations.

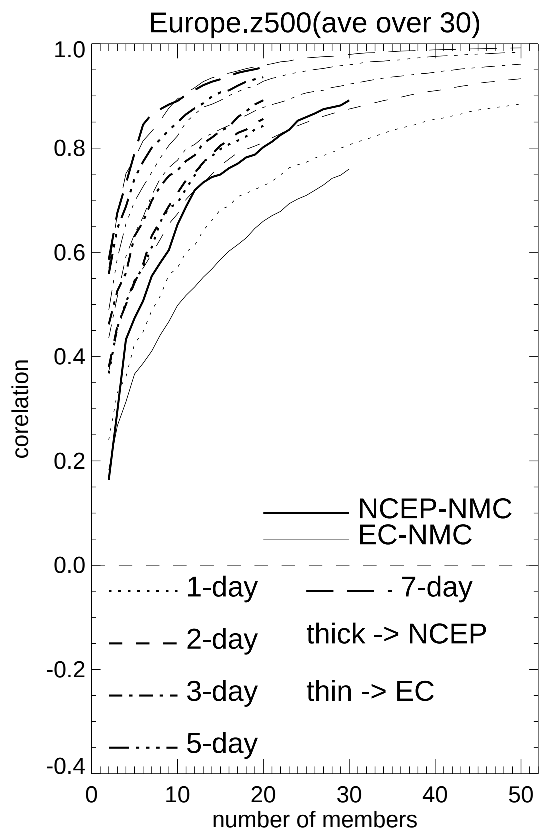

Figure 6: As in Fig.4, but for European region.

Also shown are the correlations between the 1-day forecast errors and an ensemble of forecast differences defined by the "NMC method" (Parrish and Derber, 1992). The NMC vectors are defined as the difference field between 24 and 48 hour forecasts valid at the same time. These NMC vector correlation values shown are also averages over 30 consecutive days in April. We generated 30 forecast difference fields using data from preceding period, from March 5, 2001 at 12 UTC to April 3, 2001. We call these difference fields NMC vectors.

It is useful to compare correlations from both NCEP and ECMWF at 1-day lead time with those of NMC vectors which have been widely used to build the background forecast error covariances in 3-D Var data assimilation systems at various meteorological centers including NCEP and CMC. It is clear that the correlations between the NCEP NMC vectors and NCEP forecast errors are generally higher than those between ECMWF NMC vectors and ECMWF forecast errors, which are indicated by thick and thin solid lines respectively. But both NPs and EPs can better explain their respective 1-day forecast errors than the NMC perturbations. For lead times of 2-5 days, the optimally combined NPs have slight advantages over the optimally combined EPs which perform better at 7-day lead time.

Results from a similar analysis made for the North American region are shown in Fig. 5. For lead times of 1 and 2 days, NPs perform better than EPs in terms of explaining their own center's forecast errors. For example, the NCEP ensemble can explain a similar amount of variance in the 1-day error field as the ECMWF ensemble can in 2-day errors. The correlations of EPs with ECMWF forecast errors are very close to and sometimes higher than those between NPs and NCEP errors for lead times of 3 or more days. Like in the case of the global domain, the correlations between the NCEP NMC vectors and NCEP forecast errors are clearly higher than the those between ECMWF NMC vectors and its forecast errors. The results for the European region are displayed in Fig. 6. The conclusions are very similar to those for North America.

5. CONCLUSIONS

We have calculated the correlations between individual NCEP and ECMWF perturbations and their respective forecast errors. The correlations between the forecast errors and the optimally combined perturbations have also bee computed. Attempts have been made to use one center's ensemble perturbations to explain the other center's forecast errors. We have compared the NMC perturbations with the 1-day forecast errors for both NCEP and ECMWF.

It is found that the NCEP and ECMWF ensemble perturbations perform similarly in most cases in terms of their ability to explain forecast errors. The NCEP ensemble shows a slight advantage during the first 2-3 days in most cases. But after about 2 days, the ECMWF ensemble perturbations have higher correlations in some cases. For longer lead times, the correlations from the two systems become similar, indicating that both forecast errors and ensemble perturbations may converge to patterns related to the dominant Lyapunov vector (Reynolds and Errico, 1999).

An interesting finding from our results is that one center's perturbations are not particularly successful in explaining the other center's forecast errors, especially over the large global domain. In the tropics, NCEP ensembles perform better than ECMWF's in most cases (not shown). In most cases, we find that the ensemble perturbations from either center can explain more of their forecast errors than their respective NMC vectors. This indicates that using flow dependent ensemble perturbations instead of NMC perturbations to construct the background forecast error covariances in 3-D Var systems may improve the performance of data assimilation system. All experiments in this paper have been repeated using 1000hPa geopotential height data and led to similar conclusions.

6. REFERENCES

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