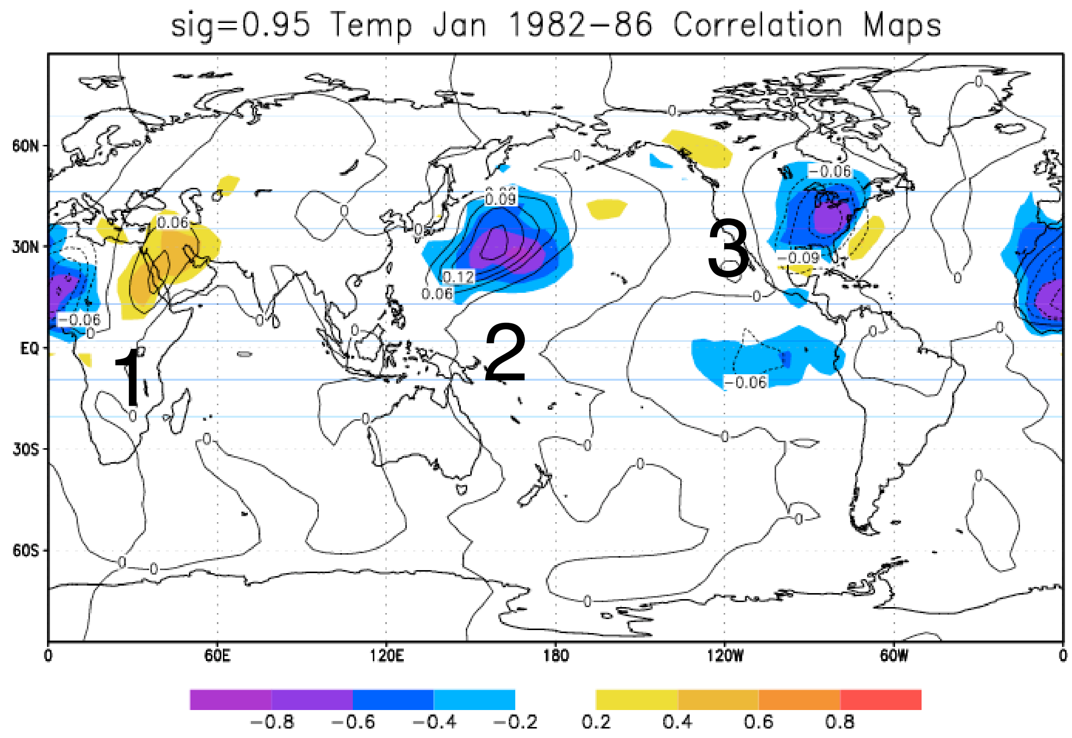
Estimating and Correcting Global Weather Model Error



Chris Danforth, Eugenia Kalnay, Takemasa Miyoshi

University of Maryland

January 18, 2006

NOAA THORPEX Workshop

Outline

- Brief review of empirical model error correction
- SPEEDY model
- Generation of 6-hour forecasts and analysis increments using NCEP reanalysis
- Separation of increments into seasonal, diurnal, and state-dependent components
- Estimation and correction of model errors
- Results: our method is effective and computationally feasible
- Conclusions

Background

Leith (1978), first to formulate state-dependent correction procedure

- given a model: $\dot{\mathbf{x}} = \mathrm{M}(\mathbf{x})$
- sought an improved model of the form: $\dot{\mathbf{x}} = \mathrm{M}(\mathbf{x}) + \mathrm{L}\mathbf{x} + \mathbf{c}$

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- the tendency error $\mathbf{g}$ of the improved model is given by

$$\mathbf{g} = \dot{\mathbf{x}}^{\mathrm{t}} - \mathrm{M}(\mathbf{x}^{\mathrm{t}}) - \mathrm{L}\mathbf{x}^{\mathrm{t}} - \mathbf{c}$$

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- derived an empirical correction by minimizing $< \mathbf{g}^{\top}\mathbf{g} >$ with respect to $\mathbf{c}$ and L
- $\mathbf{c}_{\mathrm{L}}$ is a state-independent bias estimate
- $\mathrm{L}_{\mathrm{L}}\mathbf{x}$ is a state-dependent estimate of the model error

Background

DelSole and Hou (1999)

- applied Leith's procedure to a 2-layer QG model on an 8 x 10 grid (N=160 degrees of freedom)
- perturbed the model parameters to generate 'nature'
- resulting model errors were strongly state-dependent
- Leith's state-dependent error correction extended forecast skill to within limits imposed by observational noise
- computationally prohibitive for operational use

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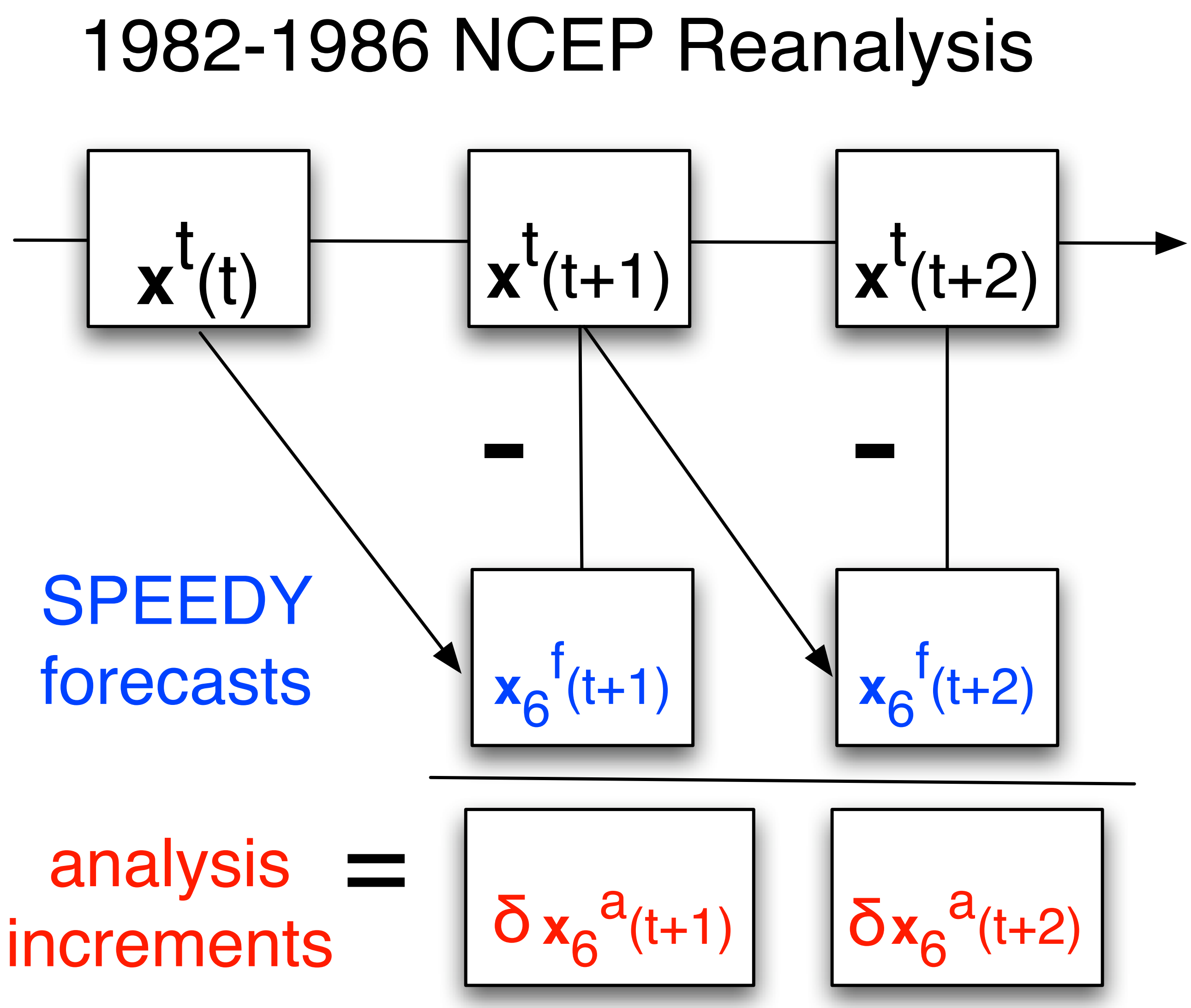
IV. Correct the state-dependent errors.

SPEEDY Model, Molteni (2003)

- primitive equations, global spectral model
- contains parameterizations of condensation, convection, clouds, radiation, surface fluxes, and vertical diffusion
- T30 horizontal resolution, 7 sigma levels
- integrates vorticity, divergence, temperature, specific humidity, and surface pressure
- post-processed into horizontal wind, temperature, specific humidity, geopotential height, and surface pressure on 96x48 grid, 7 pressure levels
- dissipation and time-dependent forcing determined by climatological SST, surface moisture, albedo, land-surface vegetation, etc.

Generating Time Series of Model Forecasts and Errors



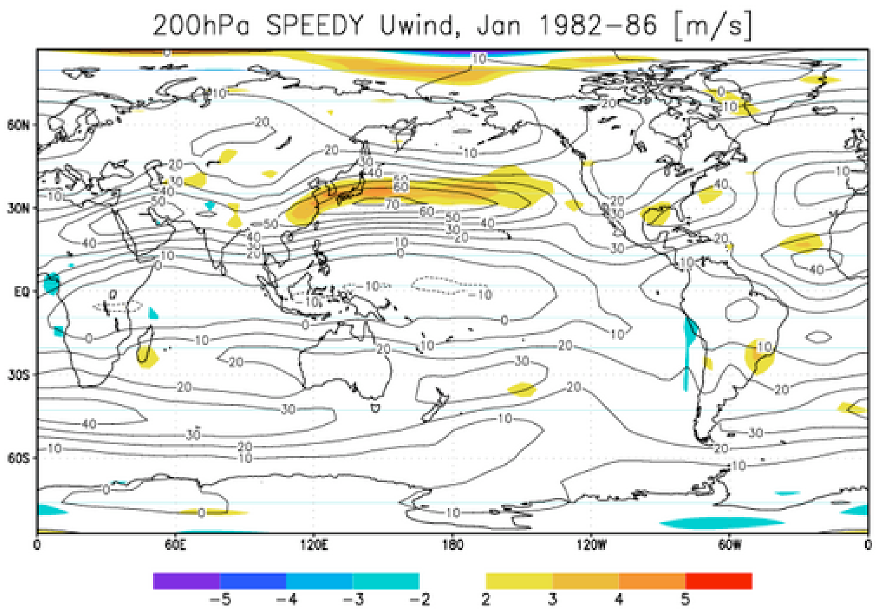
Time Series and 5-year Climatology

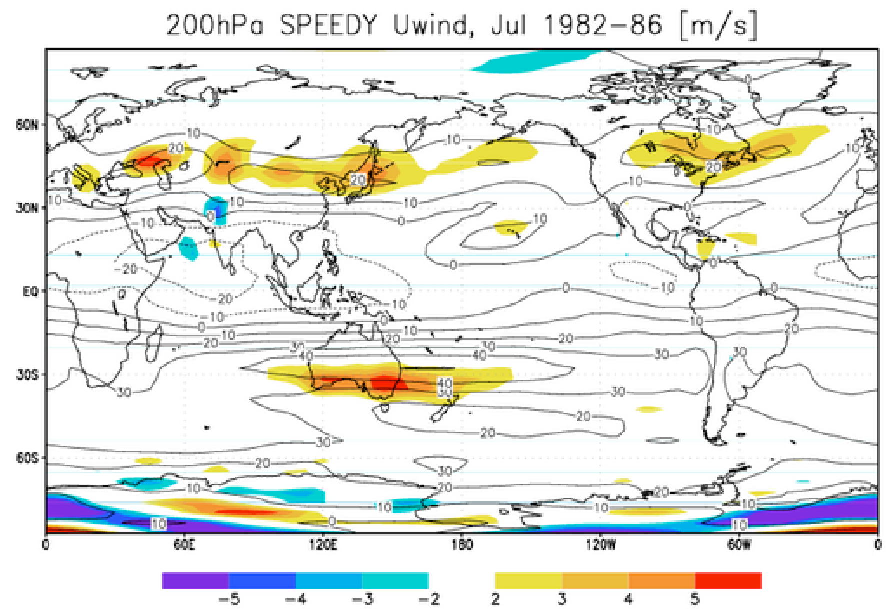
- $\mathbf{x}_6^{\mathrm{f}}(\mathrm{t})$ = time series of model states
- $\delta\mathbf{x}_6^{\mathrm{a}}(\mathrm{t})$ = corresponding analysis increments
- 5-year SPEEDY 6-hour climatology given by monthly mean $< \mathbf{x}_6^{\mathrm{f}} >$
- 5-year reanalysis climatology given by monthly mean $< \mathbf{x}^{\mathrm{t}} >$
- Bias given by monthly mean $< \delta\mathbf{x}_6^{\mathrm{a}} >$

200hPa Zonal Wind Monthly Bias

5-year Reanalysis Climatology $< \mathbf{x}^t >$ (contour), Bias $< \delta\mathbf{x}^a_6 >$ (color)

January July





- SPEEDY underestimates zonal wind on the poleward side of the winter hemisphere jet.
- Exhibits large winter polar bias.

I. Monthly Bias Correction

Generate three daily 5-day forecasts for each state in 1987 (*independent data*), verifying against NCEP reanalysis.

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2. Corrected *a posteriori*: Correct control forecast by bias $< \delta\mathbf{x}_6^a >$ at 6 hours, bias $< \delta\mathbf{x}_{12}^a >$ at 12 hours, etc.

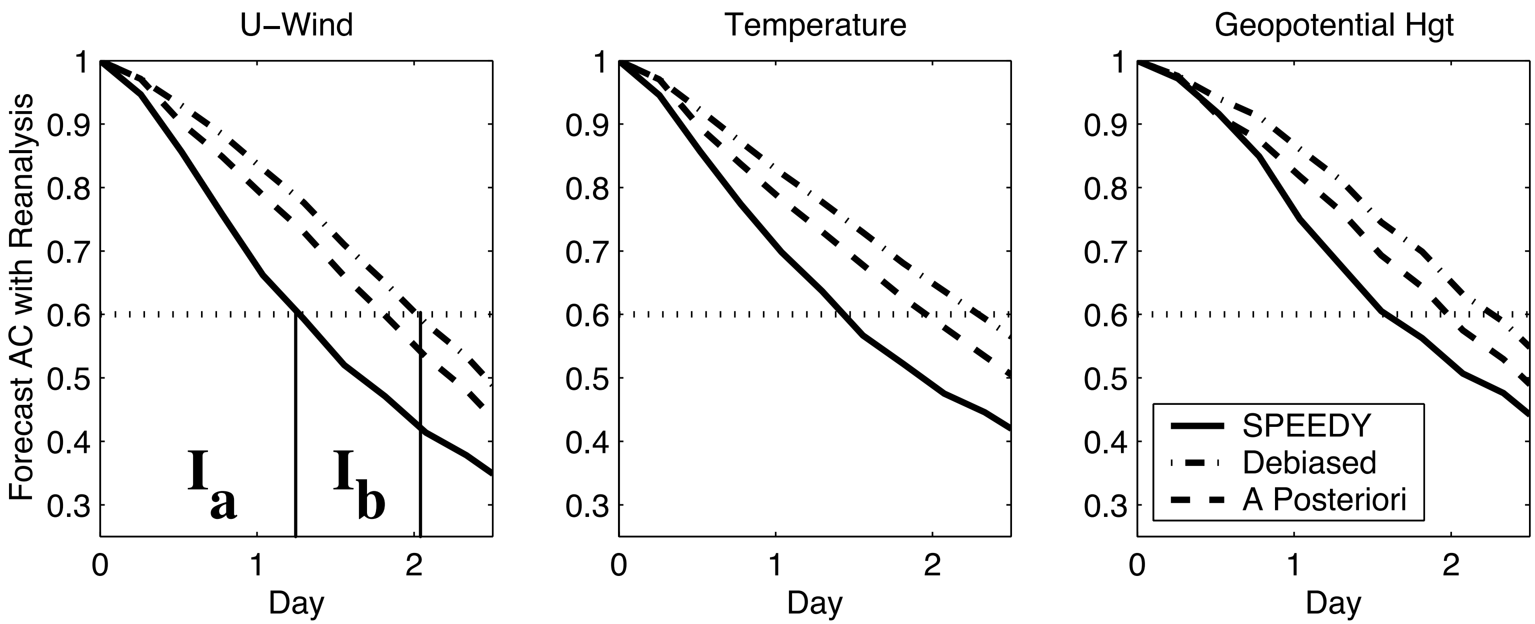
3. Corrected *online*: Integrate model, $\dot{\mathbf{x}} = \mathrm{M}(\mathbf{x}) + \frac{<\delta\mathbf{x}_6^a>}{\Delta t}$,

$< \delta\mathbf{x}_6^a >$ is a daily linear interpolation

$\left(\text{e.g. on July 1, } < \delta\mathbf{x}_6^a > = \frac{<\delta\mathbf{x}_6^a(\mathrm{Jun})> + <\delta\mathbf{x}_6^a(\mathrm{Jul})>}{2}\right)$

I. Monthly Bias Correction

500hPa November 1987 Global Mean Anomaly Correlation



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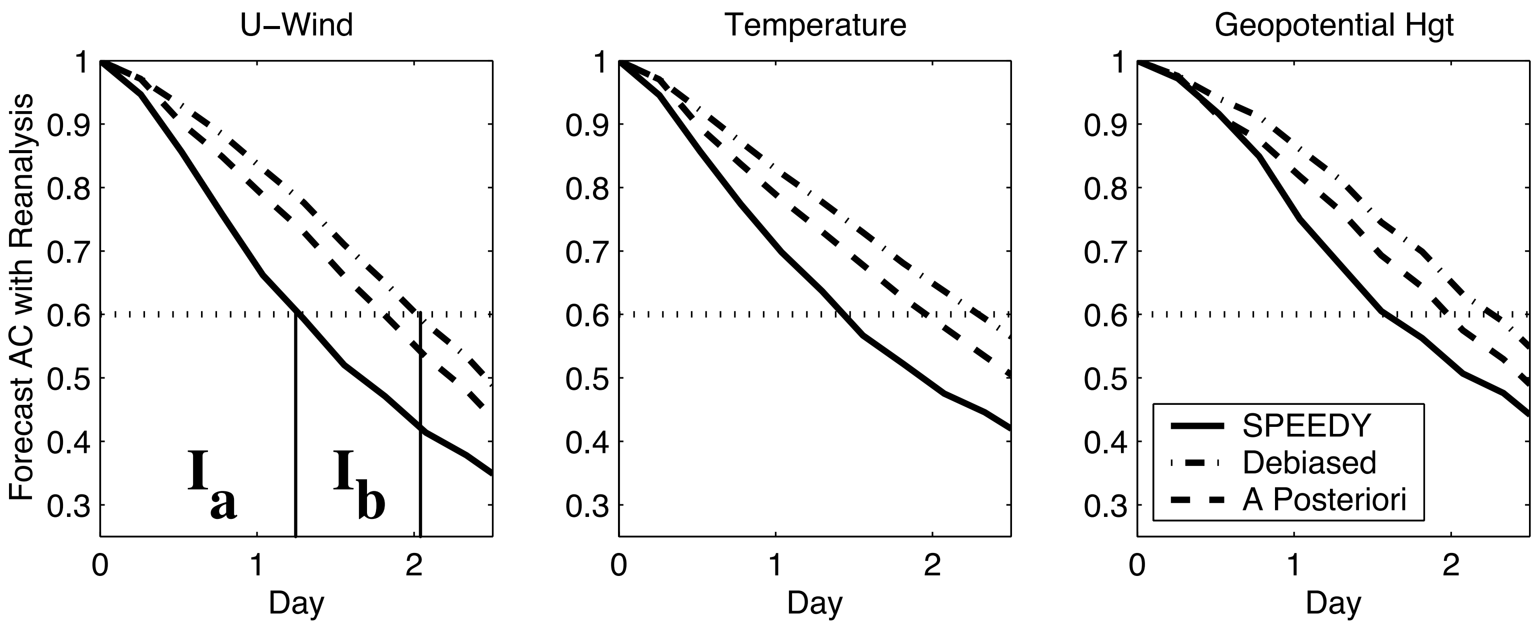
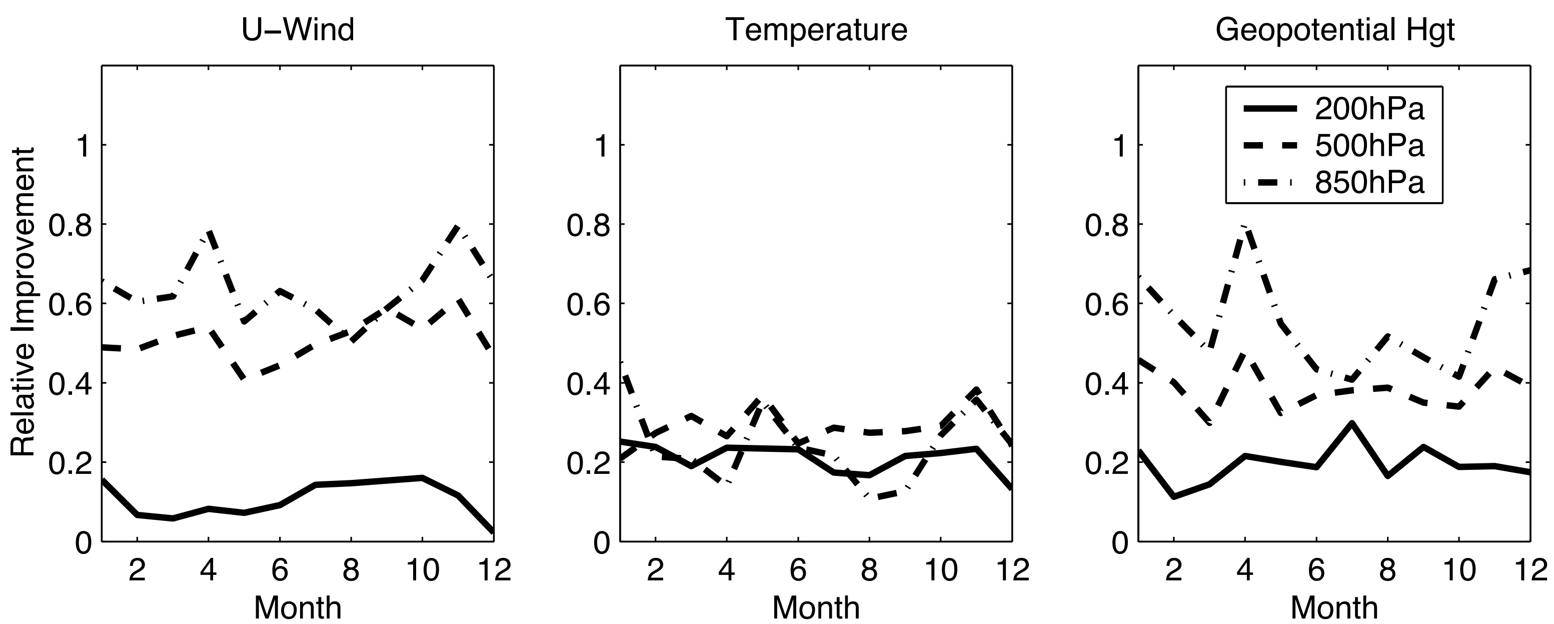


- Monthly bias correction gives substantial forecast improvement.
- Online correction performs better than a posteriori correction.

I. Monthly Bias Correction

Improvement of Online Correction Relative to Control



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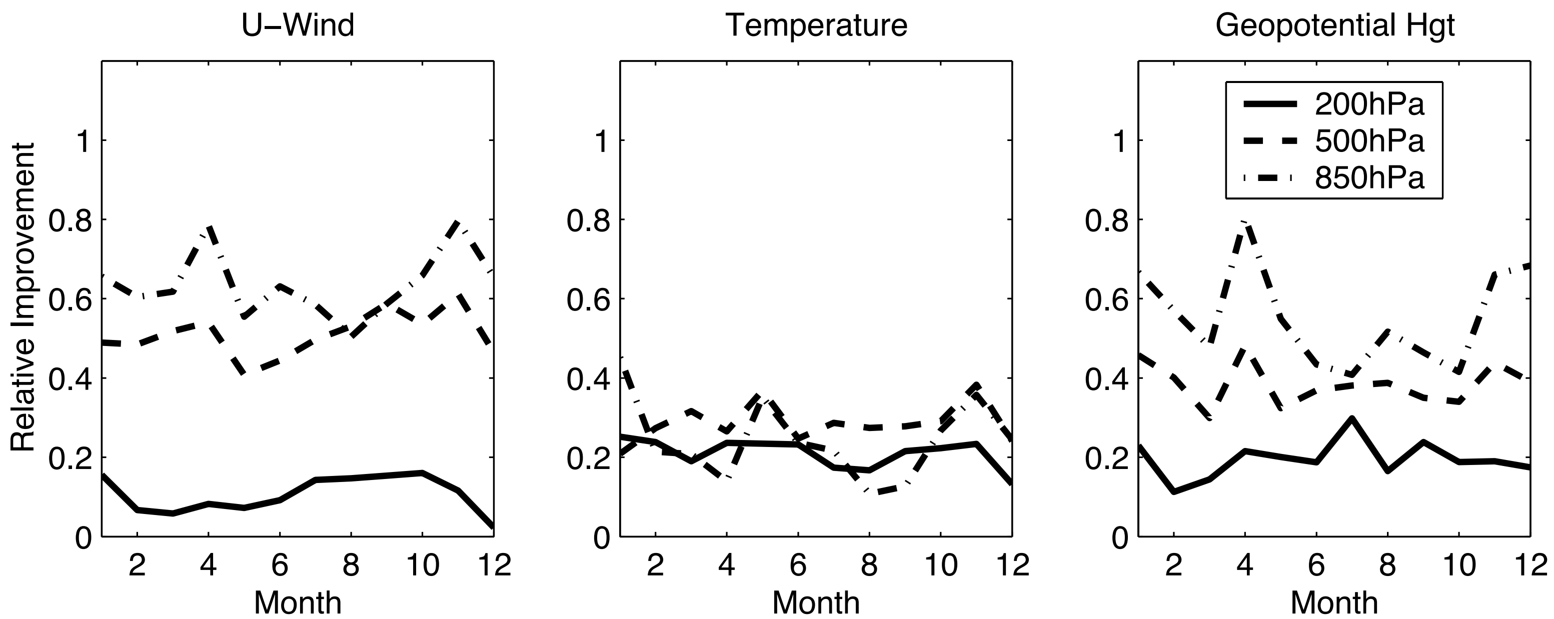
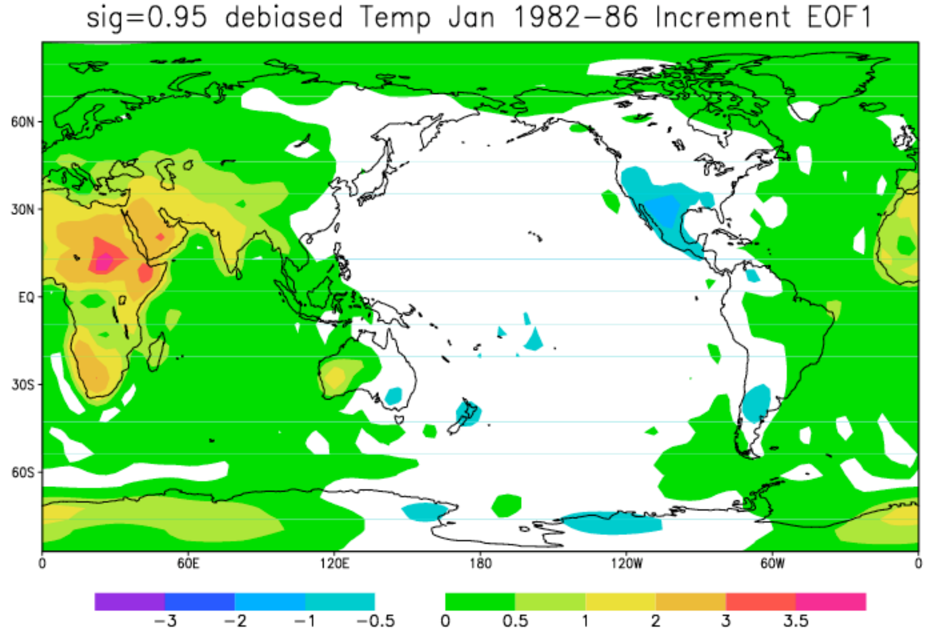
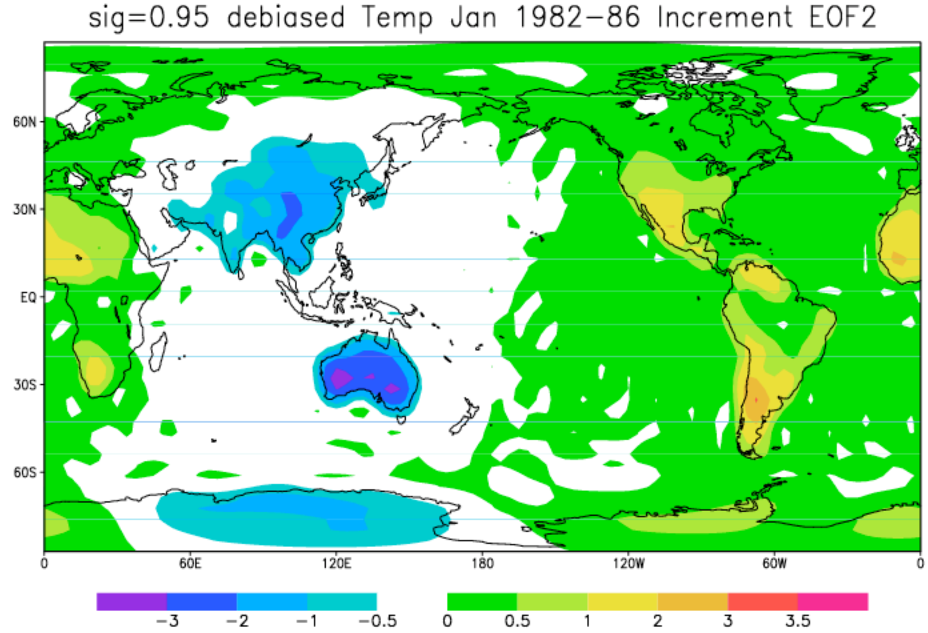


- Online correction is most effective at lower levels.
- Improvements are uniform across levels in T, across seasons by level.

II. Diurnal Bias Correction

Leading EOFs of $C_{\delta x^a \delta x^a}$, T at $\sigma = 0.95$, Jan 1982-1986



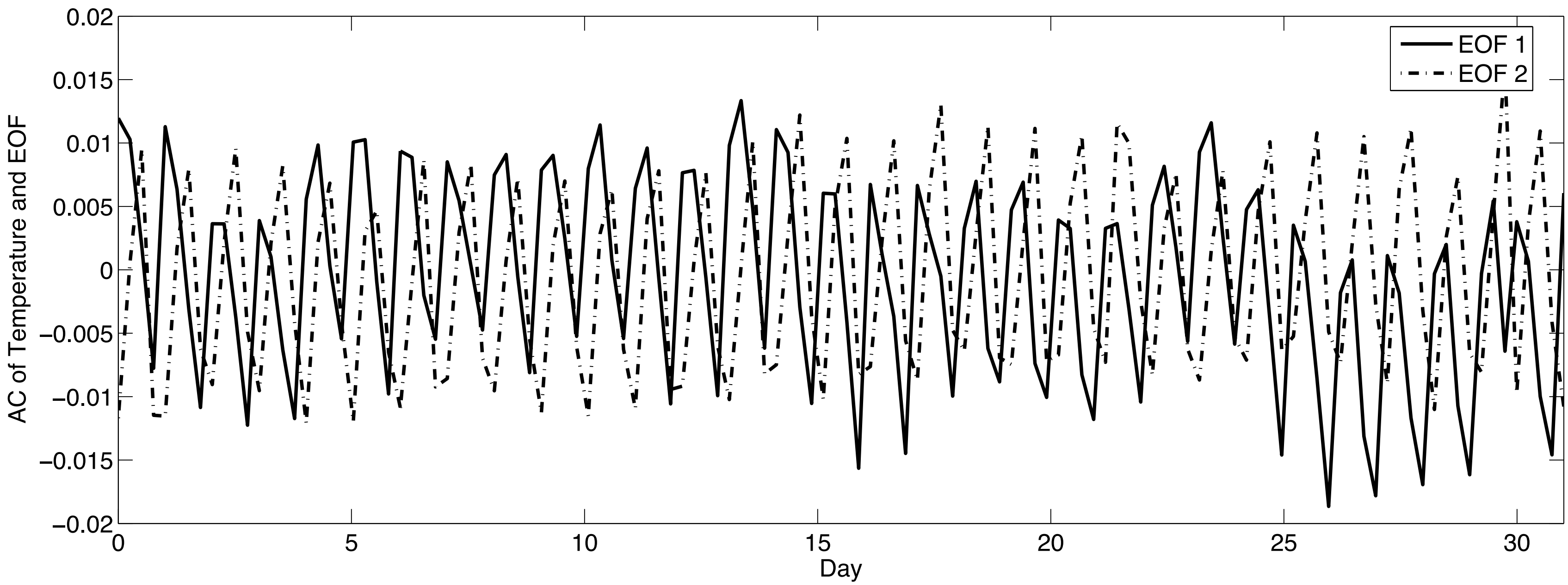


- Lack of diurnal forcing results in wavenumber 1 structure in the errors
- SPEEDY underestimates (overestimates) near surface daytime (night-time) temperatures, more prominent over land

II. Diurnal Bias Correction

Principal Components

- Project leading EOFs onto anomalous analysis increments (Jan '83)



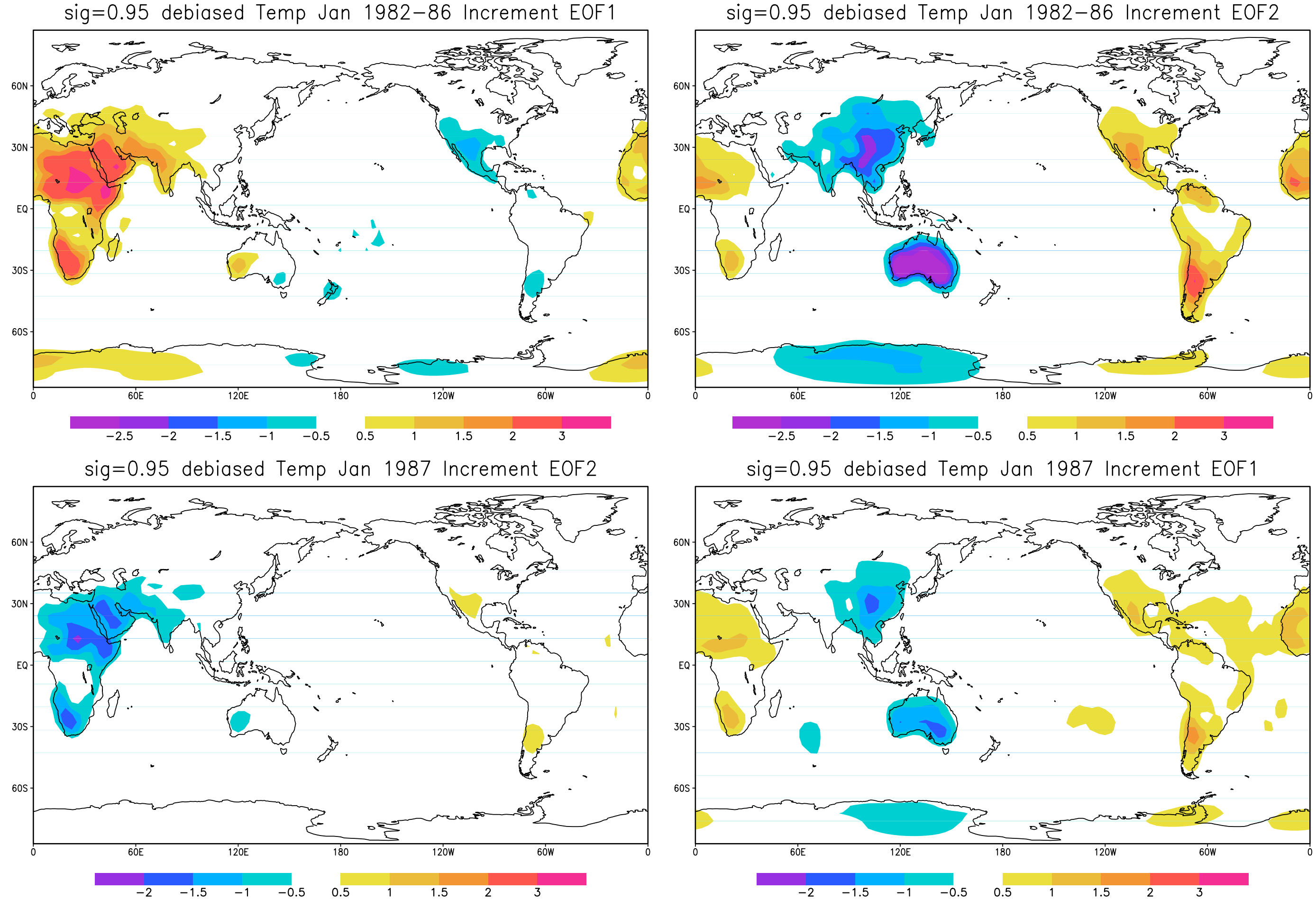
- Leading pair of EOFs out of phase by 6 hours
- Find average strength of daily cycle over Jan 1982-86
- Compute diurnal correction as a function of the time of day

II. Diurnal Bias Correction

EOFs of $C_{\delta x^a \delta x^a}$

January
1982-1986

Diurnally
Corrected
1987



- Diurnal correction substantially reduces error amplitude

III. State-Dependent Error Estimation

Leith (1978) Empirical Correction Operator

- Forecast state covariance: $C_{x^f x^f} = < \mathbf{x}_6^{f\prime} \, \mathbf{x}_6^{f\prime\top} >$
- Cross covariance: $C_{\delta x^a x^f} = < \delta\mathbf{x}_6^{a\prime} \, \mathbf{x}_6^{f\prime\top} >$

Leith's correction operator, given by $L = C_{\delta x^a x^f} \, {C_{x^f x^f}}^{-1}$, provides a state-dependent correction:

$$\dot{\mathbf{x}} = M(\mathbf{x}) + \left[L\mathbf{x}' + \mathbf{c} \right] \frac{1}{\Delta t}$$

where $\mathbf{c} = < \delta\mathbf{x}_6^a >$

Problem: Direct computation of $L\mathbf{x}^f$ requires $O(N^3)$ floating point operations *every* time step!

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Approximation of Leith correction operator:

- univariate covariances generate block diagonal structure

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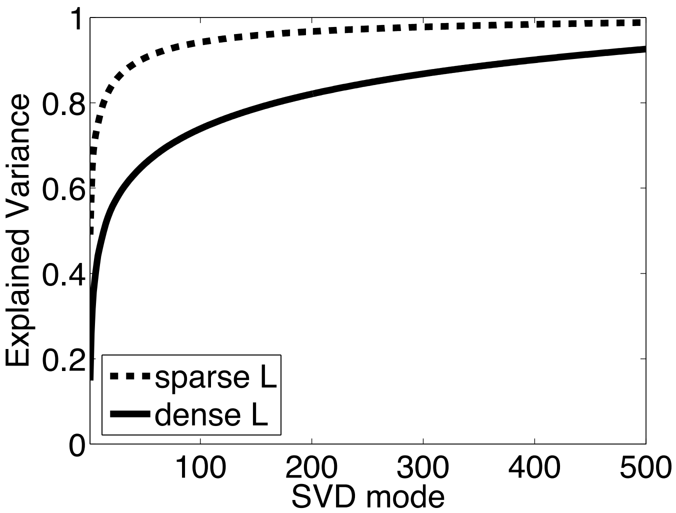

Explained variance of the SVD corresponding to *u* at sigma=0.2 for the dense and sparse Leith operators.

- 400 modes required to explain 90% of variance in dense L
- 40 modes required to explain 90% of variance in sparse L

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- SVD of the sparse analysis increment & state covariance, $C_{\delta\mathbf{x}^a\mathbf{x}^f} = U\Sigma V^\top$, identifies pairs of spatial patterns or EOFs ($\mathbf{u}_k$ and $\mathbf{v}_k$) that explain as much of possible of the mean-squared temporal covariance between the analysis increment and state anomalies.

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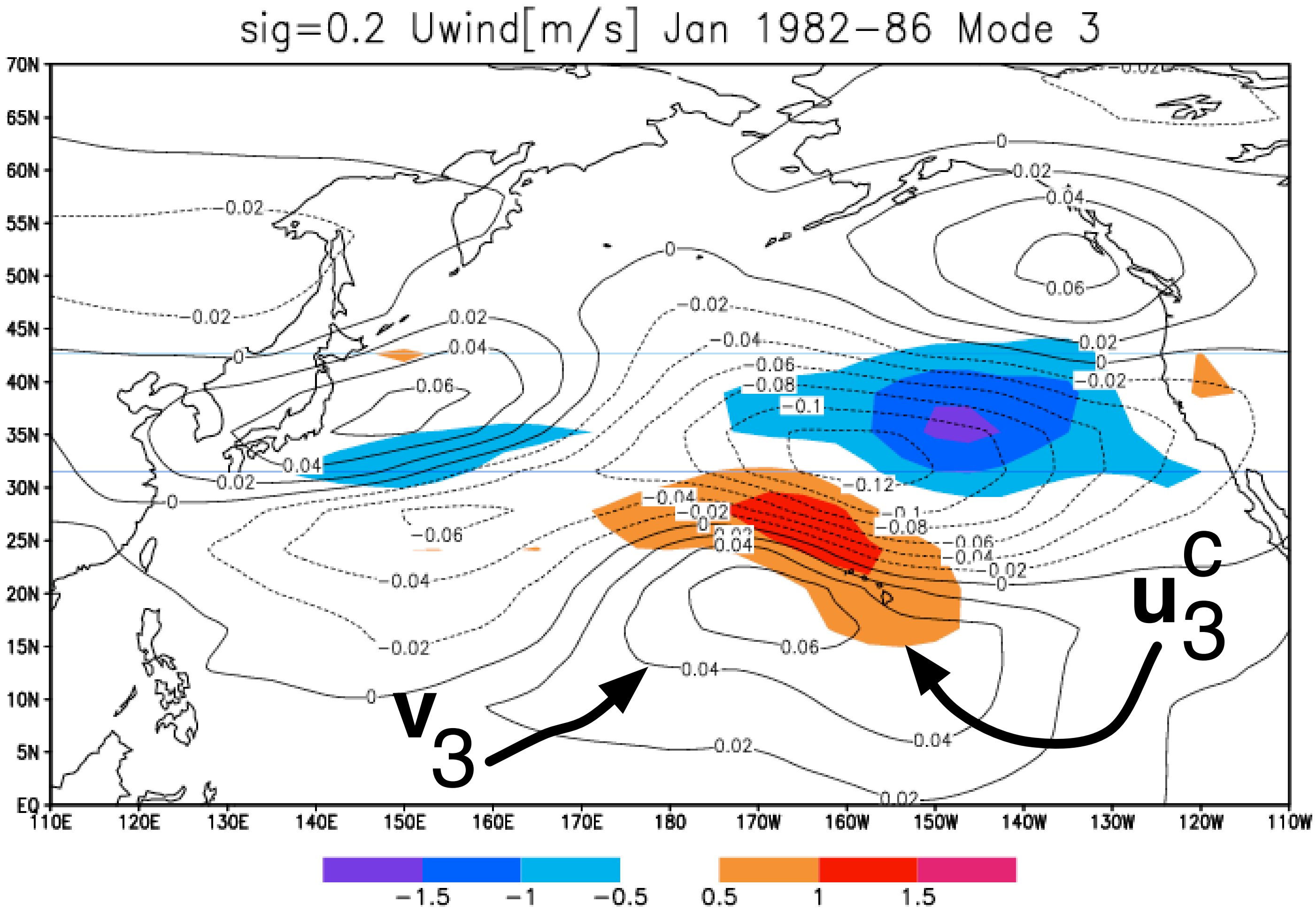
Analysis inc. (color) and state (contour) coupled signals



- $\mathbf{u}_3$ suggests shifting the anomaly $\mathbf{v}_3$ northeast (over the dependent sample)

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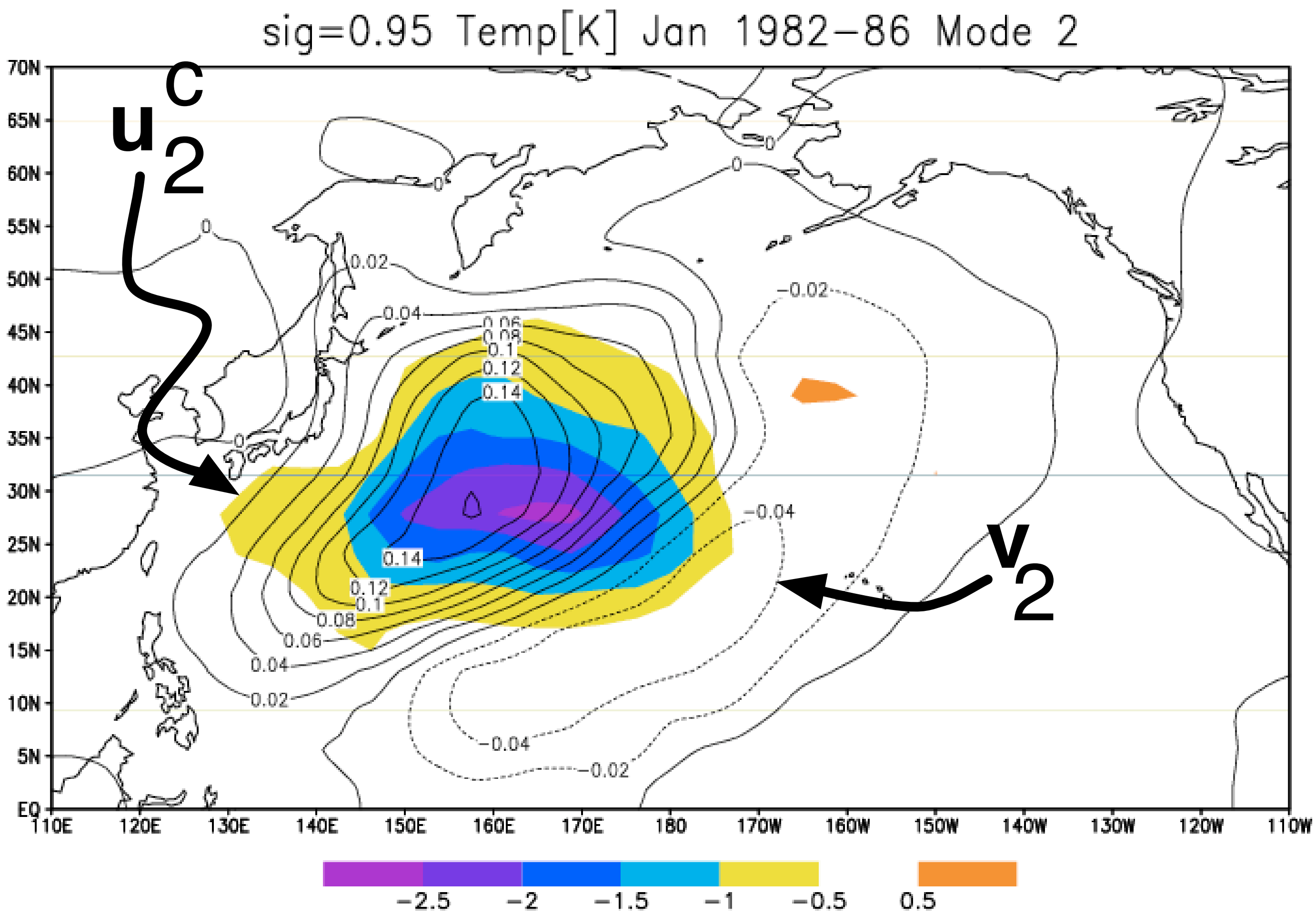

- $\mathbf{u}_2$ suggests damping the anomaly $\mathbf{v}_2$ (over the dependent sample)

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First step in our new approach:

Low-Dimensional Approximation based on regression

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- Principal Components: project EOFs onto dependent sample

$$
\begin{aligned}
a_k(t) &= \mathbf{u}_k^{\top} \cdot \delta\mathbf{x}^{a\prime}(t) \\
b_k(t) &= \mathbf{v}_k^{\top} \cdot \mathbf{x}^{f\prime}(t)
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- Heterogeneous correlation maps:

$$\rho[\delta\mathbf{x}^{a\prime}, b_k] = \left(\frac{\sigma_k}{\sqrt{< b_k^2 >}}\right)\mathbf{u}_k$$

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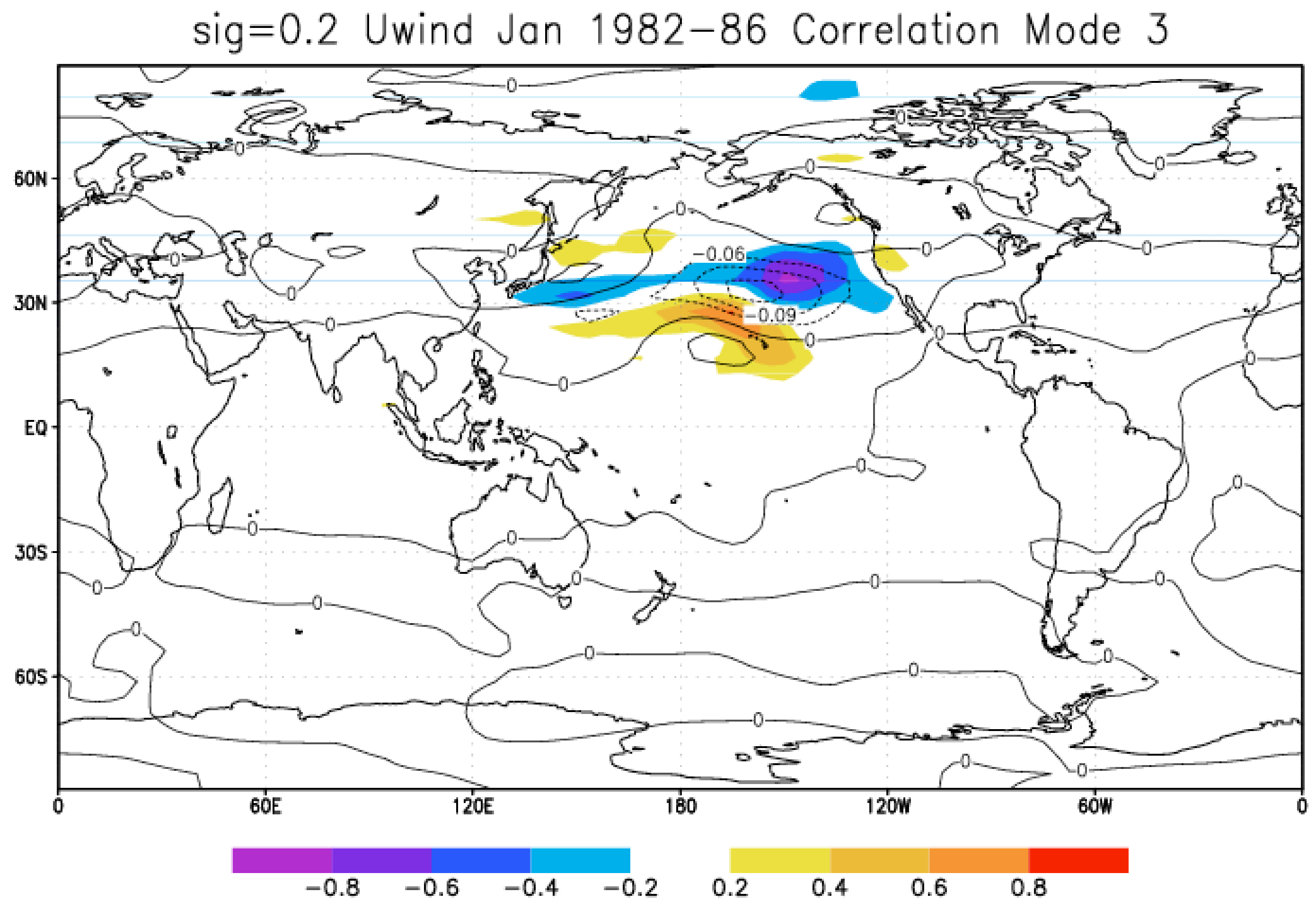

- $\mathbf{u}_3$ is predictable given the forecast anomaly $\mathbf{x}^{f\prime}$ (over the dependent sample)

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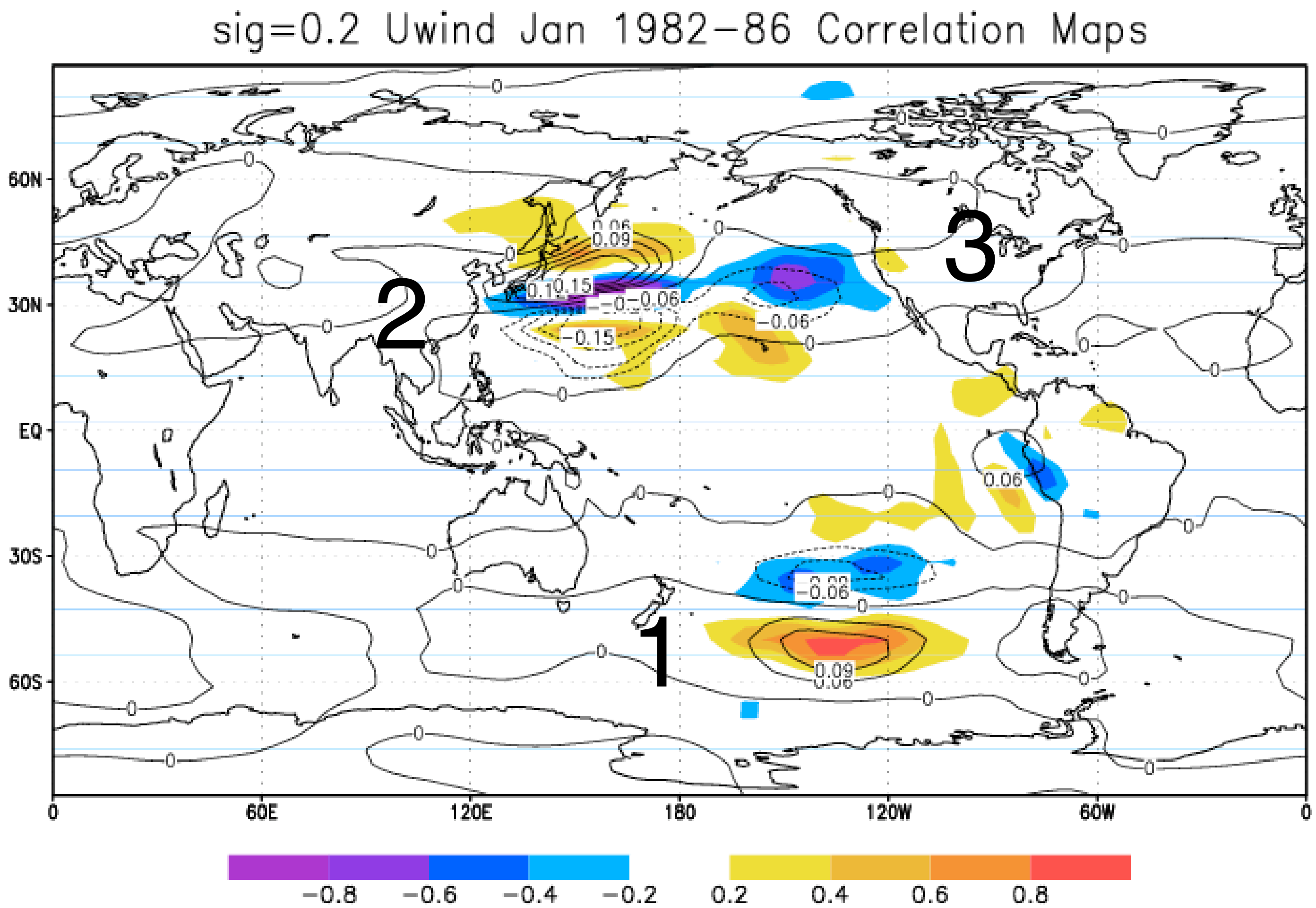

- $\mathbf{u}_k$ is predictable given the forecast anomaly $\mathbf{x}^{f\prime}$ (over the dependent sample)

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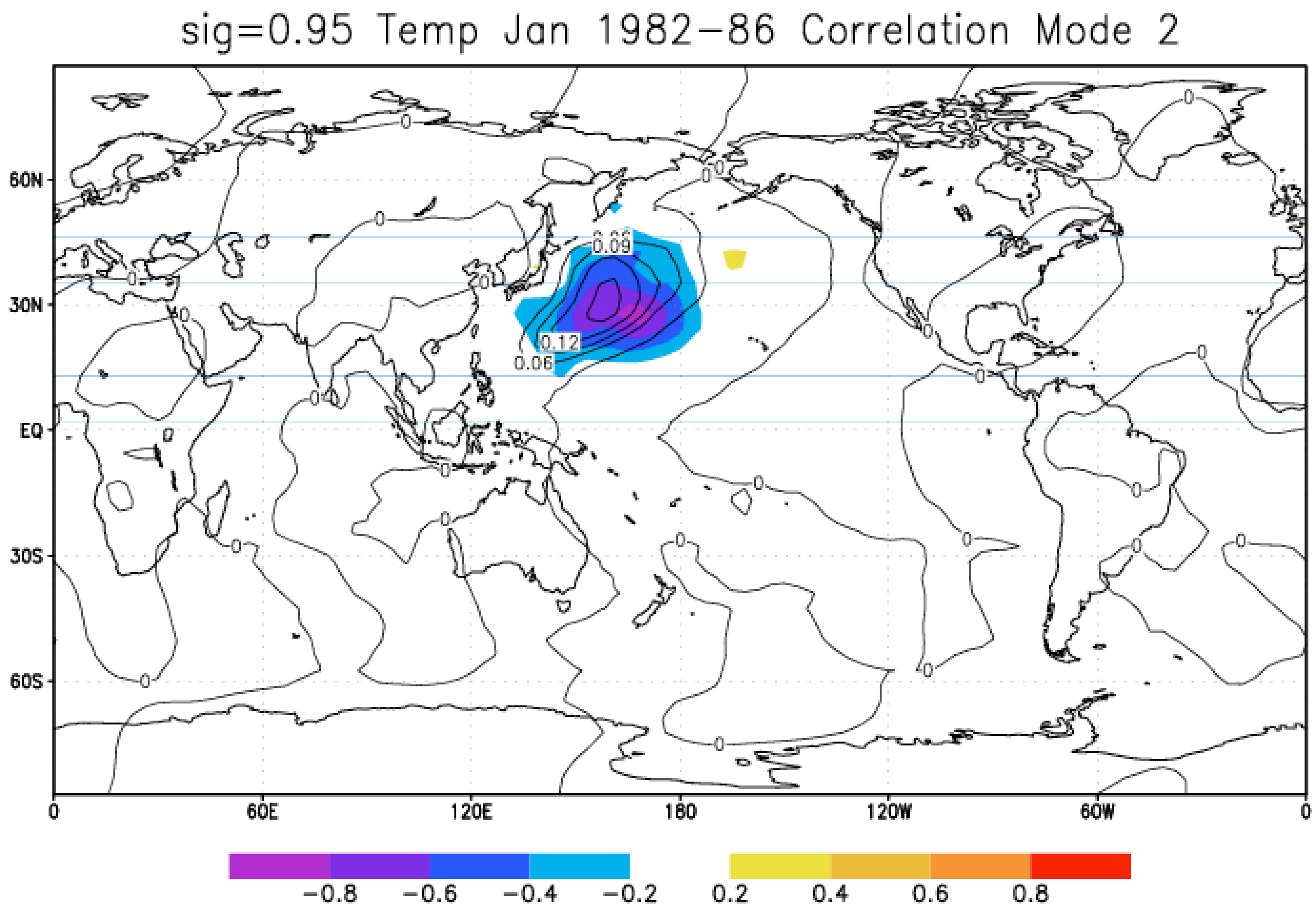

- $\mathbf{u}_2$ is predictable given the forecast anomaly $\mathbf{x}^{f\prime}$ (over the dependent sample)

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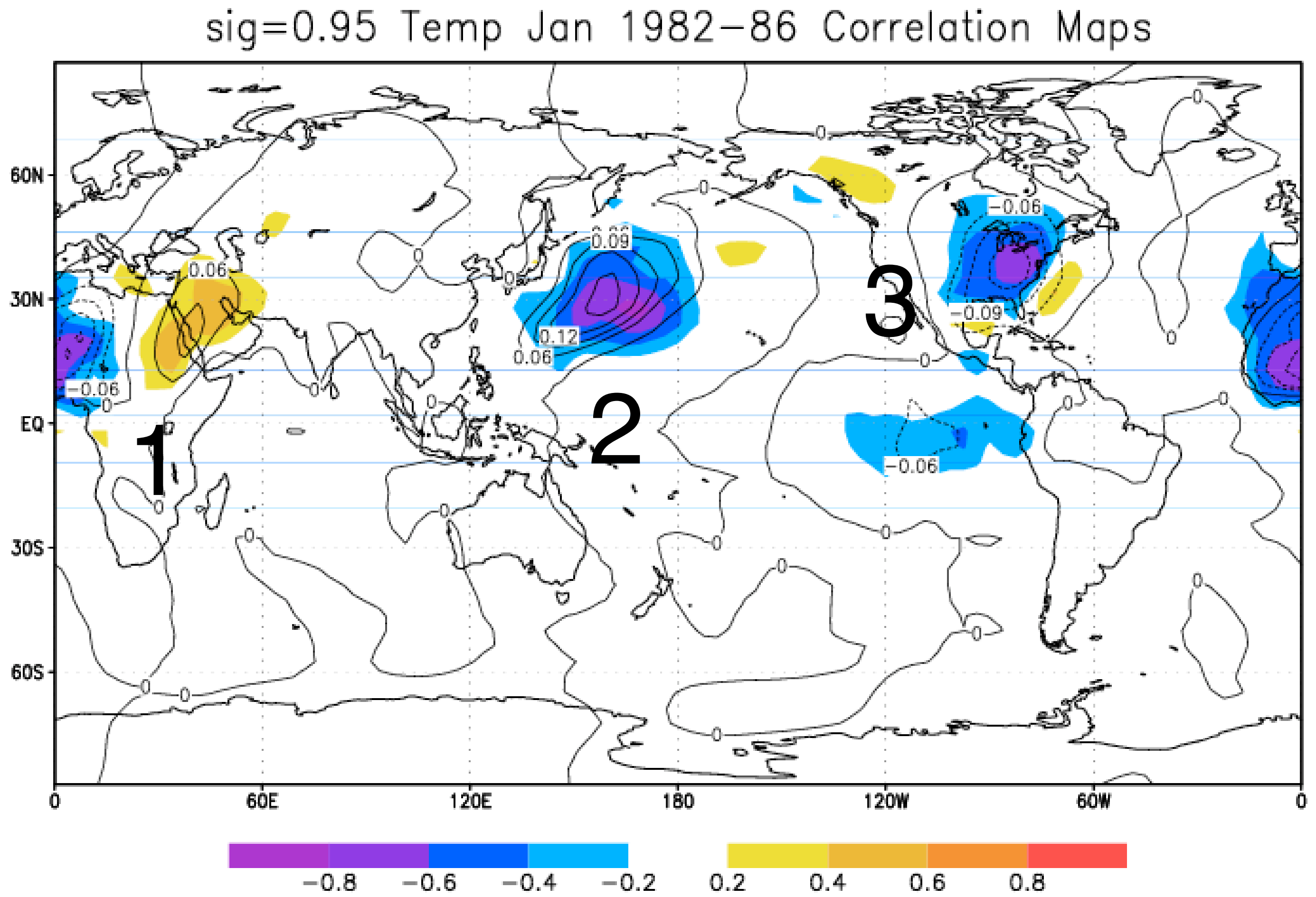

- $\mathbf{u}_k$ is predictable given the forecast anomaly $\mathbf{x}^{f\prime}$ (over the dependent sample)

III. State-Dependent Error Estimation

Second step in our new approach:

Leith's empirical correction involves solving $C_{x^f x^f}\mathbf{w} = \mathbf{x}'$ for $\mathbf{w}$ at each time step.

$$\begin{aligned} L\mathbf{x}' &= C_{\delta x^a x^f}\, {C_{x^f x^f}}^{-1}\mathbf{x}' \\ &= C_{\delta x^a x^f}\mathbf{w} \\ &= U\Sigma V^\top \mathbf{w} \end{aligned}$$

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However, only the component of $\mathbf{w}$ in the space spanned by the right singular vectors $\mathbf{v}_{\mathrm{k}}$ can contribute to the empirical correction.

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$$C_{bb} = < \mathbf{b}\mathbf{b}^\top > \qquad \text{(over dependent sample)}$$
$$b_k(T) = \mathbf{v}_k^\top \cdot \mathbf{x}'(T) \qquad \text{(at independent sample forecast time T)}$$

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The linear system $C_{bb}\gamma = \mathbf{b}$ may then be solved for γ at time T. The solution gives an approximation of $\mathbf{w}$, namely $\mathbf{w} \approx \sum_{k=1}^{K} \gamma_k \mathbf{v}_k$, which is exact if $K = N$.

IV. State-Dependent Correction

The control model:

$$\dot{\mathbf{x}} = \mathrm{M}\left(\mathbf{x}\right)$$

The state-independent *online* corrected model:

$$\dot{\mathbf{x}} = \mathrm{M}\left(\mathbf{x}\right) + < \delta\mathbf{x}_6^a > \frac{1}{\Delta t}$$

Leith's state-dependent corrected model given by:

$$\dot{\mathbf{x}} = \mathrm{M}\left(\mathbf{x}\right) + \left[< \delta\mathbf{x}_6^a > + \mathrm{L}\mathbf{x}' \right] \frac{1}{\Delta t}$$

Our low-dimensional state-dependent corrected model is given by:

$$\dot{\mathbf{x}} = \mathrm{M}\left(\mathbf{x}\right) + \left[< \delta\mathbf{x}_6^a > + \sum_{k=1}^{K} \mathbf{u}_k \sigma_k \gamma_k \right] \frac{1}{\Delta t}$$

where $\gamma = \mathrm{C}_{bb}{}^{-1}\mathbf{b}$ and $b_k = \mathbf{v}_k^\top \cdot \mathbf{x}'$

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During forecasts, a few ($K \approx 10$) dominant anomalous model state signals $\mathbf{v}_k$ can be projected onto the anomalous model state vector $\mathbf{x}'$.

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where $\gamma = {C_{bb}}^{-1} \mathbf{b}$ and $b_k = \mathbf{v}_k^\top \cdot \mathbf{x}'$

IV. State-Dependent Correction

During forecasts, a few (K≈10) dominant anomalous model state signals $\mathbf{v}_k$ can be projected onto the anomalous model state vector $\mathbf{x}'$. Then

$$\sum_{k=1}^{K} \mathbf{u}_k \sigma_k \gamma_k$$

- is the best representation of the dependent sample analysis increment anomalies $\delta\mathbf{x}^{a\prime}$ in terms of the current anomalous forecast state $\mathbf{x}'$
- may amplify, dampen, or shift the flow anomaly local to $\mathbf{u}_k$

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$$\dot{\mathbf{x}} = M(\mathbf{x}) + \left[< \delta\mathbf{x}_6^a > + \sum_{k=1}^{K} \mathbf{u}_k \sigma_k \gamma_k \right] \frac{1}{\Delta t}$$

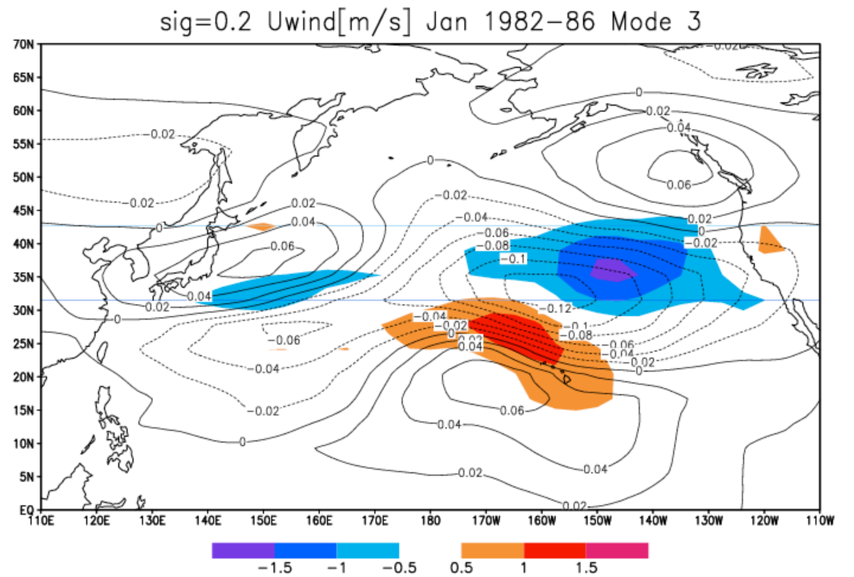
where $\gamma = C_{bb}{}^{-1}\mathbf{b}$ and $b_k = \mathbf{v}_k^\top \cdot \mathbf{x}'$

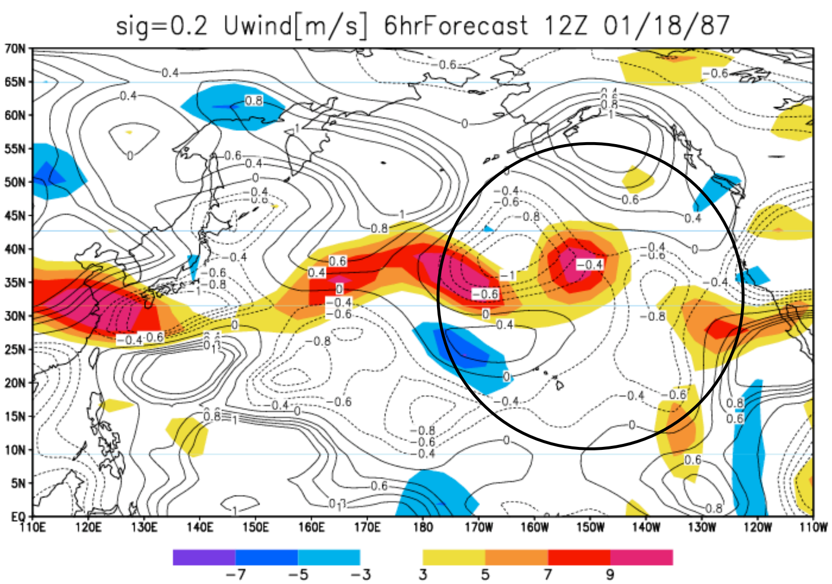
IV. State-Dependent Correction

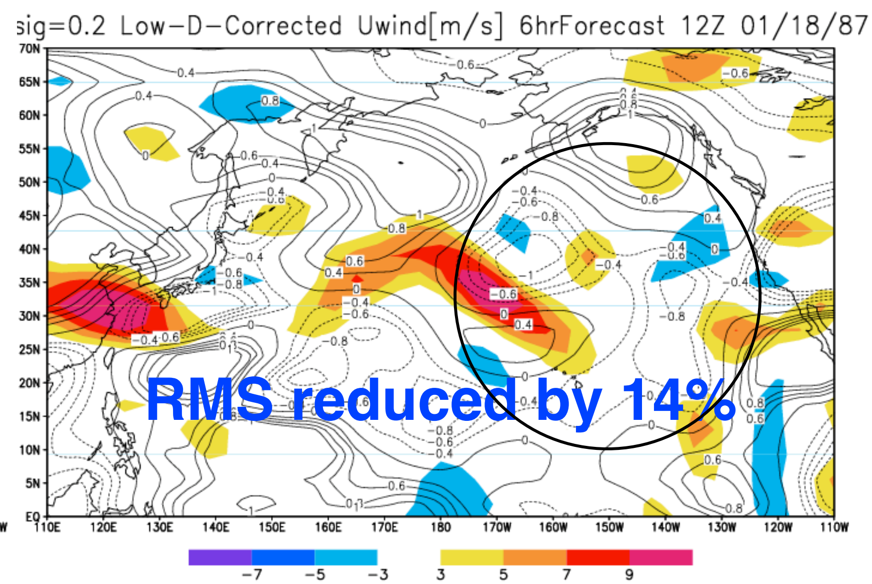
σ=0.2 U-wind
Error (shades)
and State (contour)

6-hr forecasts

debiased low-d corrected





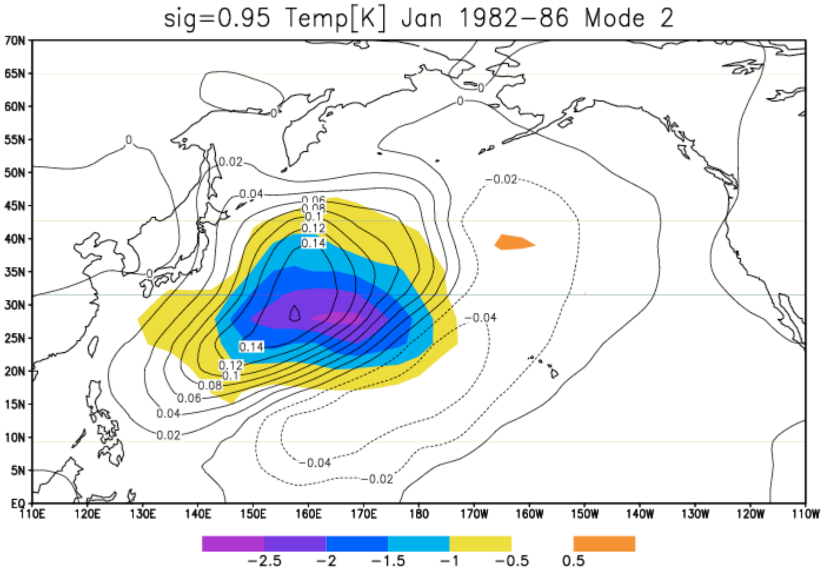


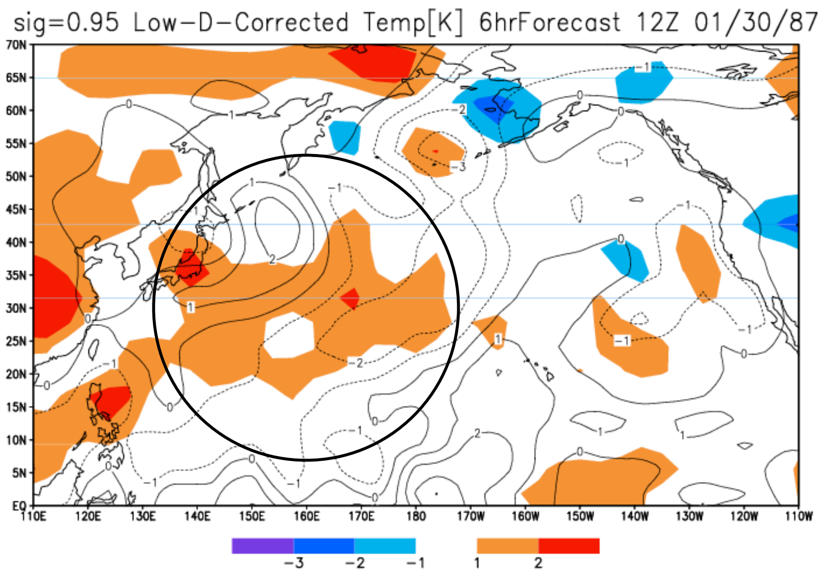
IV. State-Dependent Correction

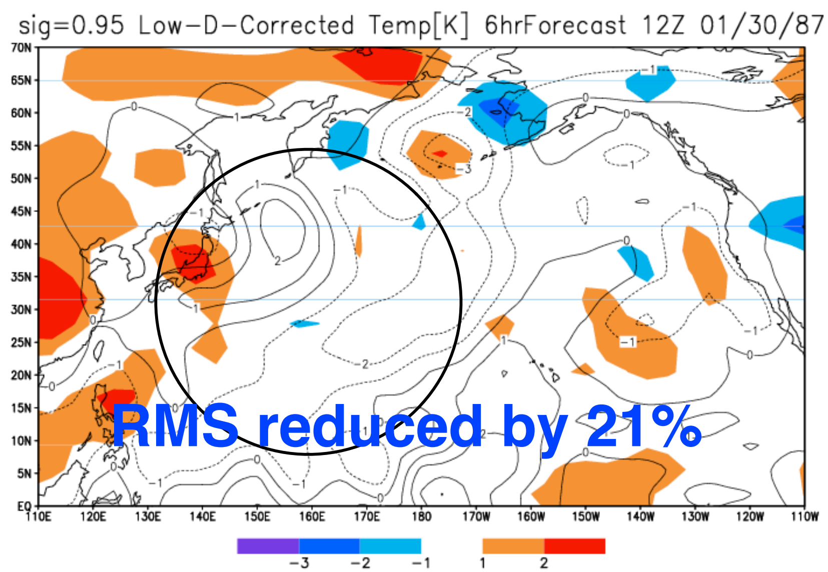
σ=0.95 Temp
Error (shades)
and State (contour)

6-hr forecasts

debiased low-d corrected







IV. State-Dependent Correction

We measure the forecast improvement using Leith's (univariate) dense and sparse correction operators and our low-dimensional approximation.

	Dense Leith	Sparse Leith	Low-Dim
Flops per time step	$O(\mathrm{N}_{\mathrm{gp}}^3)$	$O(\mathrm{N}_{\mathrm{gp}}^2)$	$O(\mathrm{N}_{\mathrm{gp}})$
Global Improvement	-8% (-4hr)	2% (1hr)	4% (2hr)
NH Extratropics Improvement	-6% (-3hr)	4% (2hr)	6% (3hr)

Chart contains average January 1987 500hPa improvement over state-independent corrected forecasts. Correction is more effective in regions where the heterogeneous correlations ρ are large.

Results

- State-independent correction of SPEEDY monthly bias dramatically improves forecasts
- Correction *during* integration outperforms correction a posteriori
- Time-dependent correction reduces amplitude of diurnal errors
- Our method of low-dimensional state-dependent correction:
  - improves forecasts, more notably where correlations are large
  - gives better results than Leith's correction operator
  - is 10 orders of magnitude cheaper! (SPEEDY implementation)
  - should work easily with existing data assimilation and ensemble schemes
  - requires only the analysis increments for sampling

Future

- Test implementation on NCEP operational model at reduced resolution with multivariate covariance.
- Implement with data assimilation and ensemble schemes
- Reduce jumps in reanalysis climatology due to changes in observing system

Research supported by NOAA THORPEX grant NA040AR4310103

 Contact: danforth@math.umd.edu