Towards non-hydrostatic spectral models

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Motivations

- Spectral models are used at leading NWP centres.
- Current semi-Lagrangian cores do not make the most out of spectral methods.
- Non-hydrostatic effects need to be evaluated with the non-hydrostatic version rather than a different model.

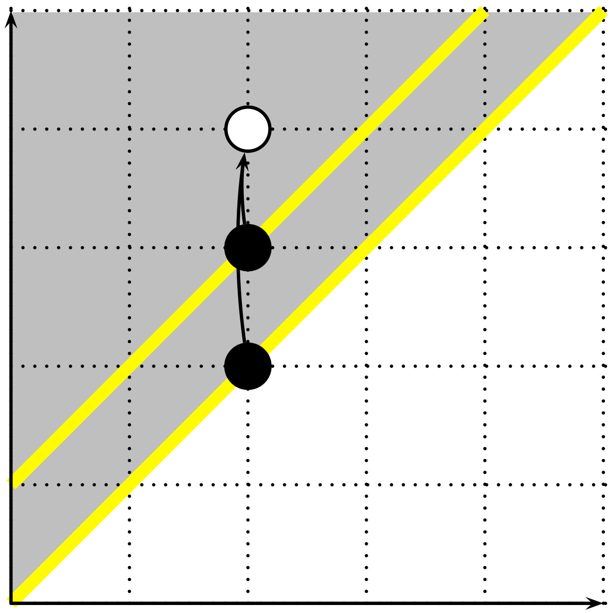
Three keys

- Accurate associated Legendre functions at high order and degree
- Accurate interpolation in semi-Lagrangian advection
- Stable and simple non-hydrostatic formulation

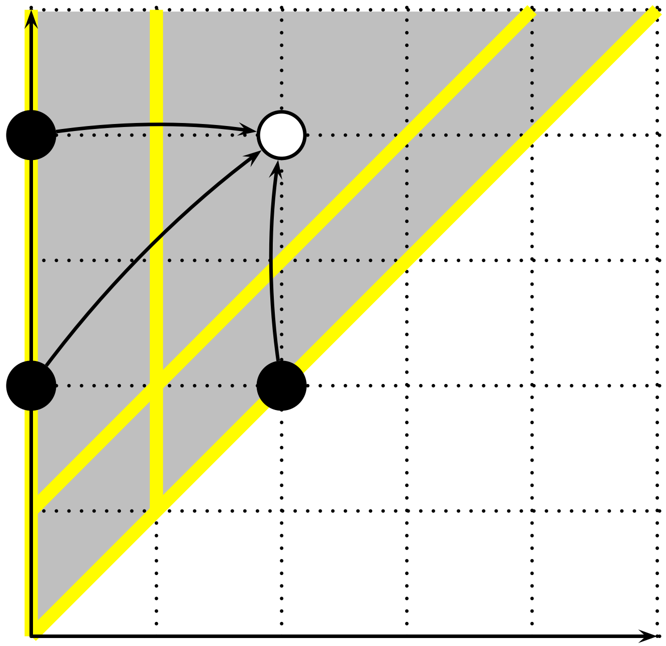
Assoicated Legendre functions

Enomoto et al. (2004; 2008);
Enomoto and Miyamoto, MSJ 2012 Spring

Conventional



Swarztrauber (1993)



Conventional three-term recurrence

$$\tilde{P}_n^m = (a_n^m \cos\theta)\tilde{P}_{n-1}^m - b_n^m \tilde{P}_{n-2}^m \tag{1}$$

$$\tilde{P}_m^m = (d_n^m \sin\theta)\tilde{P}_{m-1}^{m-1} \tag{2}$$

$$\tilde{P}_{m+1}^m = (a_{m+1}^m \cos\theta)\tilde{P}_m^m \tag{3}$$

where θ is the colatitude and

$$a_n^m = \sqrt{\frac{4n^2-1}{n^2-m^2}}, b_n^m = \frac{a_n^m}{a_n^{m-1}}, d_n^m = \sqrt{\frac{2m+1}{2m}} \tag{4}$$

The alternative four-term recurrence

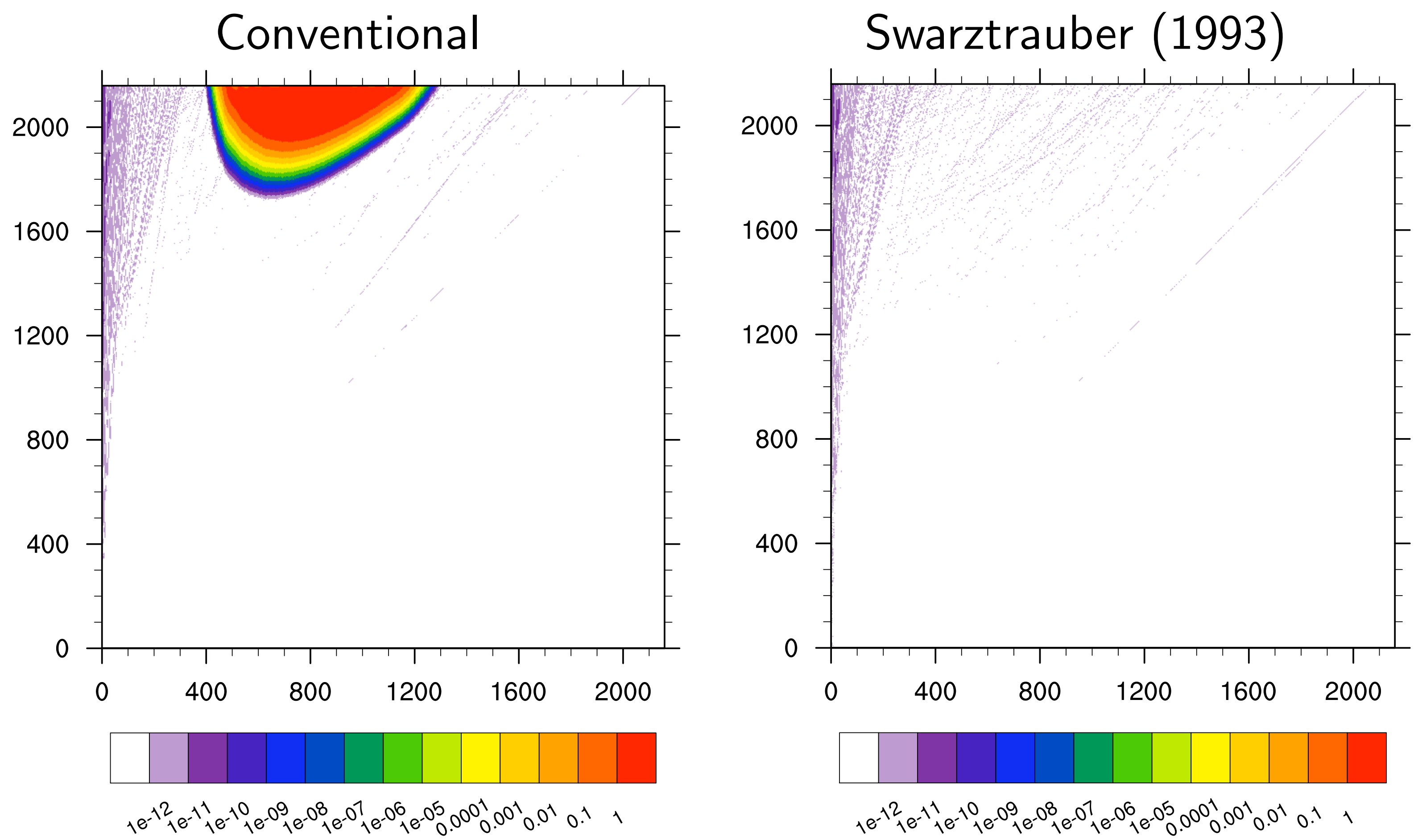
involves (m−2,n), (m-2,n−2), and (m−2, n−2) to calculate the value at (m,n):

$$\tilde{P}_n^m = \sqrt{\frac{(2n+1)(n+m-2)(n+m-3)}{(2n-3)(n+m-1)(n+m)}}\ \tilde{P}_{n-2}^{m-2} - \sqrt{\frac{(n-m+1)(n-m+2)}{(n+m-1)(n+m)}}\ \tilde{P}_n^{m-2} + \sqrt{\frac{(2n+1)(n-m)(n-m-1)}{(2n-3)(n+m-1)(n+m)}}\ \tilde{P}_{n-2}^m \qquad (5)$$

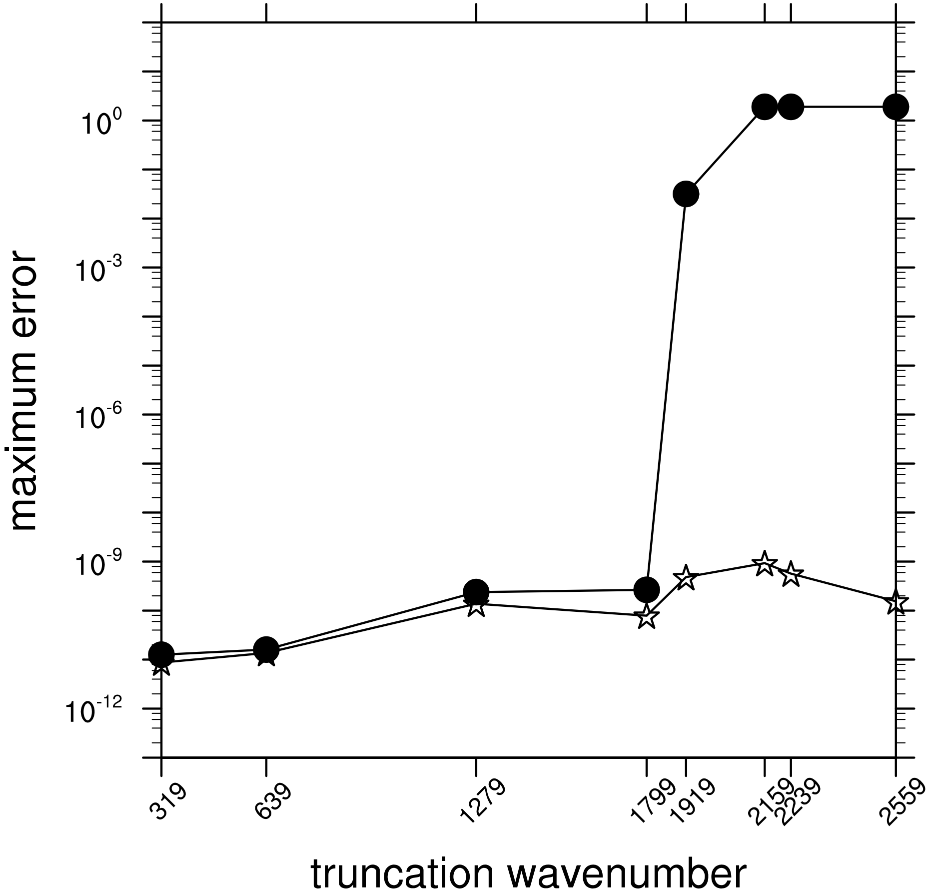
Why does the three-term recurrence fail?

- A floating-point number has can only represent a certain small number e.g. $x_{\min} = 2.23 \times 10^{-308}$ for double precision.
- $\tilde{P}_m^m \propto \sin^m \theta$ is very small near the poles at high orders.
- $|\tilde{P}_n^m|$ grows with degree n.

Legendre synthesis–analysis test at T2159



Legendre synthesis–analysis test



Extended-range arithmetic

- Smith et al. (1981) avoided overflow and underflow in the evaluation of associated Legendre functions with extended-range arithmetic at the 2x computational cost.

- Fukushima (2011) proposed an accelerated version that enables to evaluate extremely high degree as 2^{32} at only 10% increase in computational time.

Express X with a pair of a floating-point number x and an integer i.

$$X = xB^i \tag{6}$$

where B is a large power of 2.

Validation of associated Legendre functions calculated from the four-term recurrence

- The check sum

$$\int_{-1}^{1} [\tilde{P}_n^m]^2 \mathrm{d}x = 1 \tag{7}$$

  shows that the Swarztrauber (1993)'s recurrence enables accurate Legendre transforms at high order and degree.

- Associated Legendre functions evaluated by the method of Fukushima (2011) is regarded as the truth.

- The small values from the four-term recurrence is found to be in inaccurate.

Summary for associated Legendre functions

- The four-term recurrence (Swarztrauber 1993) is stable over 2000.
- The conventional three-term recurrence may be used in extended-range arithmetic to avoid overflow and underflow (Smith et al. 1981).
- An optimized evaluation with extended-range arithmetic only require 10% increase in computational time (Fukushima 2011).
- Associated Legendre functions from the four-term recurrence are validated with those from the three-term recurrence using extended-range arithmetic to be inaccurate at small values.

Interpolation

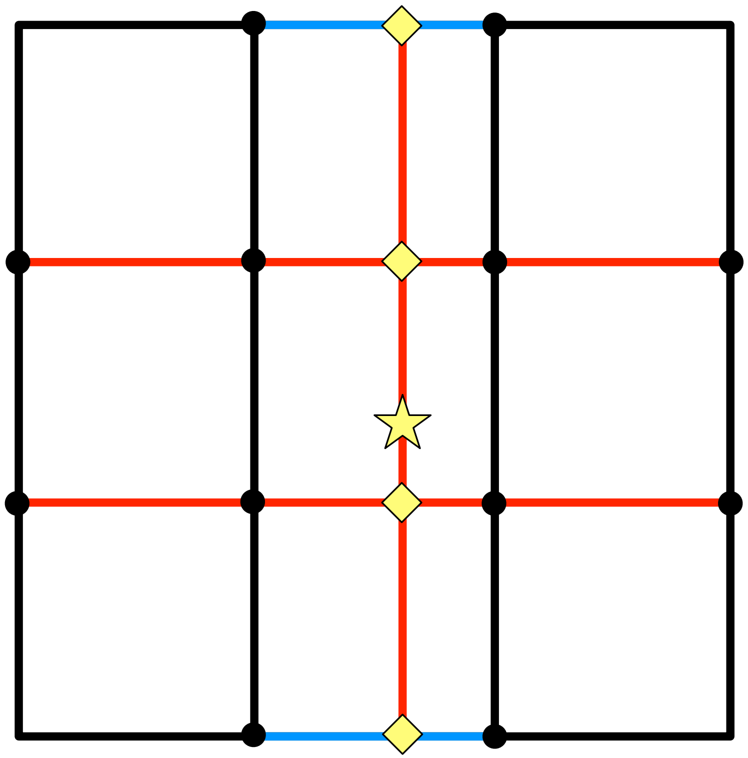
Enomoto 2008;
Lauritzen et al. 2012, in preparation

Advection

- Eulerian advection
  - is dispersive
  - requires a short time step.
- Semi-Lagrangian advection
  - allows a longer time step
  - needs interpolation, which determines the accuracy
  - is typically diffusive

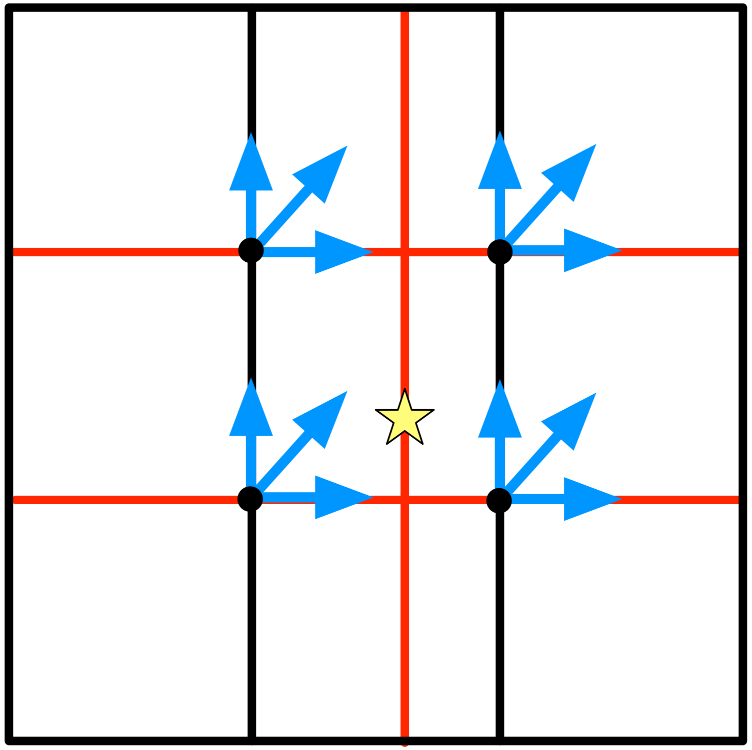
$\rightarrow$Semi-Lagrangian advection with an accurate interpolation should be less dispersive and diffusive.

Quasi-cubic interpolation



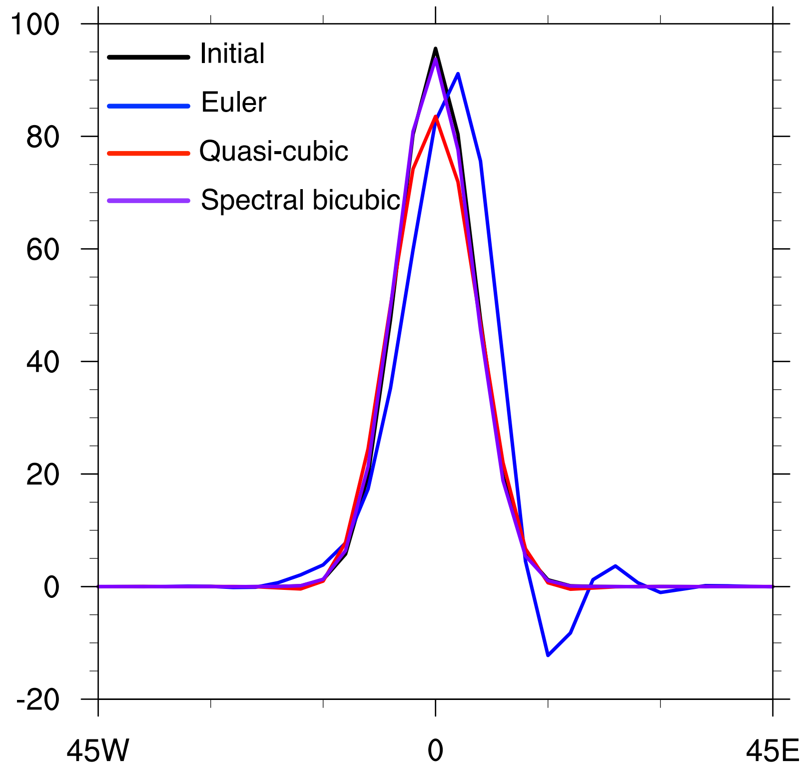
Ritchie et al. (1995)

Bicubic interpolation



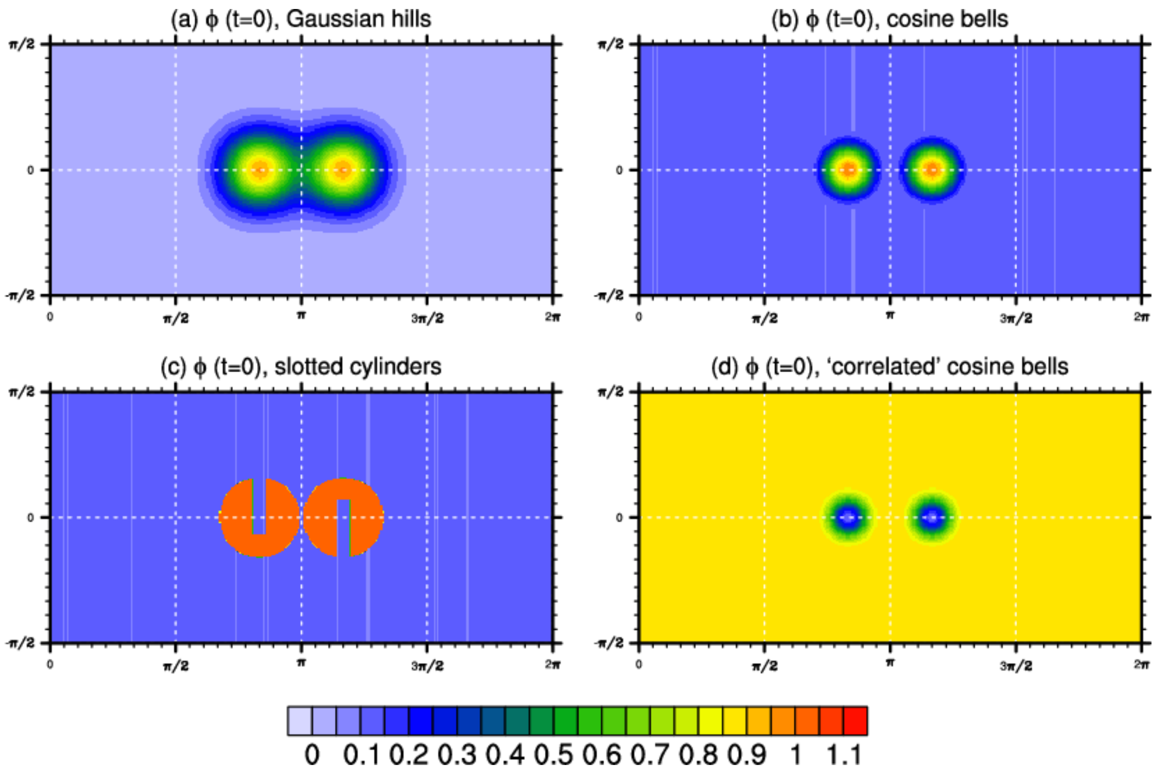
Derivatives calculated in spectral space (Enomoto 2008)

Rotation of a Gaussian hill



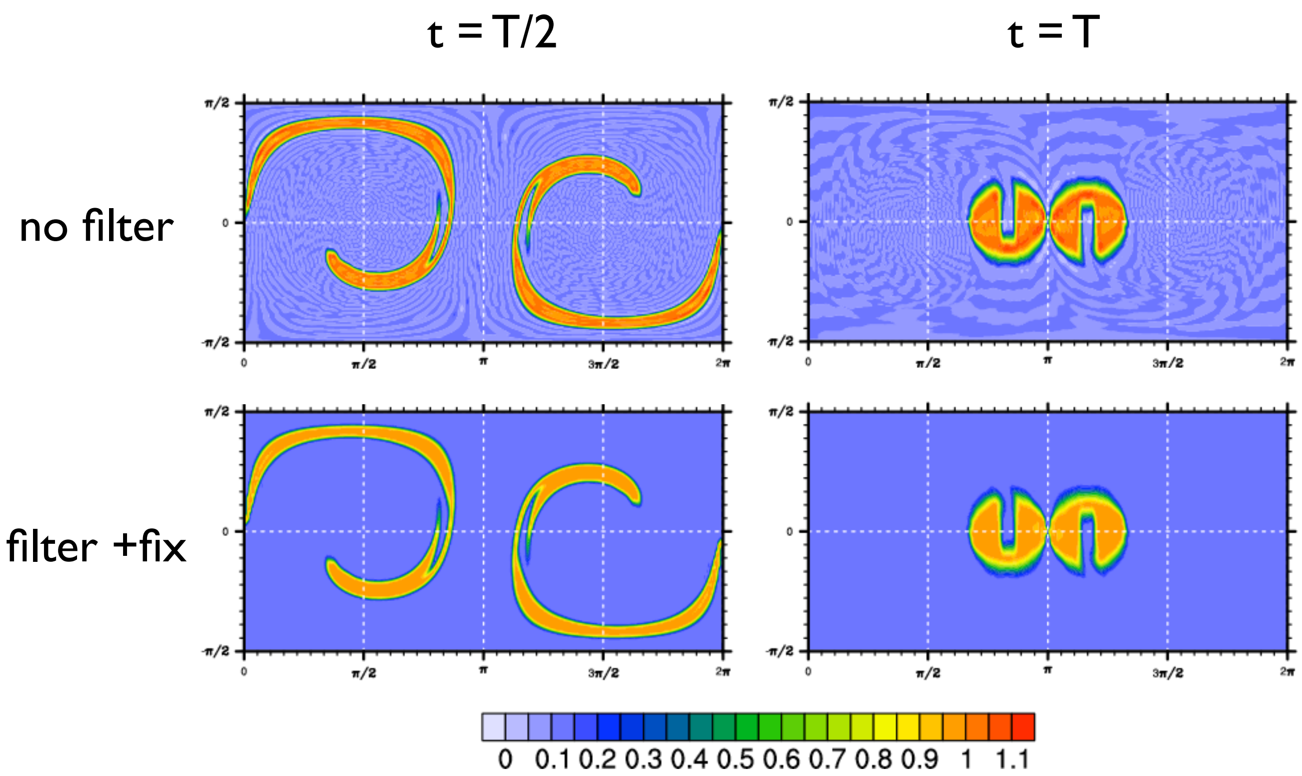
Enomoto (2008)

Standard test suite



Lauritzen and Skamarock (2010)

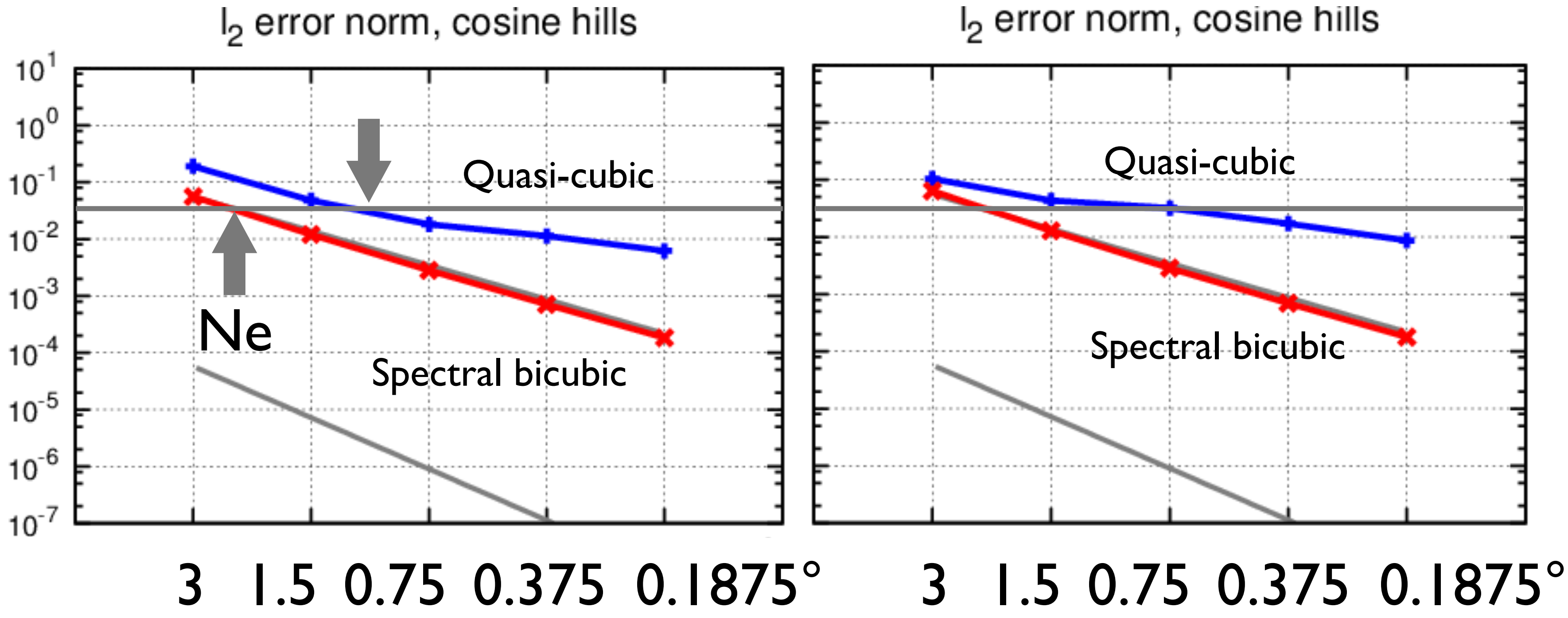
Slotted cylinders



Spectral bicubic T119 CFL 5.2

Convergence

nofilter filter+fix



T119 CFL 5.2 ℓ_2 error norm cos hills

'Minimal' resolution compared

model	reference	filter	Ne
Quasi-cubic	Ritchie et al. 1995	no	0.919
		yes	0.864
Spectral bicubic	Enomoto 2008	no	2.41
		yes	2.29

obtained from convergence of ℓ_2 error norm cos hills

Summary for interpolation

Spectral bicubic interpolation

- is easy to implement in existing spectral models.
- is accurate for smooth and non-smooth tracers.
- generates ripples but they can be removed with a short-wave filter of Sun et al. (1996).

Non-hydrostatic formulation

Enomoto and Juang, MSJ 2011 Autumn

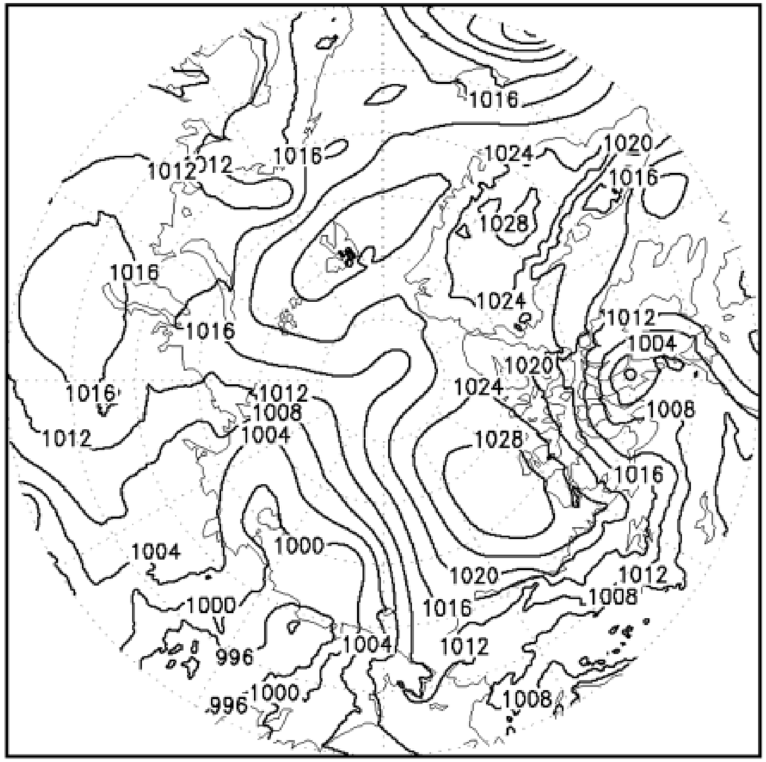
σ–co-ordinates in hydrostatic pressure $\overline{p}$ (Laprise 1992)

- Additional prognostic variables: w and p
- Monotone in the vertical unlike full pressure p (Miller 1974)
- No additional complex metric terms unlike terrain following height co-ordinates
- Easy to conserve total energy
- Adopted by Météo France ALADIN (Bubnová 1995)/Arpège (Yessard 2008), ECMWF NH-IFS (Wedi et al. 2009) and JMA GSM (Yoshimura, JMSJ spring 2011 B401)

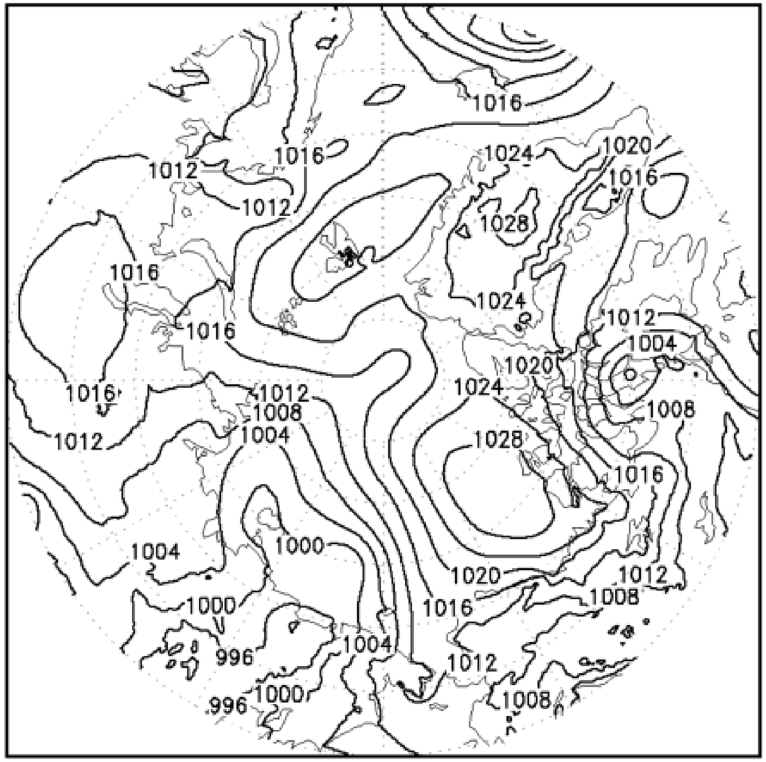
Non-hydrostatic double-fourier version of JMA GSM (Yoshimura)

TL1279L60, Δt=10 min, FT = 2 d, slp (hPa)

Spherical harmonics



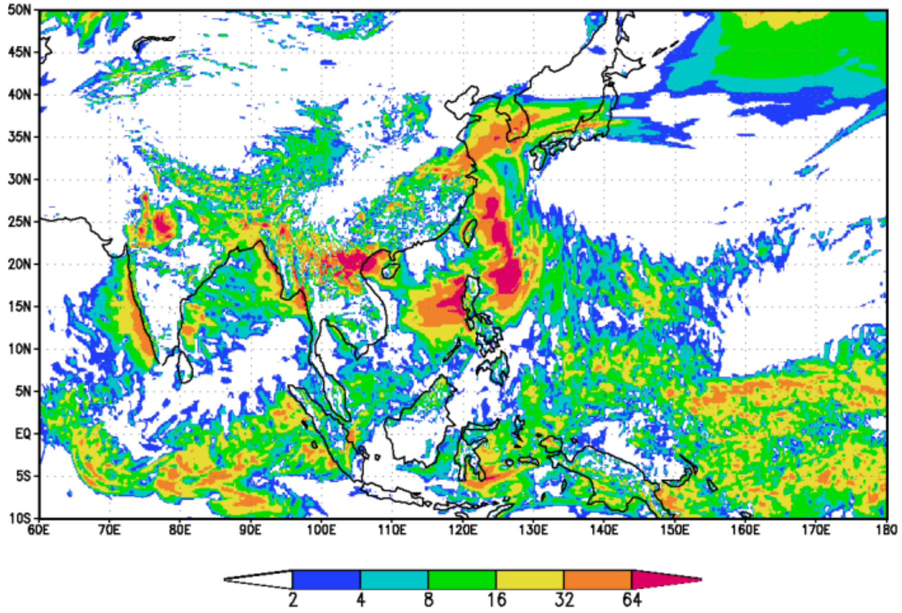
Double Fourier Series



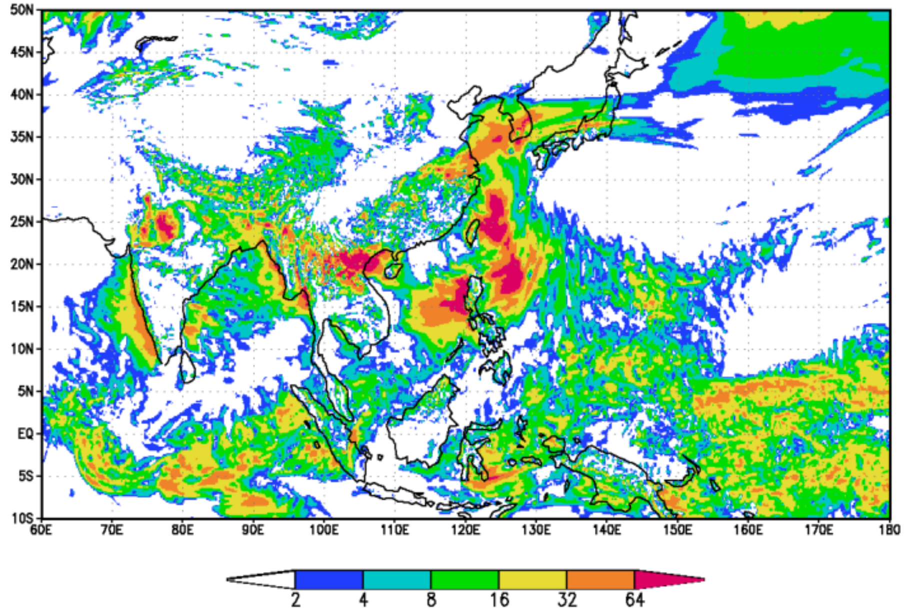
Non-hydrostatic double-fourier version of JMA GSM (Yoshimura)

TL1279L60, Δt=10 min, FT = 2 d, precipitation (mm/d)

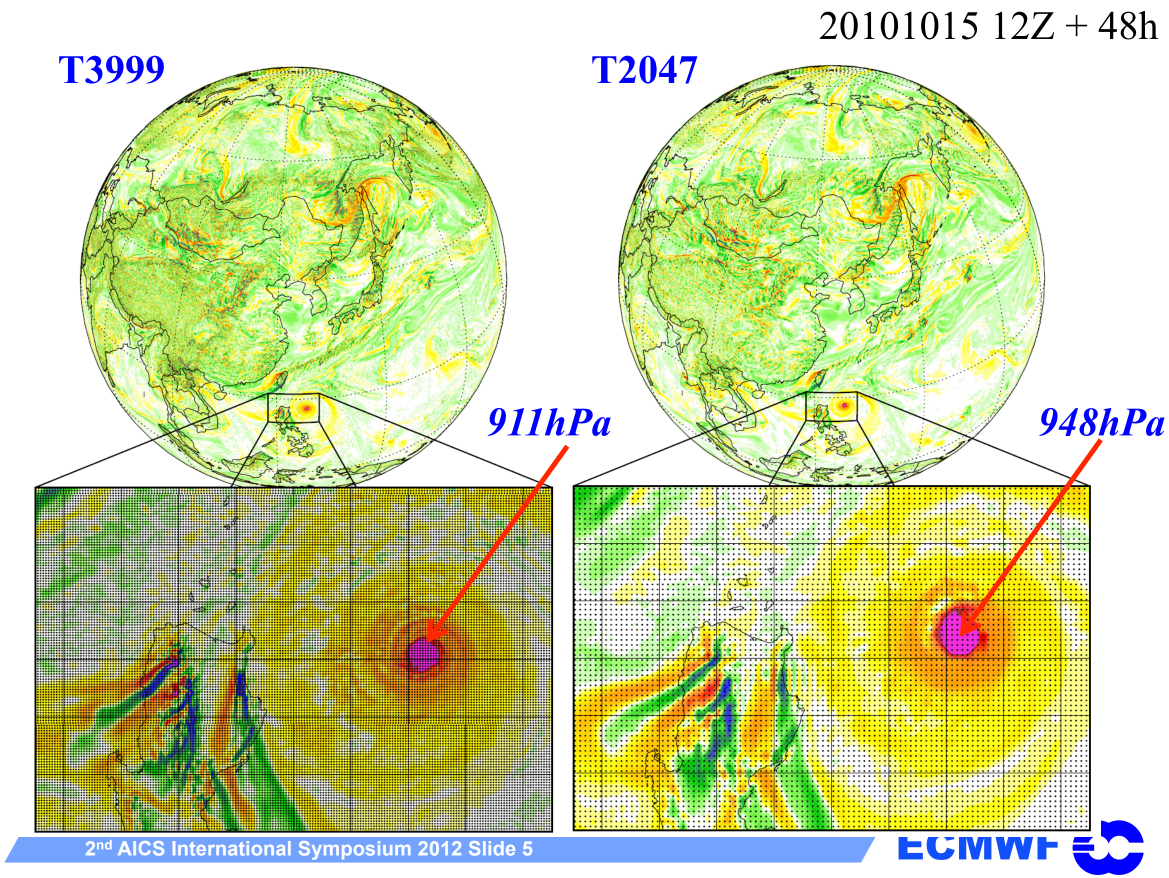
Spherical harmonics



Double Fourier Series



Non-hydrostatic IFS at ECMWF (Wedi)



You already have a gem at NCEP.

NCEP MSM (Juang 1992; 2000)

- is a non-hydrostic version of NCEP RSM (Juang and Kanamitsu 1994).
- uses horizontal discretization by double Fourier series.
- uses the perturbation method.
- shares the same physics packages with NCEP GSM.
- uses the Euler equations transformed from $z-$ to $\sigma-$ co-ordinates.

Pressure gradient terms

Laprise (1992)

$$\frac{1}{\rho}\nabla_z p = RT\nabla_\sigma \ln p + \frac{p}{\overline{p}}\frac{\partial \ln p}{\partial \ln \sigma}\nabla_\sigma \phi,\ \frac{\partial \phi}{\partial \sigma} = -\frac{RT\overline{p}_{\mathrm{s}}}{p} \tag{8}$$

Juang (1992)

$$\frac{1}{\rho}\nabla_z p = RT\nabla_\sigma \ln p + \frac{T}{\overline{T}}\frac{\partial \ln p}{\partial \ln \sigma}\nabla_\sigma \overline{\phi},\ \frac{\partial \overline{\phi}}{\partial \sigma} = -\frac{R\overline{T}}{\sigma} \tag{9}$$

The hydrostatic state of Laprise (1992)

Define

$$\frac{\partial \phi}{\partial p} = -\frac{1}{\rho}, \tag{10}$$

means

$$\overline{\rho} = \rho. \tag{11}$$

Since $\overline{\rho} = \overline{p}/R\overline{T}, \rho = p/RT$,

$$\frac{T}{\overline{T}} = \frac{p}{\overline{p}} \tag{12}$$

Hydrostatic variables

Choice of the hydrostatic temperature $\overline{T}$ and surface pressure $\overline{p}_{\mathrm{s}}$:

1. Time independent (Juang 1994; Gallus and Rančić 1996)

2. Both $\overline{T}$ and $\overline{p}_{\mathrm{s}}$ are determined externally (Juang 1992).

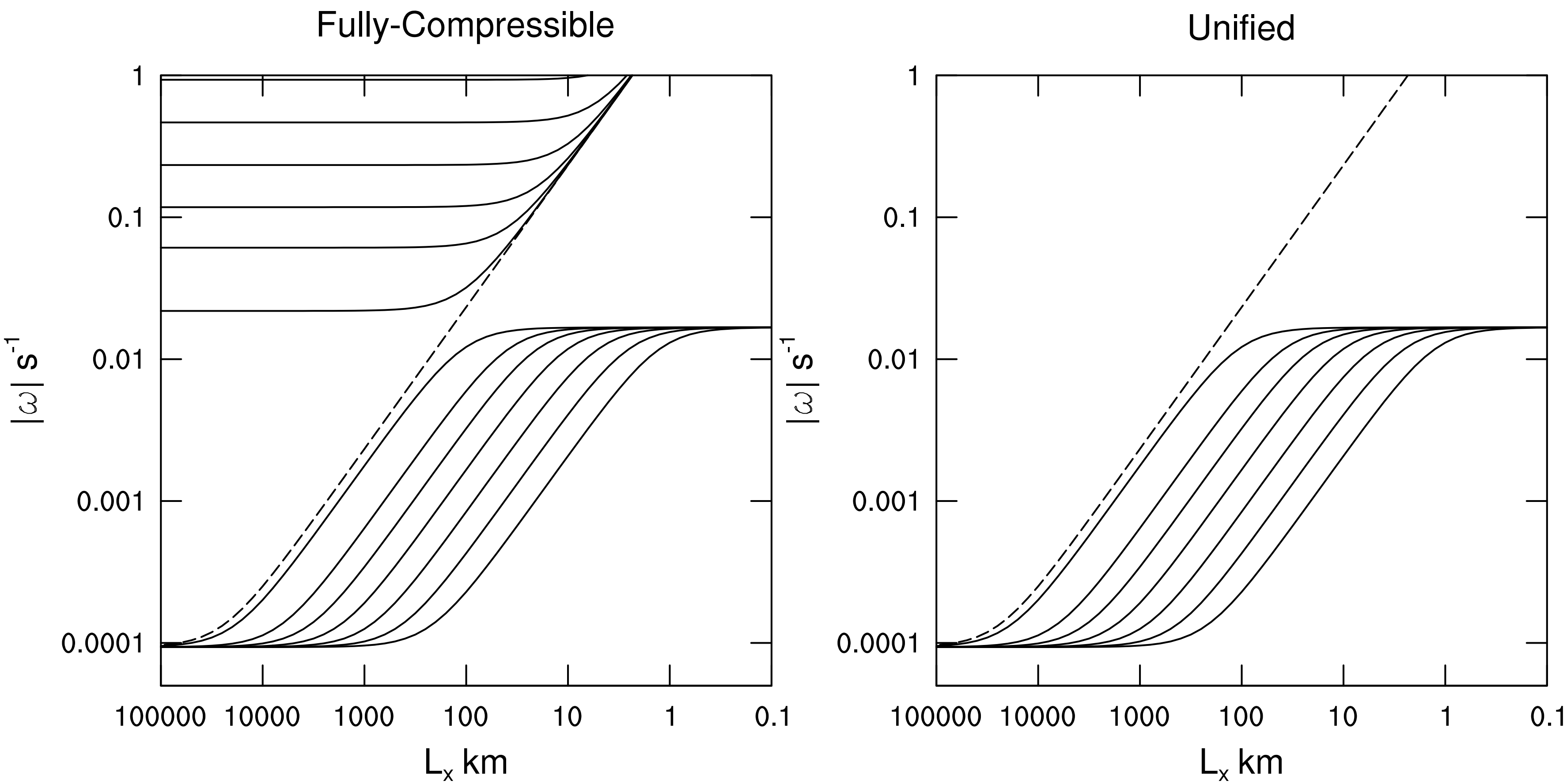
3. Impose $\overline{T}$ externally but predict $\overline{p}_{\mathrm{s}}$ internally (Juang 2000, default of MSM).

4. Predict both $\overline{T}$ and $\overline{p}_{\mathrm{s}}$ internally (Juang 2000).

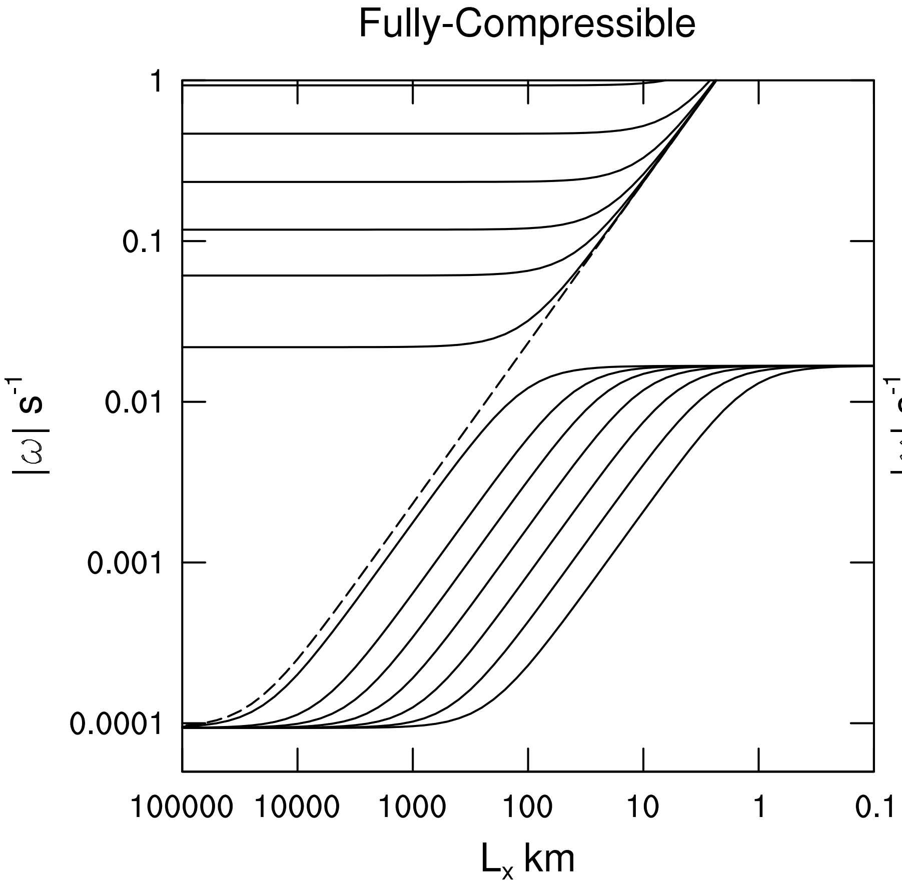
The unified system by Arakawa and Konor (2009)

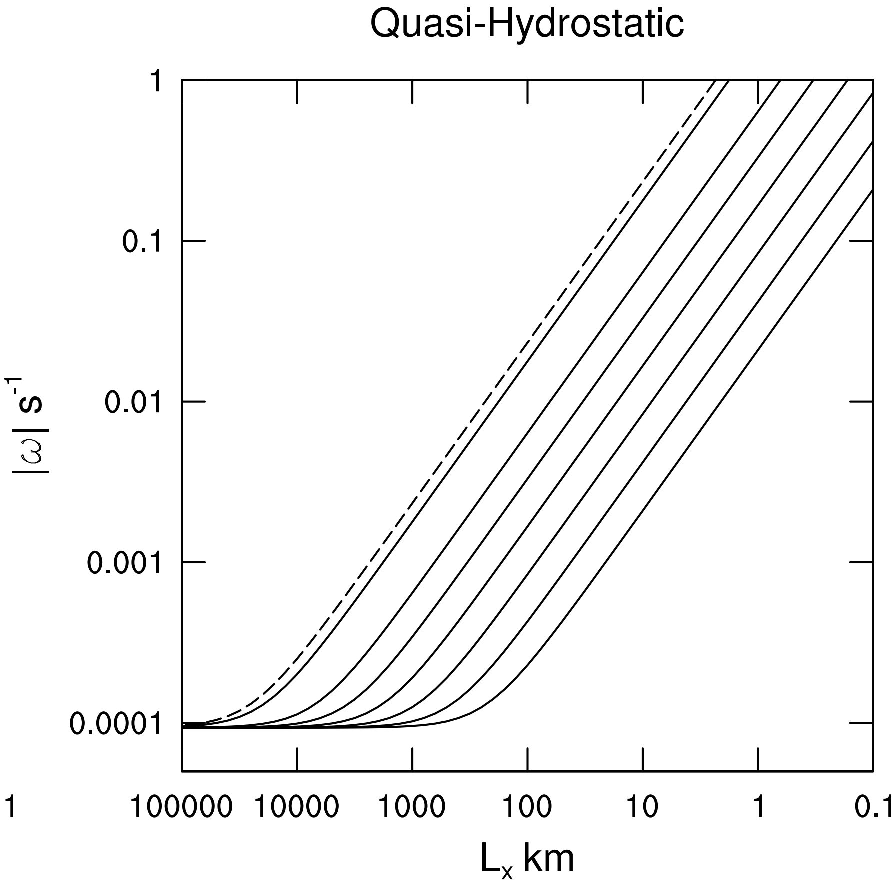
- unifies anelastic and primitive (quasi-hydrostatic) systems.
- uses the hydrostatic density $\overline{\rho}$ in the continuity equation.
  - by ignoring non-hydrostatic pressure tendency $\partial(p - \overline{p})/\partial t$: $p - \overline{p}$ is obtained by solving an Helmholz equation.
- removes sound waves without distortion of planetary waves.

Sound and gravity waves

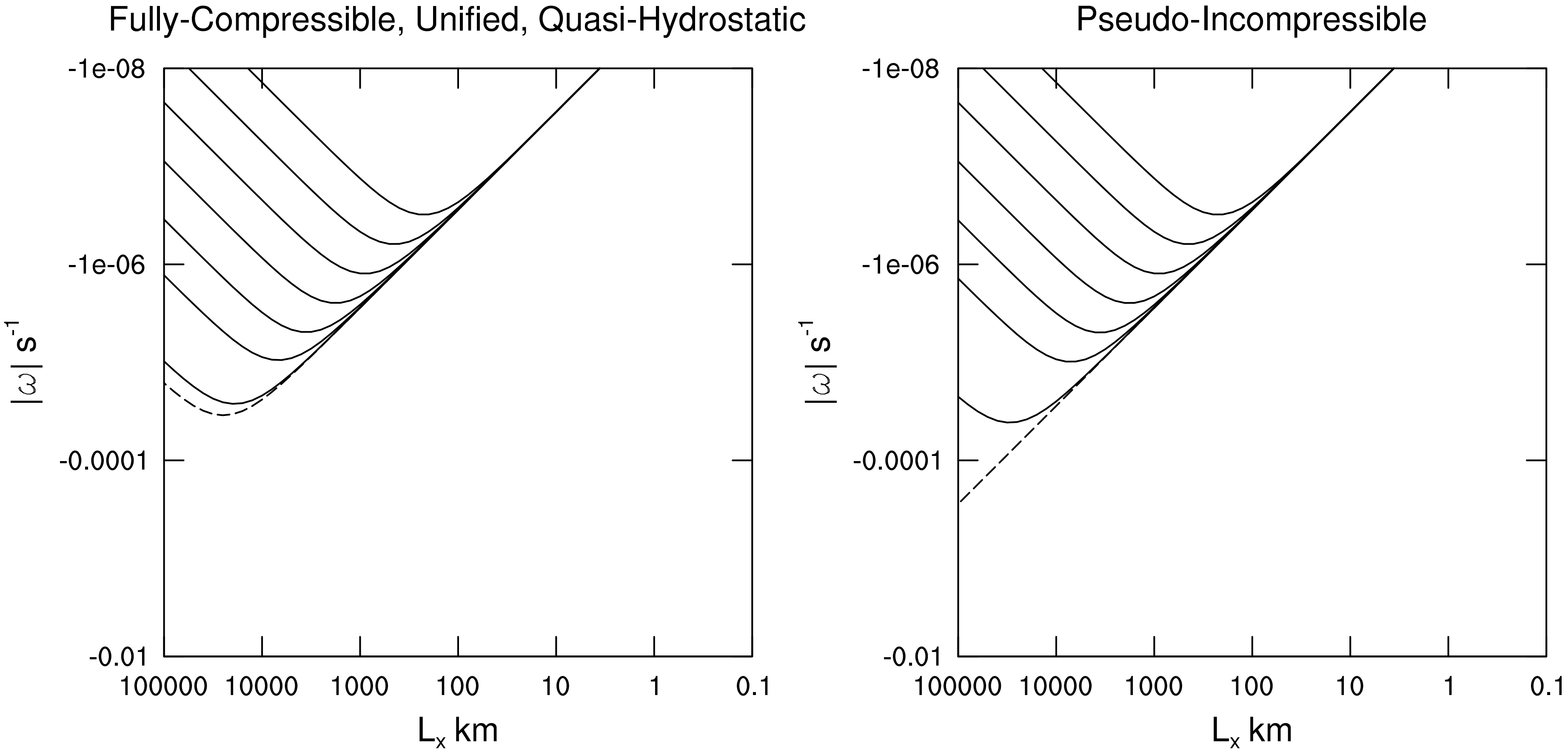


Sound and gravity waves





Rossby waves



The hydrostatic state of Arakawa and Konor (2009)

Using the hydrostatic Exner function $\overline{\pi} = (\overline{p}/p_{\mathrm{ref}})^{\kappa}$, define

$$\frac{\partial \overline{\pi}}{\partial z} \equiv -\frac{g}{c_p \theta}, \tag{13}$$

which means

$$\overline{\theta} = \theta. \tag{14}$$

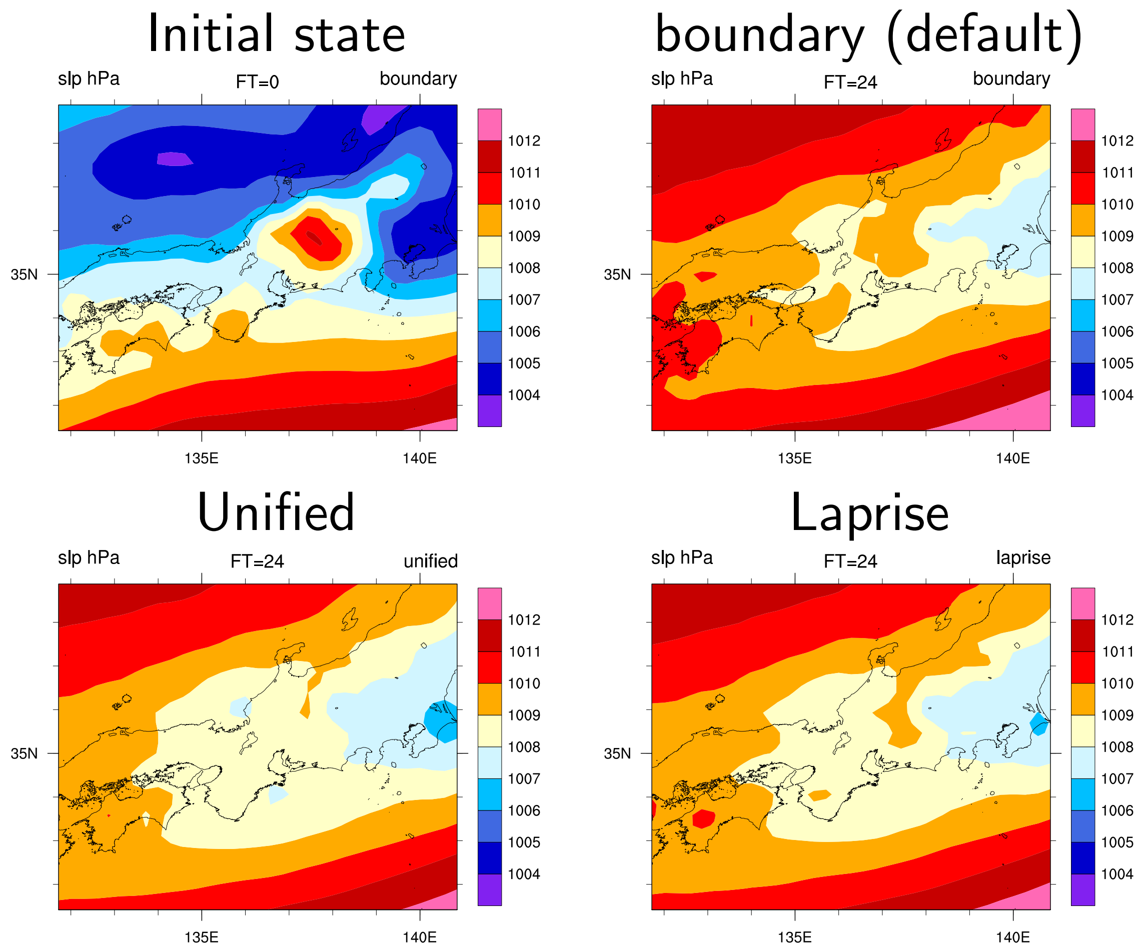
Since $\theta = T/\pi$ and $\overline{\theta} = \overline{T}/\overline{\pi}$, hydrostatic temperature $\overline{T}$ may be written as

$$\overline{T} = T\left(\frac{\overline{p}}{p}\right)^{\kappa}. \tag{15}$$

Forecast experiment using NCEP MSM

- Initial time: 0 UTC 11 August 2011, 24-hour forecast
- Horizontal resolution: $\Delta h = 26$ km, vertical levels: 42, time step: $\Delta t = 60$s
- Initial and boundary conditions: NCEP GFS
- Projection: polar stereo
- Domain: (132–141E, 31–39N) centred at (136E, 35N)
- Problem size: 32×32×42

slp hPa FT=24 INIT=2011081800

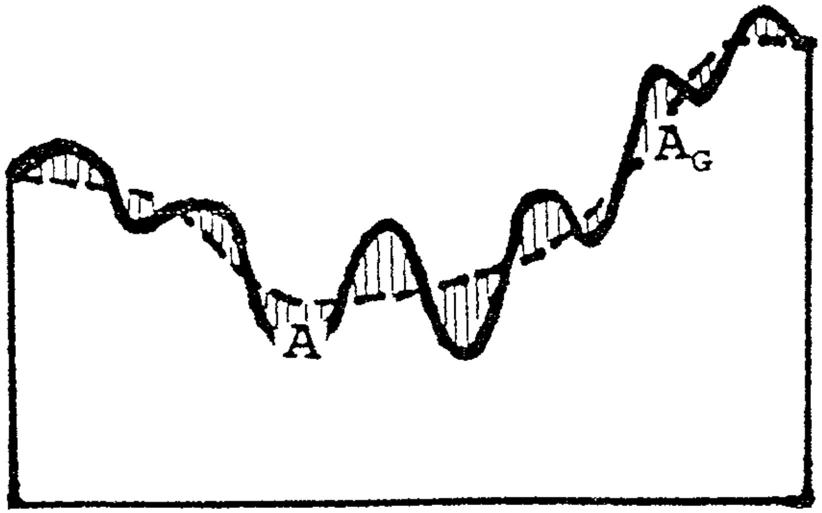


Summary for non-hydrostatic formulation

- The governing equation of MSM is the Euler equations in σ–co-ordinates transformed from those in z–co-ordinates (Juang 1992).
- Sound wave can be removed without distorting planetary wave by the use of the hydrostatic density $\overline{\rho}$ in the continuity equation (Arakawa and Konor 2009).
- The hydrostatic temperature state $\overline{T}$ may be defined by the hydrostatic assumption of Arakawa and Konor (2009) or Laprise (1992).
- Preliminary forecast experiments shows that both hydrostatic assumption are stable.

The perturbation method

predicts deviations from from the global model.



PERTURBATION METHOD

$$\frac{\partial A'}{\partial t} = \frac{\partial A}{\partial t} - \frac{\partial A_{\mathrm{b}}}{\partial t} \tag{16}$$

The governing equations

$$
\begin{aligned}
\frac{\partial \boldsymbol{v}'}{\partial t} &= -m^2\boldsymbol{v}\cdot\nabla\boldsymbol{v} - \dot{\sigma}\frac{\partial \boldsymbol{v}}{\partial \sigma} - E\nabla m^2 + f\boldsymbol{k}\times\boldsymbol{v} \\
&\quad - R(\overline{T}+T')\nabla(\overline{Q}_{\mathrm{s}}+Q') \\
&\quad - \left(1+\frac{T'}{\overline{T}}\right)\left(1+\frac{\partial Q'}{\partial \ln\sigma}\right)\nabla\overline{\phi} + \boldsymbol{F} - \frac{\partial \boldsymbol{v}_{\mathrm{b}}}{\partial t} && (17) \\
\frac{\partial w'}{\partial t} &= -m^2\boldsymbol{u}\cdot\nabla w - \dot{\sigma}\frac{\partial w}{\partial \sigma} \\
&\quad - g\left[1-\left(1+\frac{T'}{\overline{T}}\right)\left(1+\frac{\partial Q'}{\partial \ln\sigma}\right)\right] + F_z - \frac{\partial w_{\mathrm{b}}}{\partial t} && (18)
\end{aligned}
$$

$$\frac{\partial \overline{Q}_{\mathrm{s}}'}{\partial t} = -m^2 \int_0^1 \left[u\frac{\partial \overline{Q}_{\mathrm{s}}}{\partial x} + v\frac{\partial \overline{Q}_{\mathrm{s}}}{\partial y} + \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) \right] \mathrm{d}\sigma - \frac{\partial Q_{\mathrm{sb}}}{\partial t} \quad (19)$$

$$\begin{aligned} \frac{\partial Q'}{\partial t} = & -m^2 u \frac{\partial \overline{Q}_{\mathrm{s}} + Q'}{\partial x} - m^2 v \frac{\partial \overline{Q}_{\mathrm{s}} + Q'}{\partial y} - \dot{\sigma}\frac{\partial Q'}{\partial \sigma} - \frac{\dot{\sigma}}{\sigma} \\ & - \gamma \nabla_3 \cdot \boldsymbol{v} + \gamma \frac{F_T}{T} - \frac{\partial \overline{Q}_{\mathrm{s}}}{\partial t} \end{aligned} \quad (20)$$

$$\begin{aligned} \frac{\partial T'}{\partial t} = & -m^2 u \frac{\partial \overline{T} + T'}{\partial x} - m^2 v \frac{\partial \overline{T} + T'}{\partial y} - \dot{\sigma}\dot{\sigma}^{\kappa} \frac{\partial (\overline{T} + T')\dot{\sigma}^{-\kappa}}{\partial \sigma} \\ & - \frac{RT}{c_v} \nabla_3 \cdot \boldsymbol{v} + F_T - \frac{\partial \overline{T}_{\mathrm{b}}}{\partial t} \end{aligned} \quad (21)$$

$$\frac{\partial q'}{\partial t} = -m^2 u \frac{\partial q}{\partial x} + m^2 v \frac{\partial q}{\partial y} - \dot{\sigma} q \sigma + F_q - \frac{\partial q_{\mathrm{b}}}{\partial t} \quad (22)$$

Vertical velocity and divergence

$$\dot{\sigma} = \frac{\sigma}{R\overline{\overline{T}}}\left[gw + \frac{\partial\overline{\phi}}{\partial t} + m^2\left(u\frac{\partial\overline{\phi}}{\partial x} + v\frac{\partial\overline{\phi}}{\partial y}\right)\right] \tag{23}$$

$$\nabla_3 \cdot \boldsymbol{v} = m^2\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) + \frac{\sigma}{R\overline{\overline{T}}}\left[m^2\left(\frac{\partial u}{\partial \sigma}\frac{\partial\overline{\phi}}{\partial x} + \frac{\partial v}{\partial \sigma}\frac{\partial\overline{\phi}}{\partial y}\right) - g\frac{\partial w}{\partial \sigma}\right] \tag{24}$$

$$\overline{\phi} = \overline{\phi}_{\mathrm{s}} + \int_{\sigma}^{\sigma_{\mathrm{s}}=1}\frac{R\overline{T}}{\sigma}\mathrm{d}\sigma \tag{25}$$

$$Q' \equiv \ln p - \ln \overline{p} = Q - \overline{Q}_{\mathrm{s}} - \ln \sigma \tag{26}$$

gives

$$\left(\frac{\overline{p}}{p}\right)^{\kappa} = \exp(-\kappa Q'). \tag{27}$$

Using this identity

$$\begin{aligned} \nabla \overline{T} &= \left(\frac{\overline{p}}{p}\right)^{\kappa} (\nabla T - \kappa T \nabla Q') \\ &= \exp(-\kappa Q') \left(\nabla T - \kappa T \nabla Q'\right). \end{aligned} \tag{28}$$

$\nabla \overline{T}$ is used in $\dot{\sigma}$ and $\nabla_3 \cdot \boldsymbol{v}$.