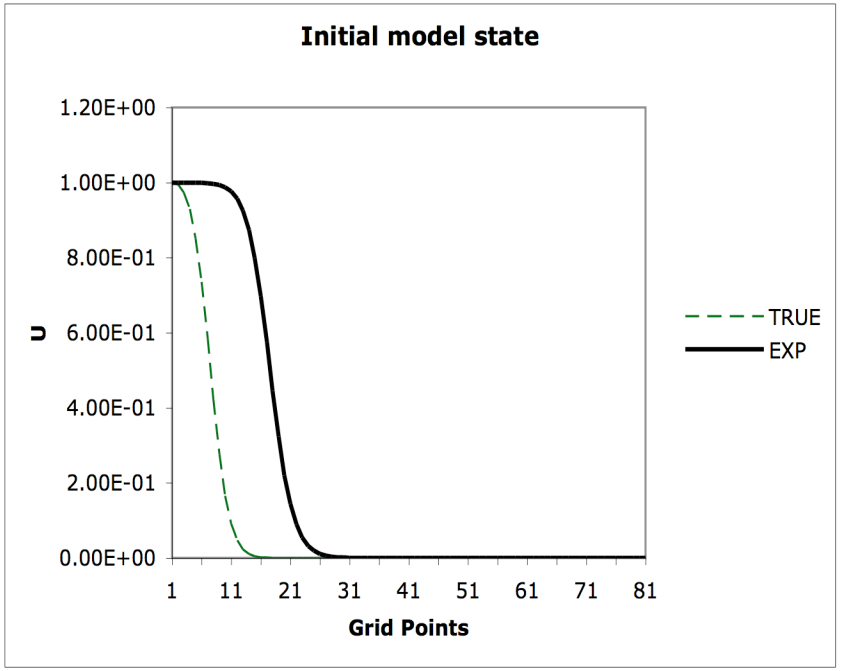
Non-differentiable observation operator in MLEF minimization

1-d Burgers model simulating a shock-wave

- *Nstate=81*
- *Nobs=81* (simulated observations)

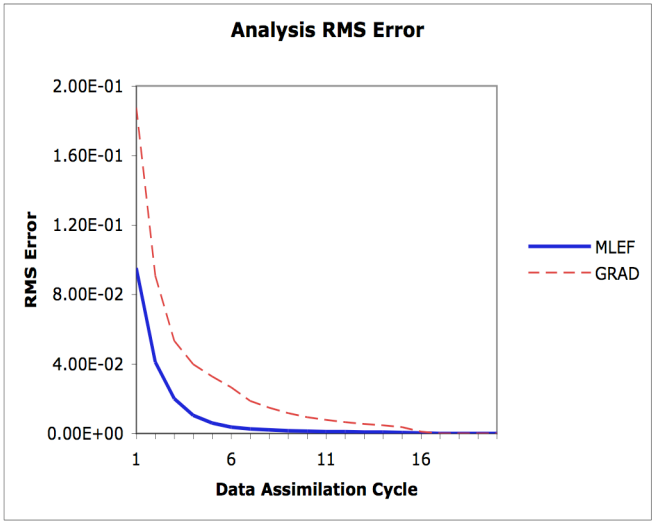


$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = \nu\frac{\partial^2 u}{\partial x^2}$$

Non-differentiable observation operator

$$H(x) = \begin{cases} x^3 & for \quad x \geq 0.5 \\ -x^3 & for \quad x < 0.5 \end{cases}$$

Discontinuity in the function and all its derivatives

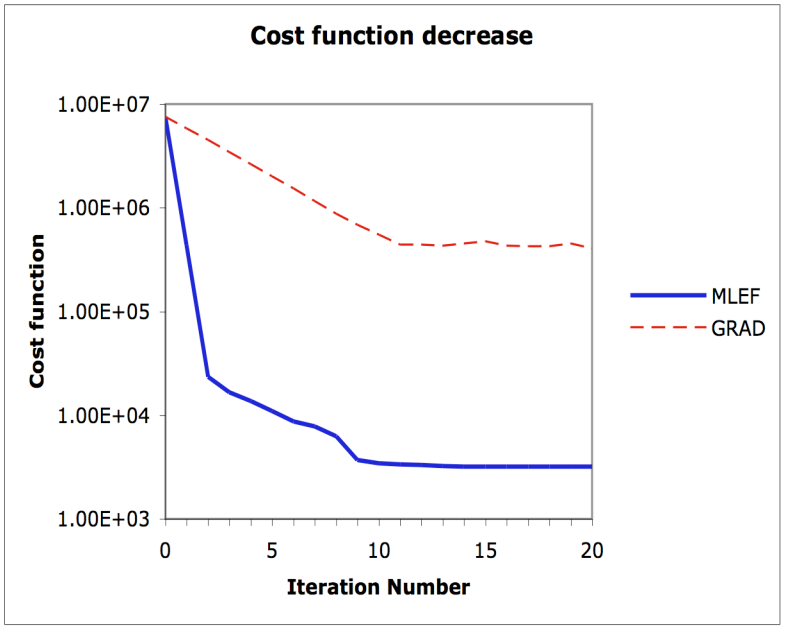


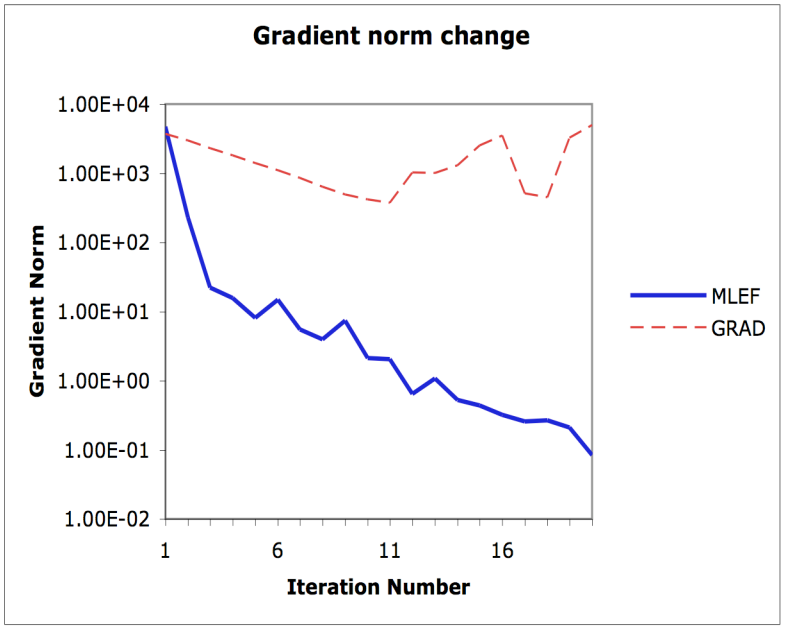
Analysis RMS error in the MLEF and GRAD experiments

MLEF $H(\boldsymbol{x} + \boldsymbol{p}_i^f) - H(\boldsymbol{x})$

GRAD $\left(\frac{\partial H}{\partial \boldsymbol{x}}\right) \boldsymbol{p}_i^f$

MLEF vs. gradient-based minimization (GRAD)





Cost function

Gradient norm (GRAD)

Generalized Gradient norm (MLEF)

- **Cost function and "gradient" norm show the benefit of the generalized conjugate-gradient minimization (MLEF)**