Ensemble Kalman Filter: Handling nonlinearity with the Running in Place (RIP) and Quasi Outer-Loop (QOL) methods

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Outline

- Background
  - Schemes of Running-in-Place (RIP)/Quasi-Outer Loop (QOL)
- Results with the Lorenz63 model
- Comparisons between *iterative* EnKFs
- Applications of LETKF-RIP to regional data assimilation
- Summary/Discussions

Issues of nonlinearities in data assimilation

• The Kalman Filter assumption that the ensemble forecast perturbations are Gaussian is not valid if there is nonlinear growth

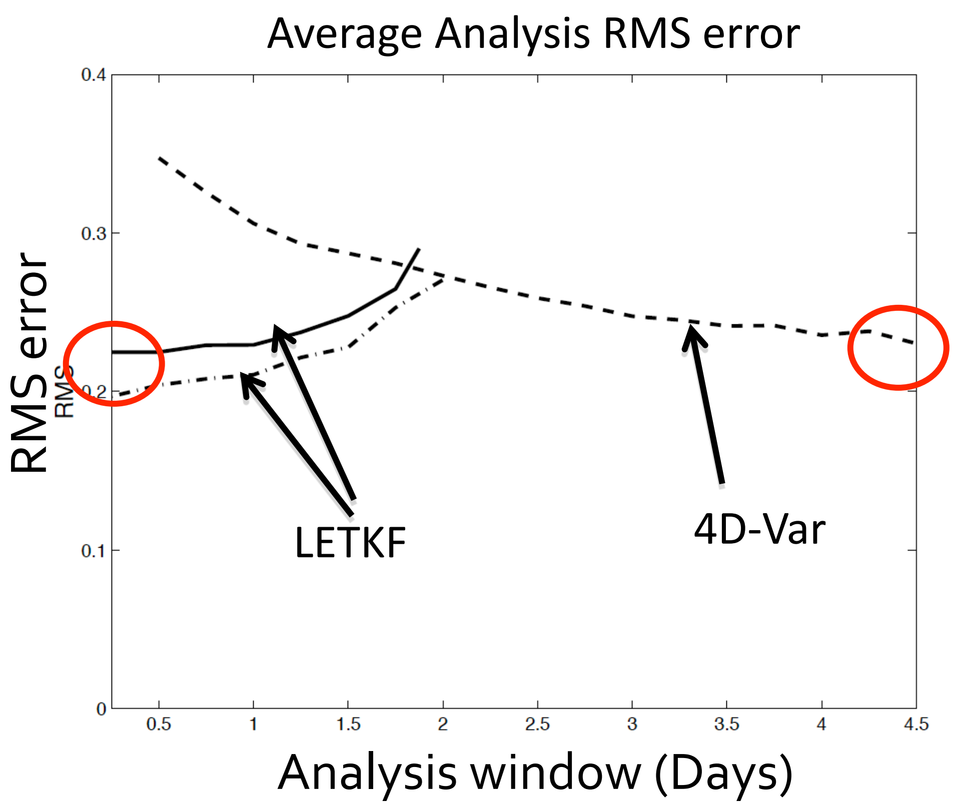
•Nonlinearity depends on **model dynamics**, **observations** (accuracy, operators, sampling frequency) and model error

- •Being able to use the nonlinear operators (M and H), EnKF handles some nonlinearity.

•Nonlinearity will increase the difficulty of the data assimilation, particularly for EnKF.

LETKF v.s. 4D-Var

with the Lorenz 40-variable model (Fertig et al., 2007)



EnKF does not handle well long windows because ensemble perturbations become non-Gaussian. 4D-Var simply iterates and produces a more accurate analysis.

Issues of nonlinearities in data assimilation

•Nonlinearity will increase the difficulty of the data assimilation, particularly for EnKF.

- A disadvantage of ensemble-based KF is that ensemble perturbations become non-Gaussian under strong nonlinearity, and therefore needs short assimilation windows.
- 4D-Var is a smoother: it keeps iterating until it fits the observations within the assimilation window as well as possible.
- EnKF doesn't have the important outer loop as in the incremental 3D-Var and 4D-Var, widely used in operational centers (ECMWF, NCEP, GMAO…)

Outer-loop in the incremental 4D-Var

Adjustments for the background trajectory and sensitivity matrix related to the linearization of the observation operator

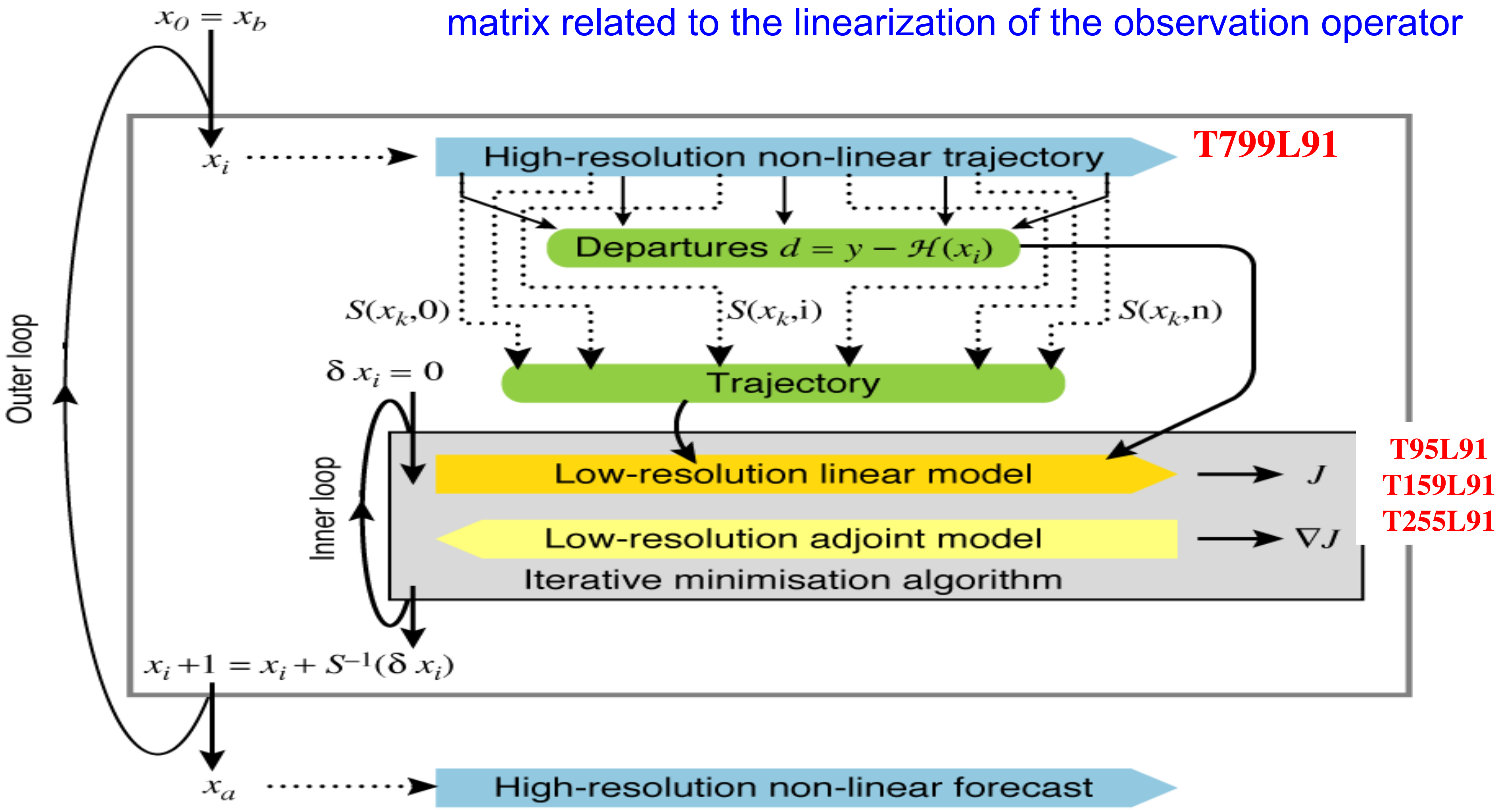


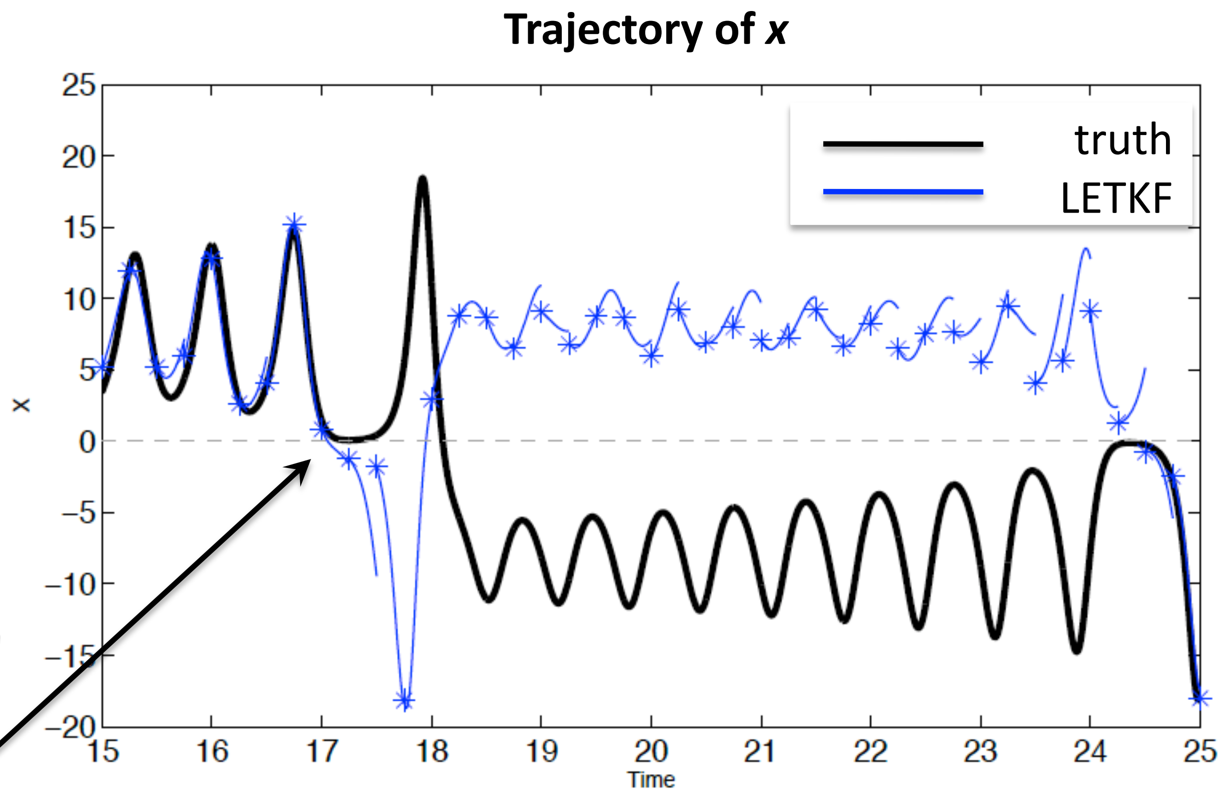
Figure from ECMWF, Anderson, 2005

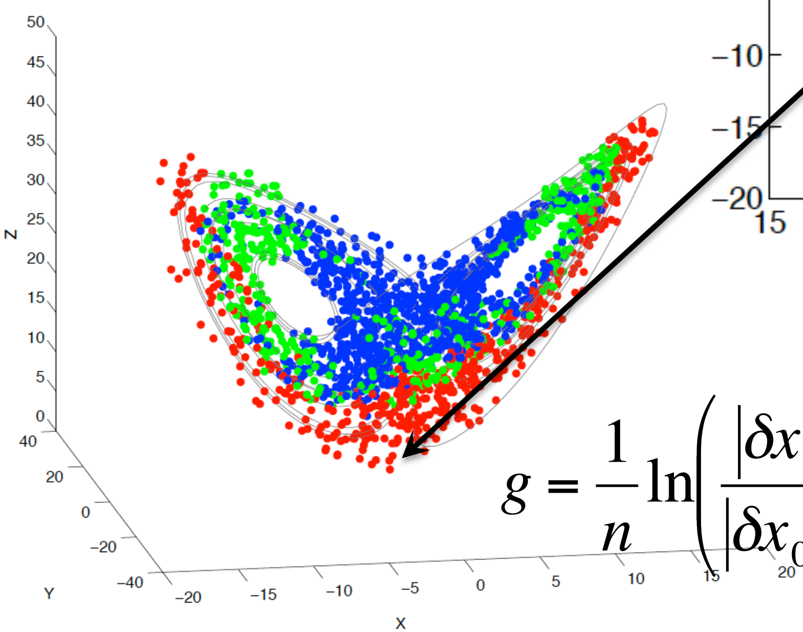
Dealing with nonlinearities within EnKF

- EnKF is a sequential data assimilation system where, after the new data is used at the analysis time, it should be discarded. **Only if the previous analysis and the new background are the most likely states given the past observations.**
- For cases with strong nonlinear growth (e.g. the EnKF spin-up or the sudden change of the background dynamics), background ensemble can't represent the state uncertainty and <u>the most likely state is unlikely to happen!!</u>
  - Filter divergence can take place.

Filter divergence:

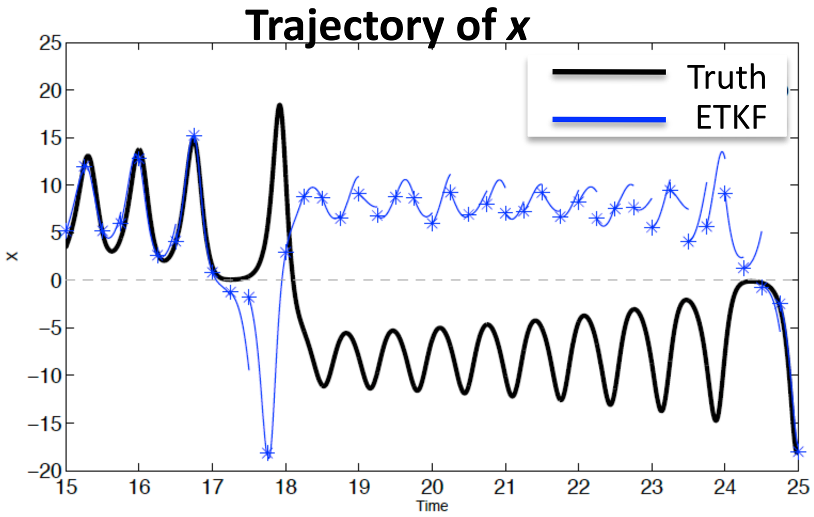
when the trajectory is about to change regime





$$g = \frac{1}{n}\ln\left(\frac{|\delta x|}{|\delta x_0|}\right)$$ **perturbation grow rate**

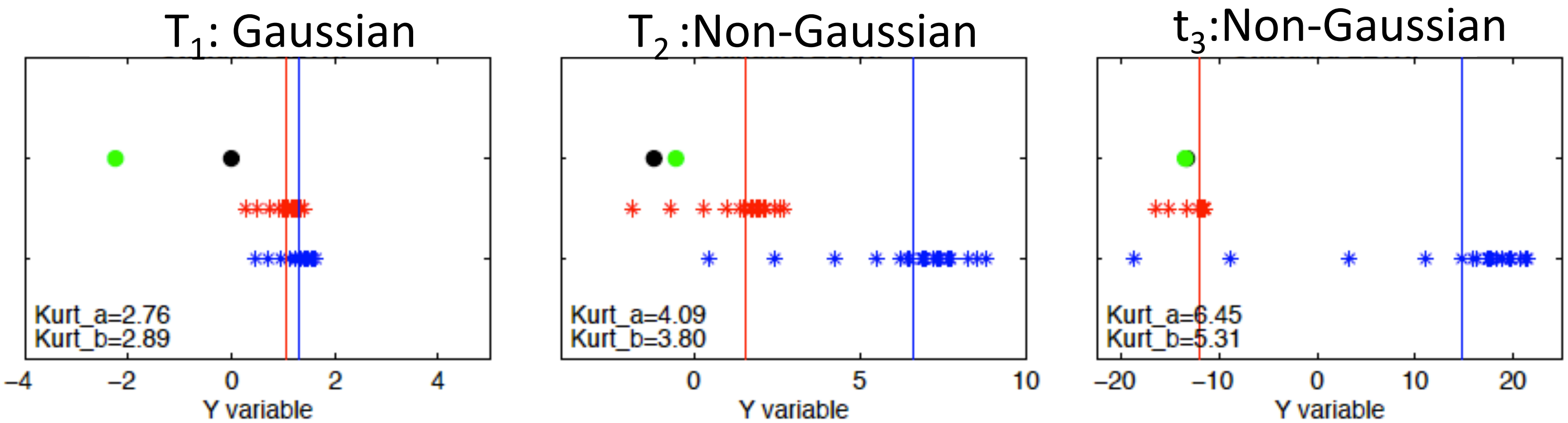
Nonlinearity vs. Non-Gaussianity in EnKF



•Nonlinearity will distort the ensemble distribution and make it less Gaussian

•With non-Gaussian ensemble, the background ensemble quickly degrades.

•Sampled error statistic lost track of the true dynamics



• truth, • observation, * analysis ensemble , * background ensemble

Dealing with nonlinearities within EnKF

- EnKF is a sequential data assimilation system where, after the new data is used at the analysis time, it should be discarded. **Only if the previous analysis and the new background are the most likely states given the past observations.**
- For cases with strong nonlinear growth (e.g. the EnKF spin-up or the sudden change of the background dynamics), background ensemble can't represent the state uncertainty and the most likely state is unlikely to happen!!
- **During strong nonlinearity, we wish to increase the influence of observations and should use the observations more than once if we can extract more information**.

Kalman Filter and RIP with linear dynamics

- RIP is an algorithm that uses the same observation multiple times
- Using RIP produces the same analysis means as the optimal KF.
- The estimated error from KF-RIP with re-using the observations N times is the same as the one that would be computed from KF with higher observation accuracy, i.e., with an observation error variance divided by N.

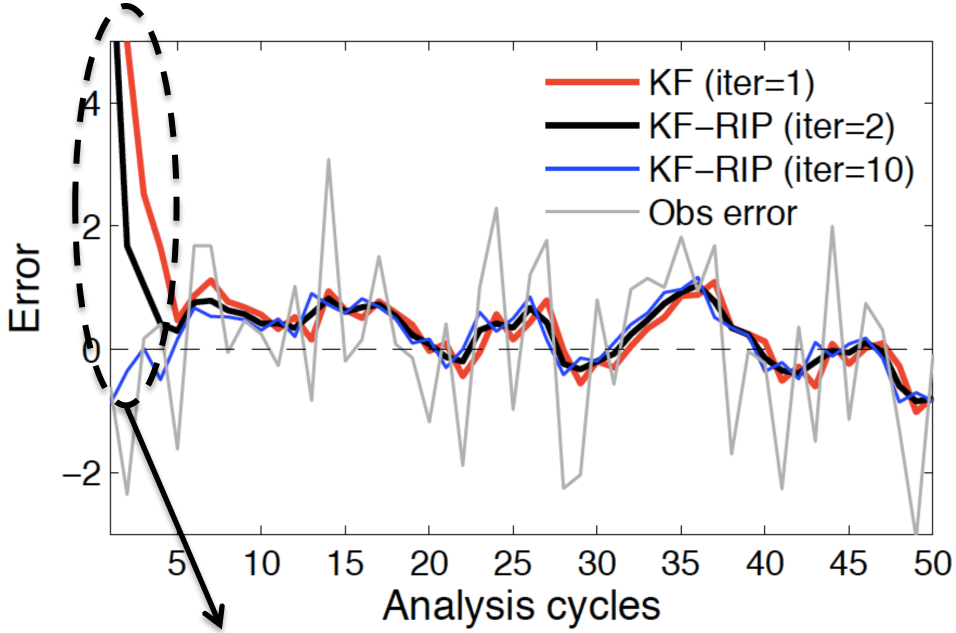
KF vs. KF-RIP with a linear model

A linear model for state and error variance

$$x_n = x_{n-1} + \alpha$$

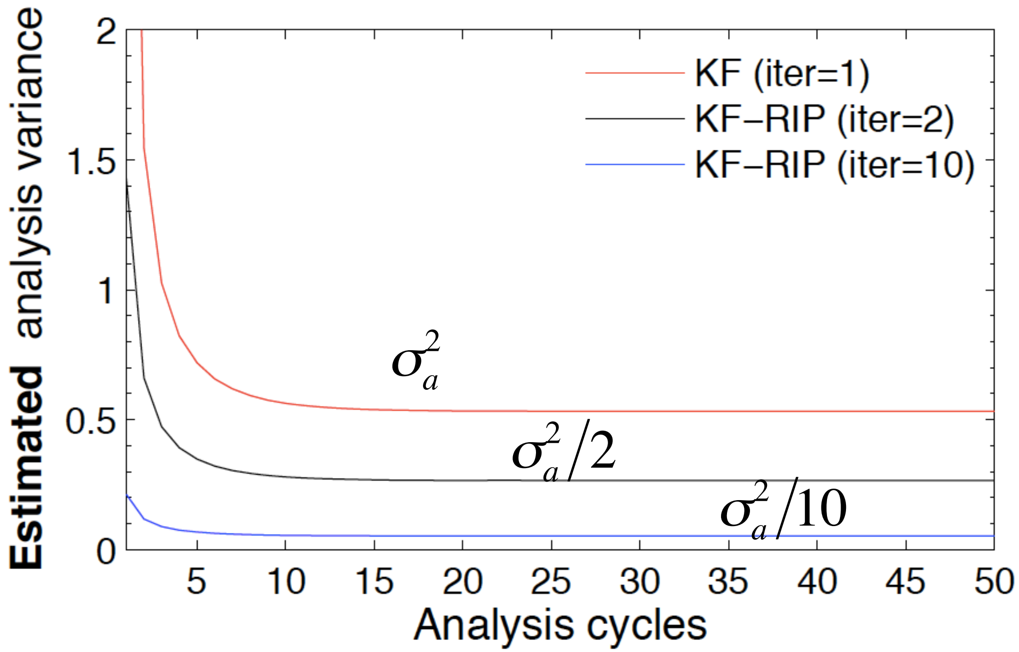
$$\sigma_n^2 = C\sigma_{n-1}^2$$

Using RIP produces the same analysis means as the optimal KF.



RIP accelerates the spin-up

RIP as multi-step analysis correction



- With N iteration, the estimated error variance from KF-RIP is N times smaller than the one from KF.

- The estimated variance from KF-RIP is the same as the one that would be computed from KF with higher observation accuracy, i.e., with an observation error variance divided by N, the number of the iterations.

- The small estimated error variance from KF-RIP is used to achieve small increment for multi-step analysis correction

Increase the influence of observations by reducing their error covariance R

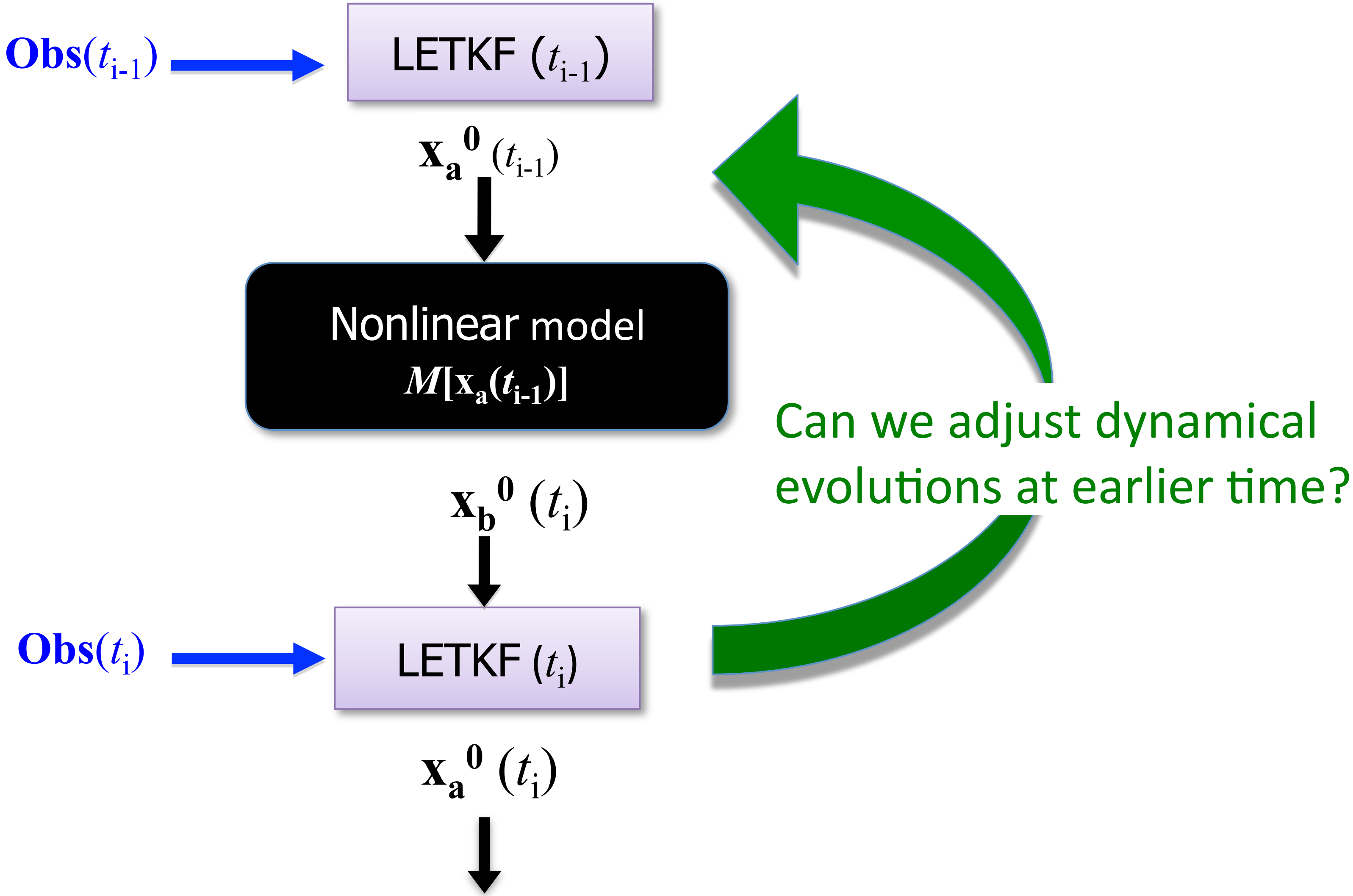
☐ **“Hard way”:**

- Reduce the observation error and assimilate this observation once.
- Compute the analysis increment at once

☑ **“soft way”: (RIP/QOL)**

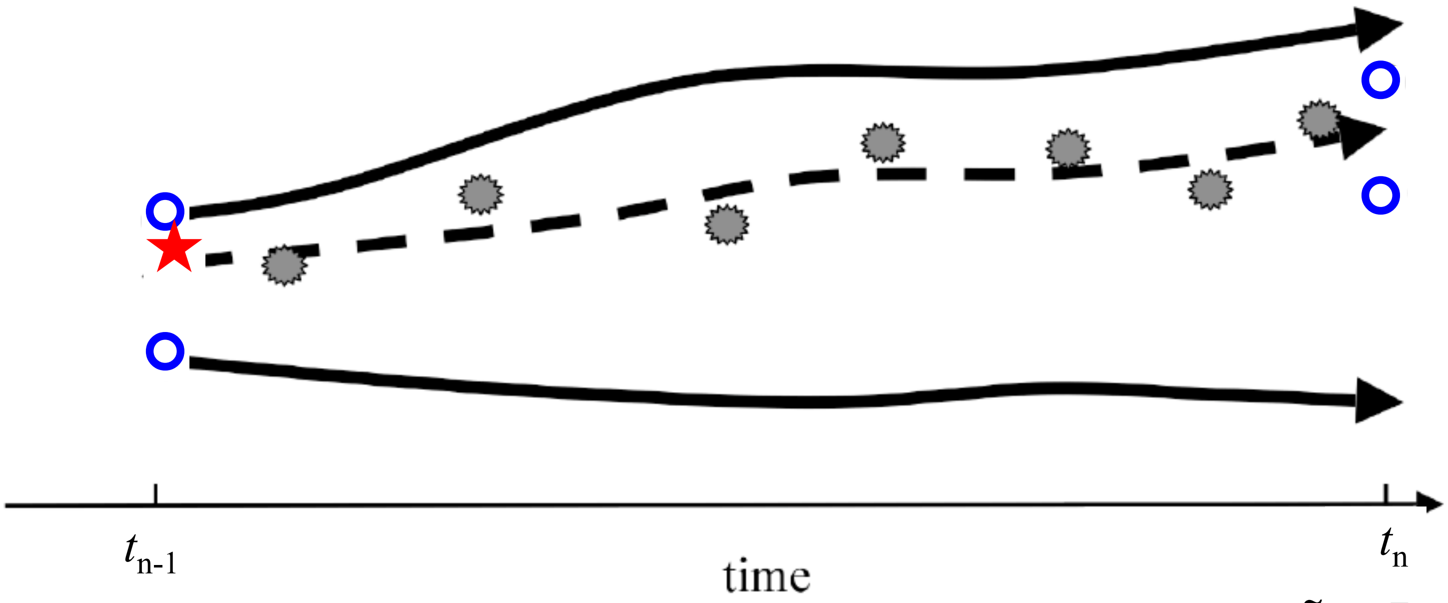
- Use the original observation error and assimilate the same observation multiple times.
- The total analysis increment is achieved as the sum of multiple smaller increments (advantageous with nonlinear cases).

Standard LETKF framework



No-cost smoother for 4D-LETKF

(Kalnay et al, 2007, Yang et al. 2008)



$$\tilde{\bar{\mathbf{x}}}_a(t_{n-1}) = \bar{\mathbf{x}}_a(t_{n-1}) + \mathbf{X}_a(t_{n-1})\bar{\mathbf{w}}_a(t_n)$$

$$\tilde{\mathbf{X}}_a(t_{n-1}) = \mathbf{X}_a(t_{n-1})\mathbf{W}_a(t_n)$$

$$\bar{\mathbf{w}}_a = \tilde{\mathbf{P}}_a \mathbf{Y}_b^T \mathbf{R}^{-1}(\mathbf{y} - H(\bar{\mathbf{x}}));$$

$$\mathbf{W}_a = [(K-1)\tilde{\mathbf{P}}_a]^{\frac{1}{2}}$$

- No-cost LETKF smoother (★): apply at t_{n-1} the same weights found optimal at t_n , works for 3D- or 4D-LETKF
- Propagate information about the observation and “error of day” at tn to tn-1

Iterative algorithm for re-using observations (no-cost smoother + stopping criterion)

Analysis step at t_n

$$\bar{\mathbf{x}}_a^l(t_n) = \bar{\mathbf{x}}_b^l(t_n) + \mathbf{X}_b^l(t_n)\overline{\mathbf{w}}(t_n)$$

$$\mathbf{X}_a^l(t_n) = \mathbf{X}_b^l(t_n)\mathbf{W}(t_n)$$

Smooth step at t_{n-1}

$$\tilde{\bar{\mathbf{x}}}_a^l(t_{n-1}) = \tilde{\bar{\mathbf{x}}}_a^{l-1}(t_{n-1}) + \tilde{\mathbf{X}}_a^{l-1}(t_{n-1})\overline{\mathbf{w}}(t_n)$$

$$\tilde{\mathbf{X}}_a^l(t_{n-1}) = \tilde{\mathbf{X}}_a^{l-1}(t_{n-1})\mathbf{W}(t_n) + \mathbf{E}^{l-1}$$

Forecast time from t_{n-1} to t_n

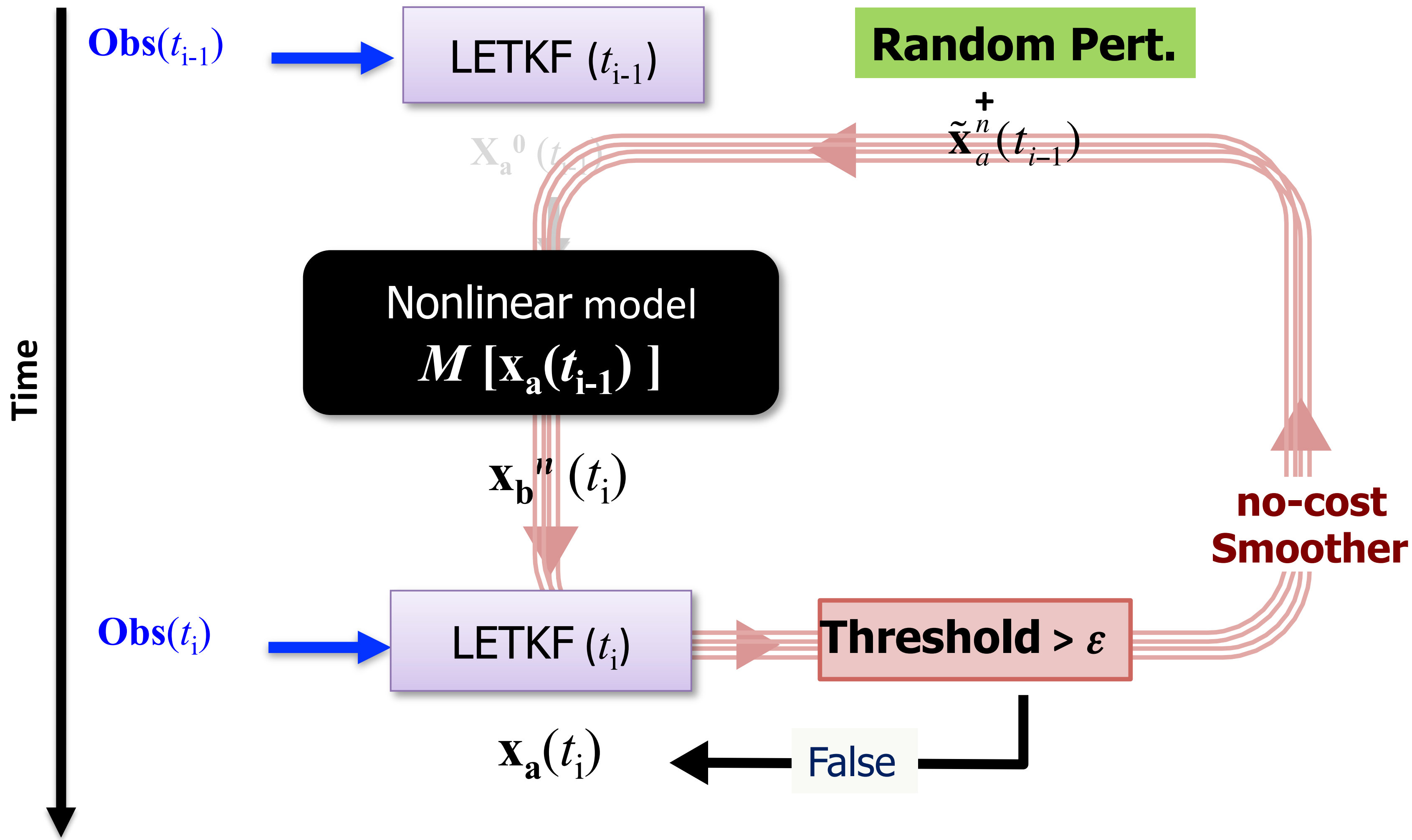
$$\mathbf{x}_b^l(t_n) = M_{t_{n-1}\to t_n}[\tilde{\bar{\mathbf{x}}}_a^{l-1}(t_{n-1}) + \tilde{\mathbf{X}}_a^{l-1}(t_{n-1})]$$

threshold

$$\varepsilon = \frac{\left|\mathbf{y}_o(t_n) - H[\bar{\mathbf{x}}_b^{l-1}(t_n)]\right| - \left|\mathbf{y}_o^l(t_n) - H[\bar{\mathbf{x}}_b^l(t_n)]\right|}{\sigma_o} > \varepsilon_s$$

The iteration continues only if we can extract extra information from the same observations

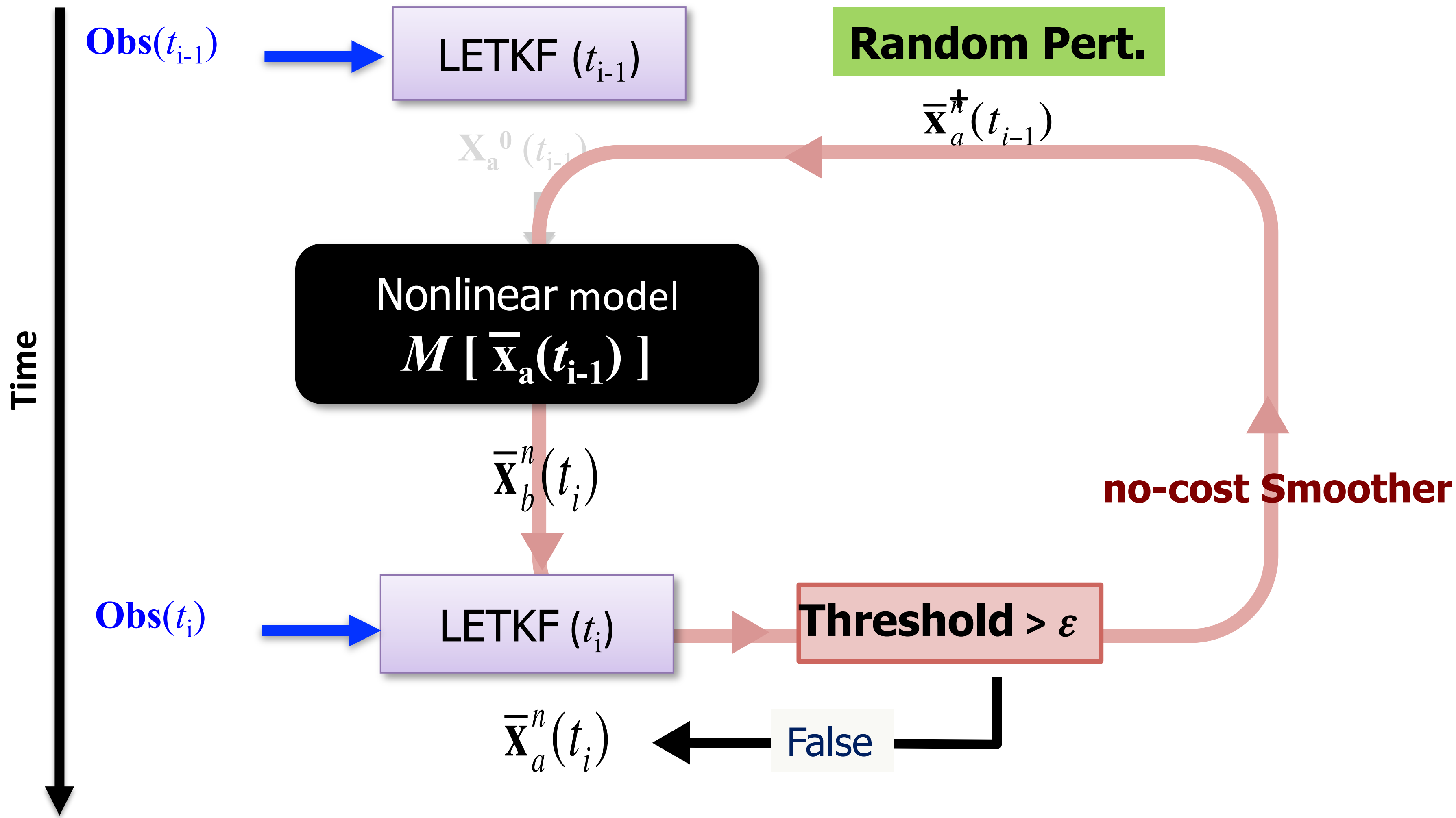
"Running in place" in the LETKF framework



re-evolve the whole ensemble to catch up the true dynamics, represented by OBS

"Quasi Outer-loop" in the LETKF framework

(simplified RIP)



adjust the nonlinearity of the mean trajectory

Dealing with nonlinearities with EnKF

We'll focus on nonlinear dynamics, and propose two new methods based on the LETKF framework for using long windows

	RIP	QOL
	generalized outer-loop	simplified RIP
improvement	mean and covariance	mean
cost	expensive	less expensive
iteration number	< 10	1~2

Experimental setting

- Nonlinear model: Lorenz 3-variable model
- assimilation setup
  - DA methods: LETKF, RIP, QOL and 4D-Var
  - observation error variance= 2.0
  - assimilation window
    - Linear window (frequent observation): 8 timestep
    - Nonlinear window (infrequent observation): 25 timestep

	4D-Var	LETKF
Linear window **(obs every 8 timesteps)**	0.31	0.30
Nonlinear window **(obs every 25 timesteps)**	0.53 Assim window=75	0.68

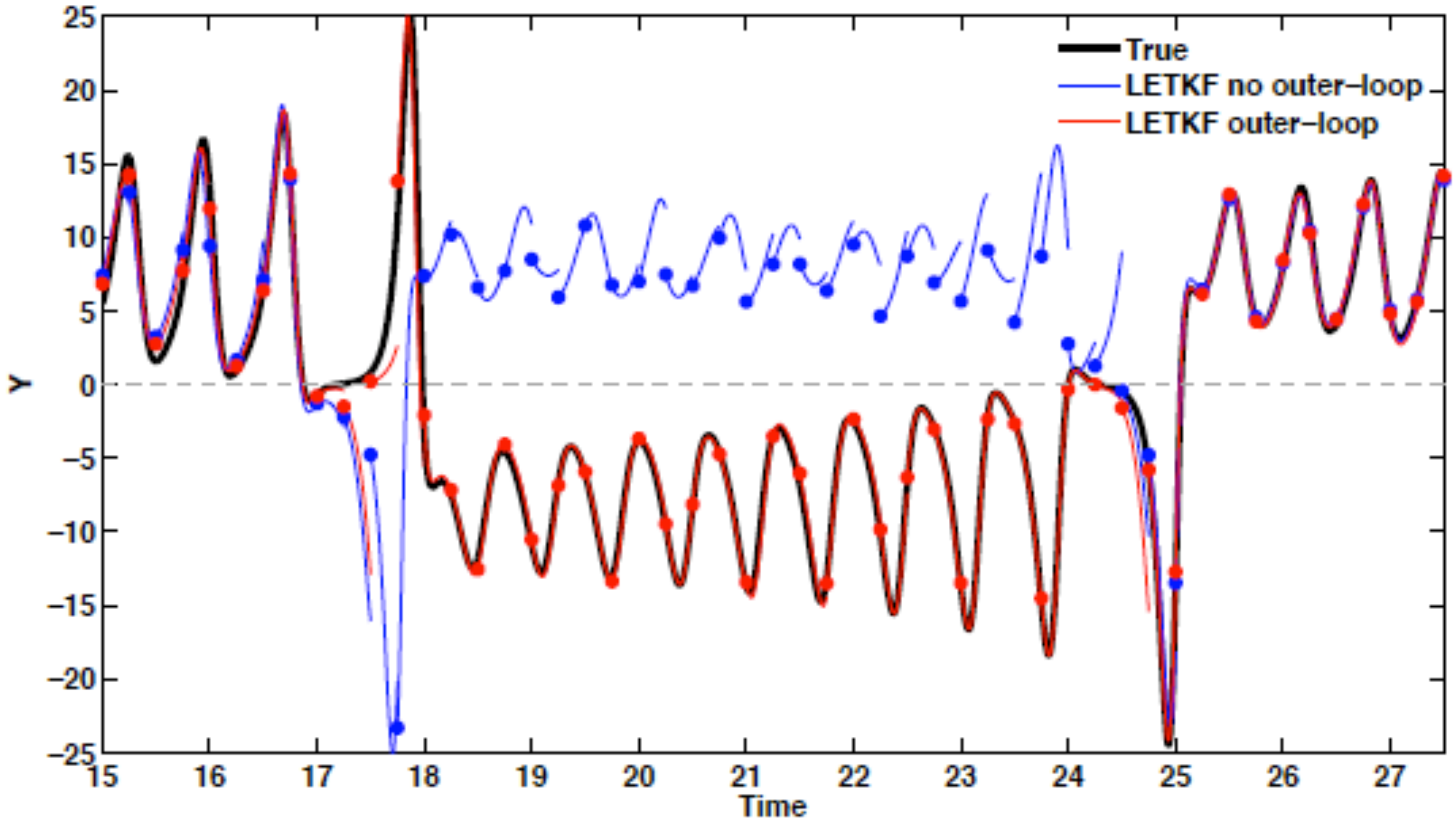
- Long window + Quasi-static variational analysis (Pires et al., 1996) -> 4D-Var wins!
- The standard LETKF can't handle the long assim. window.

Results with Lorenz 3-variable model

	4D-Var	LETKF standard	LETKF +QOL	LETKF +RIP
obs every 8 time-step (linear window)	0.31	0.30	0.27	0.27
obs every 25 time-step (nonlinear window)	**0.53** (assim window=75)	0.68	0.47	0.35

- With the QOL, LETKF analysis with nonlinear window is much improved, even better than 4D-Var!
- RIP gives even more improvement than the QOL because it improves both the mean and the covariance.

Trajectory of variable y



with RIP/QOL filter divergence is avoided

fewer observations

	x	y	z	xy	xz	yz	xyz
ETKF	2.9	1.67	7.16	1.01	1.53	0.78	0.68
QOL	1.98	1.23	5.94	0.82	1.16	0.60	0.47
RIP	1.57	0.97	3.81	0.56	0.66	**0.40**	**0.35**

•With fewer observations (constraint), the model trajectory is strongly affected by the nonlinear evolution of the initial errors.

•RIP and QOL use the observations more efficiently for the under-observed cases.

•Performance: RIP > QOL > standard ETKF

Comparisons between iterative EnKFs

1. RIP/QOL
2. Ensemble Randomized Maximum Likelihood (EnRML, Gu and Oliver, 2007)
   – Same framework as the 4D-Var: Improve only the sensitivity matrix, **H**x, and the background trajectory for computing the innovation, y_o-$H(x_b)$
   – Minimize the cost-function with the reduced adjustment Gauss-Newton method

Ensemble Randomized Maximum Likelihood

Implement MLH with the stochastic EnKF

(1) Minimizing the cost-function is solved for the ensemble member (k), with perturbed observations at tn

$$J(\mathbf{x}_0^k) = \frac{1}{2}\left[\mathbf{x}_0^k - \mathbf{x}_{b0}^k\right]^T \mathbf{P}_{b0}^{-1}\left[\mathbf{x}_0^k - \mathbf{x}_{k0}^k\right] + \frac{1}{2}\left[H(\mathbf{x}_n^k) - \mathbf{y}_{on}^k\right]^T \mathbf{R}^{-1}\left[H(\mathbf{x}_n^k) - \mathbf{y}_{on}^k\right]$$

$$\mathbf{x}_0^{k,i+1} = \beta_i \mathbf{x}_{b0}^k + (1-\beta_i)\mathbf{x}_0^{ki} - \beta_i\left(\frac{1}{K-1}\right)\mathbf{X}_{b0}^k(\mathbf{Y}_{bn}^k)^T\left(\mathbf{R} + \left(\frac{1}{K-1}\right)\mathbf{Y}_{bn}^k(\mathbf{Y}_{bn}^k)^T\left[\mathbf{x}_n^{k,i} - \mathbf{y}_{on}^{k,i} - (\mathbf{x}_n^{k,i} - \mathbf{x}_{bn}^k)\right]\right]$$

(2) estimate the data mismatch for both $\mathrm{x}_0^{k,i+1}$ and $\mathrm{x}_0^{k,i}$ with OMF_i

$$\mathbf{OMF}_i = \frac{1}{2}\left[H(M[\mathbf{x}_0^{k,i}]) - \mathbf{y}_{on}^k\right]^T \mathbf{R}^{-1}\left[H(M[\mathbf{x}_0^{k,i}]) - \mathbf{y}_{on}^k\right]$$

(3) if $\mathbf{OMF}_{l+1} < \mathbf{OMF}_l$, $\mathrm{x}_0^{k,i+1} = \mathrm{x}_0^{k,i}$ and increase β_l,
otherwise keep $\mathbf{x}_0^{k,l}$ and decrease β_l

(4) If the criteria is not satisfied, repeat (1) to (3). Criteria to stop the iteration:

- $(\mathbf{OMF}_l - \mathbf{OMF}_{l+1})/\mathbf{OMF}_l < 10^{-4}$
- Maximum iteration number is 20

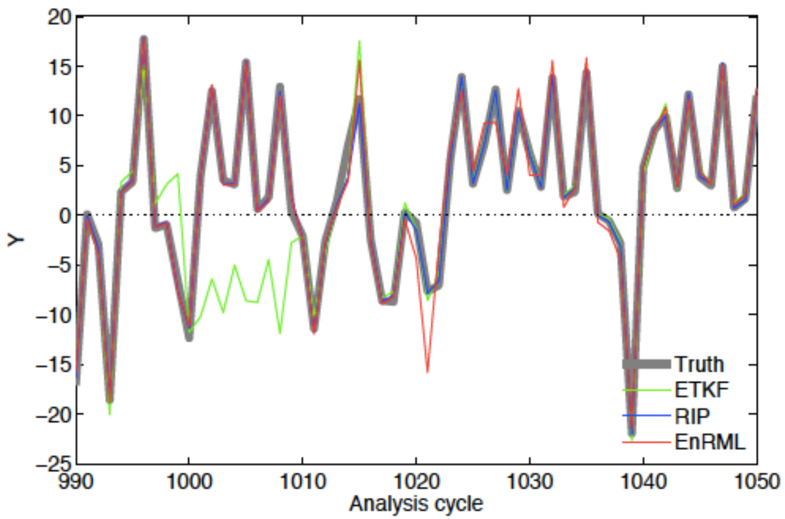
RMS error with iterative EnKFs (K=24)

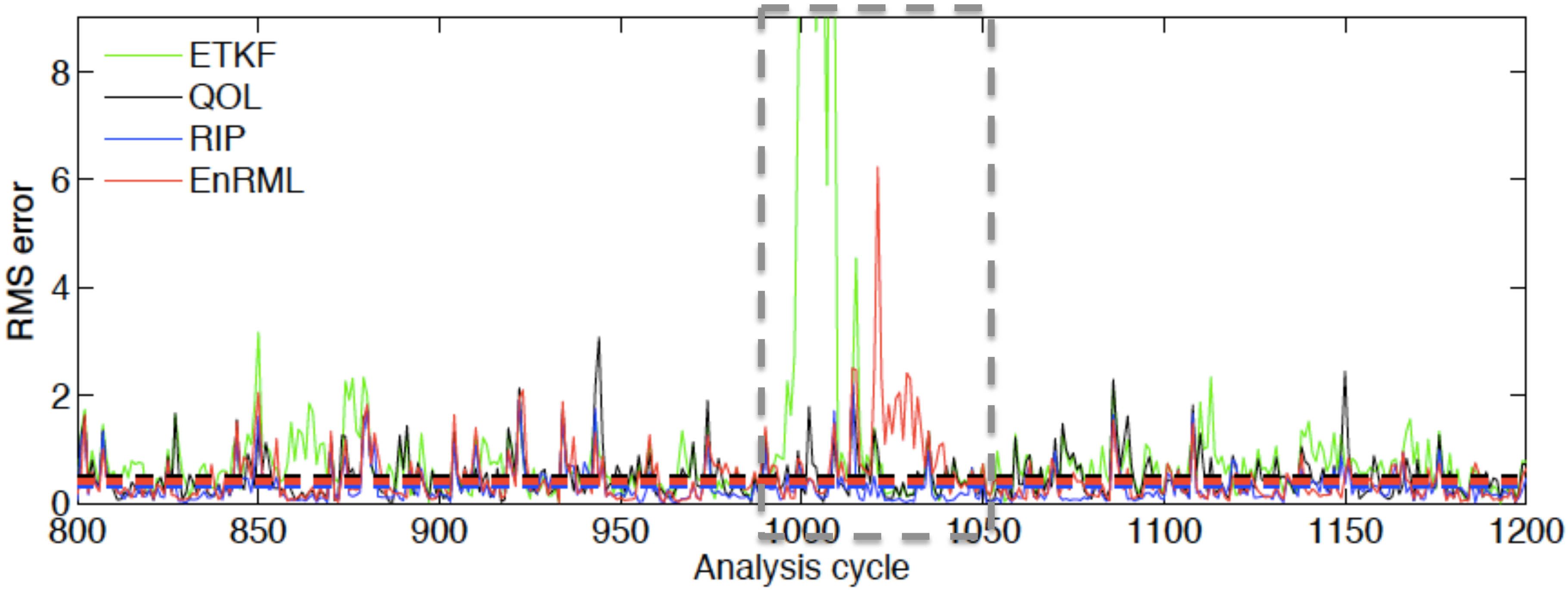
(EnRML doesn't work with 3 ensemble member, while
RIP and QOL already reach optimal value at K=3)

	QOL	RIP	EnRML	
				W/O adjusting the minimizing step
RMS error	0.49	0.33	0.41	1.45

- Experiment with EnRML is performed with an assimilation window of 25 time-step with observations arranged at the end of the window
- With infrequent observation, RIP performs better than EnRML.

- The minimization of EnRML can still fail with strong nonlinearity
- RIP/QOL follow the true trajectory better
- However, when EnRML is well behaved, it is slightly more accurate than RIP/QOL.





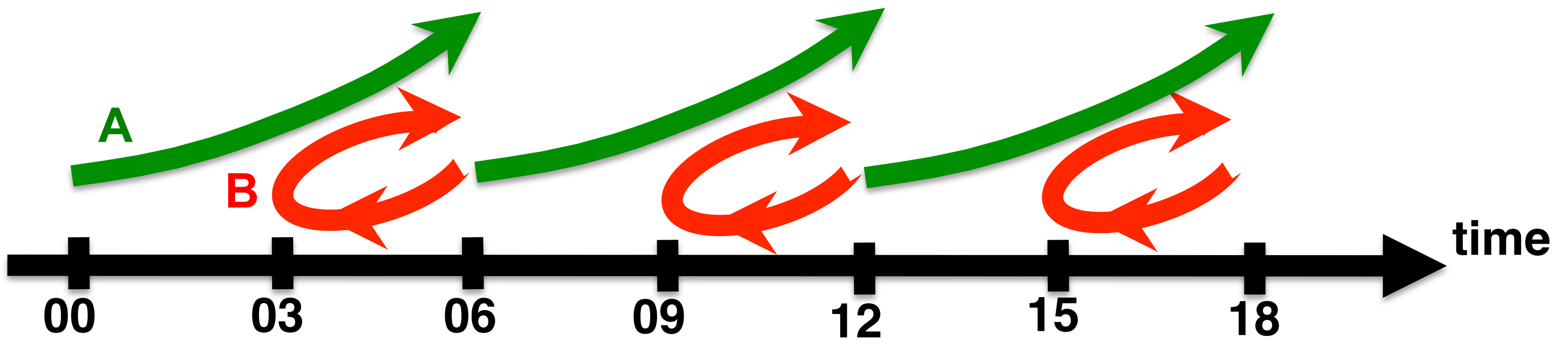
Application of LETKF-RIP to typhoon assimilation/prediction

OSSE experiment setup:

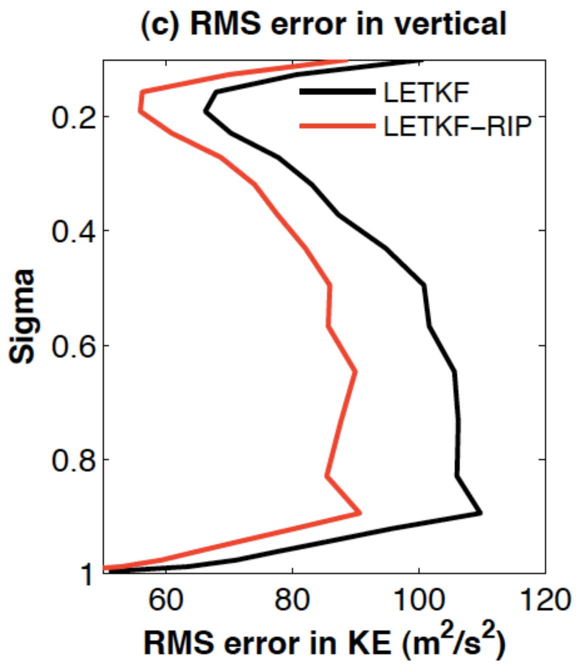
- Regional Model: Weather Research and Forecasting model (WRF, 25km)
- Assimilation scheme: LETKF and LETKF-RIP with 36 ensemble members
- Observations: radiosonde, dropsondes and surface ocean wind

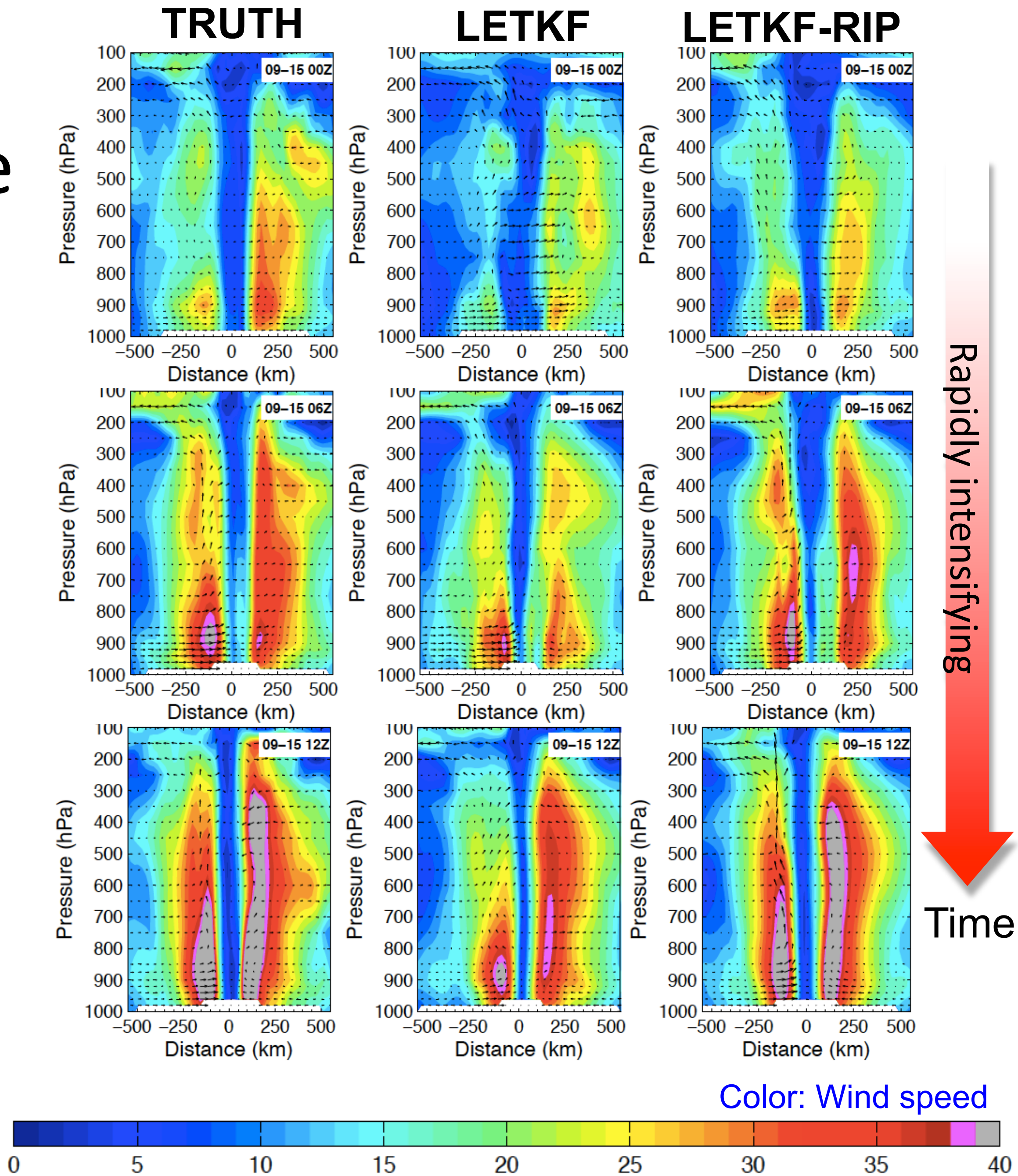
LETKF-RIP setup

1) Computed the LETKF weights at analysis time (00,06,12,18Z)
2) Use these weight to reconstruct the ensemble (U, V) at (03,09,15,21Z)
3) perform the 3-hr ensemble forecasts
4) Re-do the LETKF analysis (only one iteration is tested)

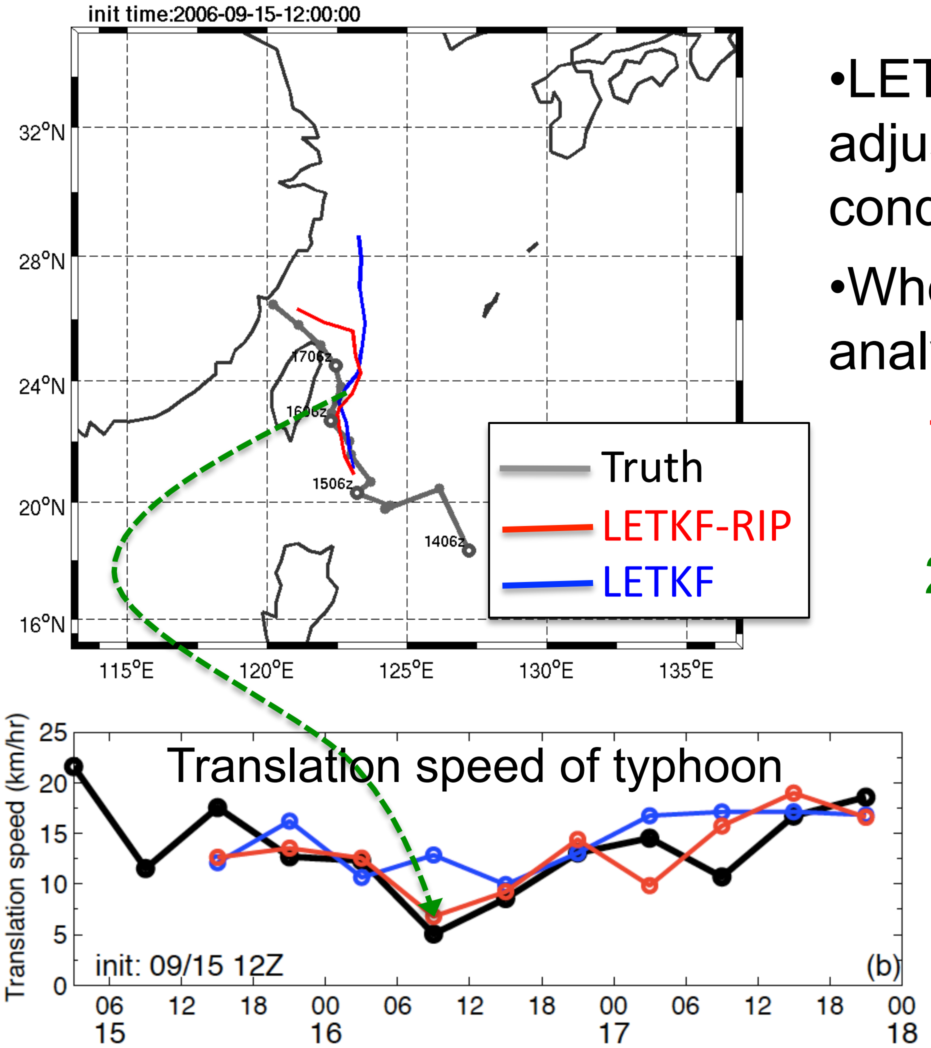


Typhoon vertical structure (analysis)



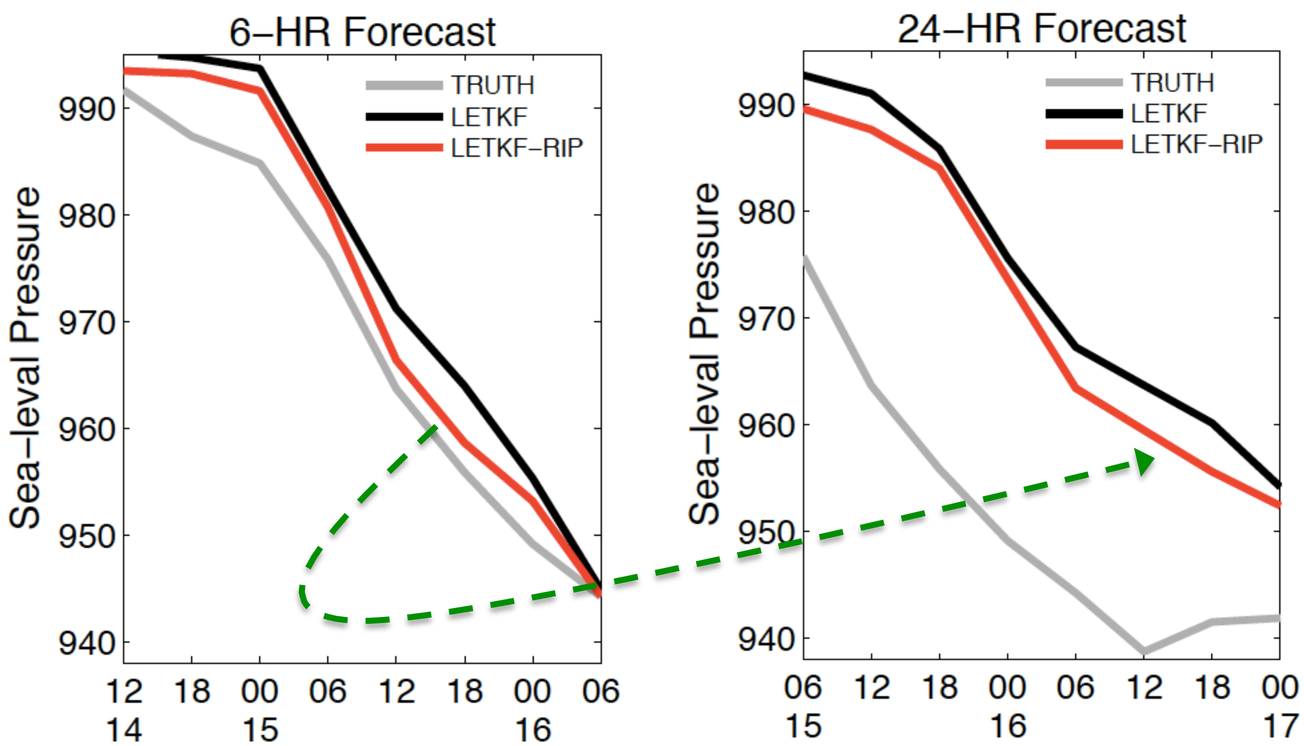


Typhoon prediction:
represent the environmental condition



- LETKF-RIP is able to accelerate the adjustment of the environmental condition for typhoon development:
- When initialized with the LETKF-RIP analysis, the improvements include:
  1. Capture the west-ward turning direction the typhoon track.
  2. Capture the slow typhoon movement speed when approaching Taiwan

Typhoon prediction: typhoon intensity



- The typhoon intensity can be also spun-up by the LETKF-RIP
- The advantage is still valid for the 24-hour forecast

*Data Assimilation of the Global Ocean using 4D-LETKF and MOM2

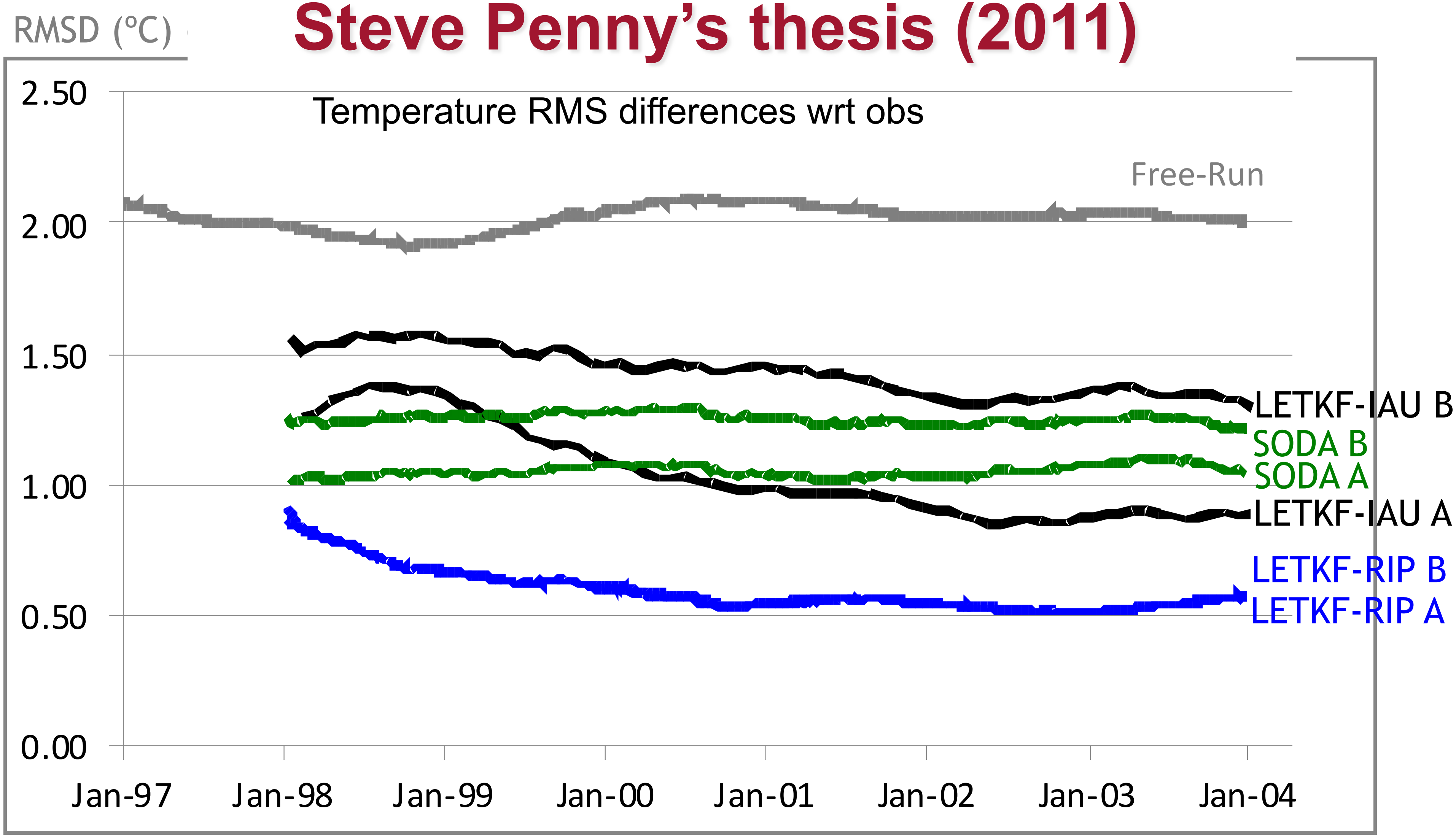
Steve Penny's defense

April 15, 2011

With:

- Eugenia Kalnay
- Jim Carton
- Brian Hunt
- Kayo Ide
- Takemasa Miyoshi
- Gennady Chepurin

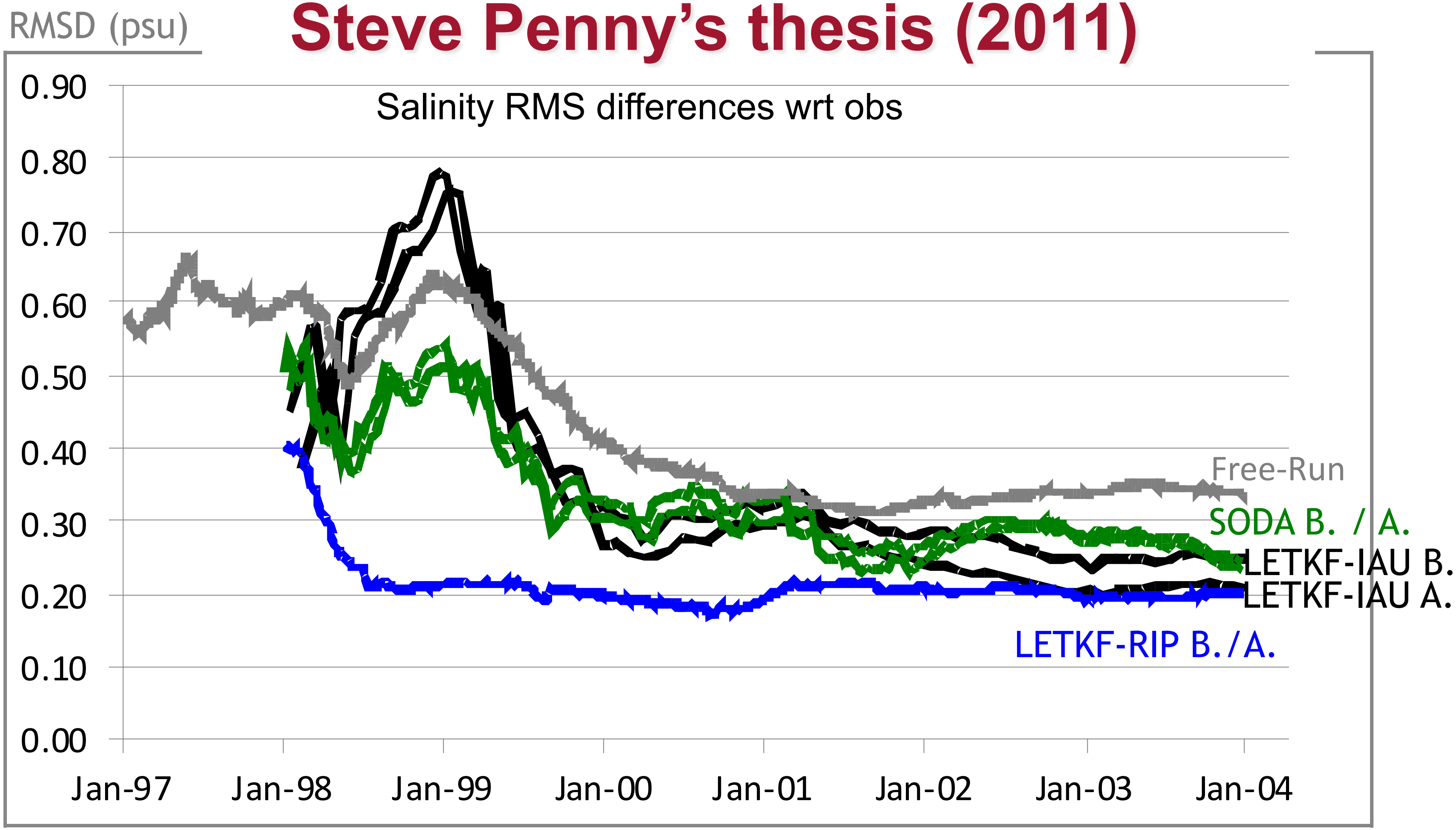
Steve Penny's thesis (2011)



Ocean Reanalysis (7 years):

LETKF-IAU, LETKF-RIP, compared with SODA (OI)
12-month running mean

Steve Penny's thesis (2011)



Ocean Reanalysis (7 years):

LETKF-IAU, LETKF-RIP, compared with SODA (OI)

12-month running mean

Summary

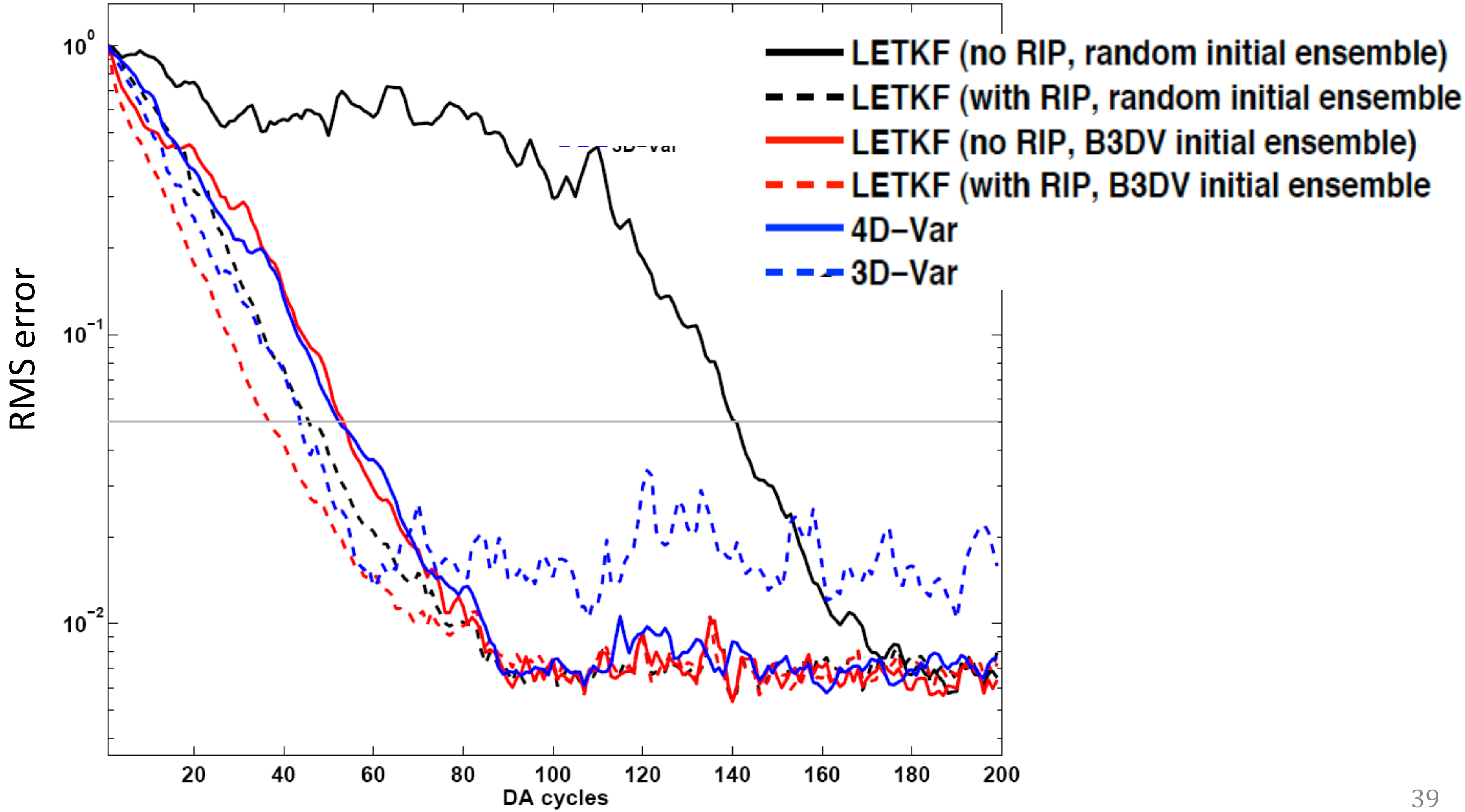
- As in the variational methods, an outer-loop with LETKF (EnKF) allows to improve the nonlinear evolution of the background trajectory and better fit the observations.
  - Both the RIP and QOL methods are able to improve the nonlinearity of the model trajectory, so less non-Gaussian distribution occurs.
- Despite violating the Kalman Filter rule that observations should be used only once, the QOL and RIP methods are clearly very successful to use observation more than once.
  - Multi-step analysis correction with small ensemble spread
- The RIP analysis is actually more accurate than the EnRML analysis, a iterative EnKF based on Gauss-Newton minimization.

Running in place (Kalnay and Yang, 2011)

- During the spin-up, we propose to use the observations repeatedly “ONLY IF” we could extract extra information. But we should avoid overfitting the observations.
- With RIP, we improve both the accuracy of the mean state and the flow-dependent error structures.
- Elements for RIP
  - No-cost smoother (vs. adjoint model in 4D-Var)
  - An appropriate scheme to avoid over-fitting

RIP-LETKF with the QG model
(Kalnay and Yang, 2010)

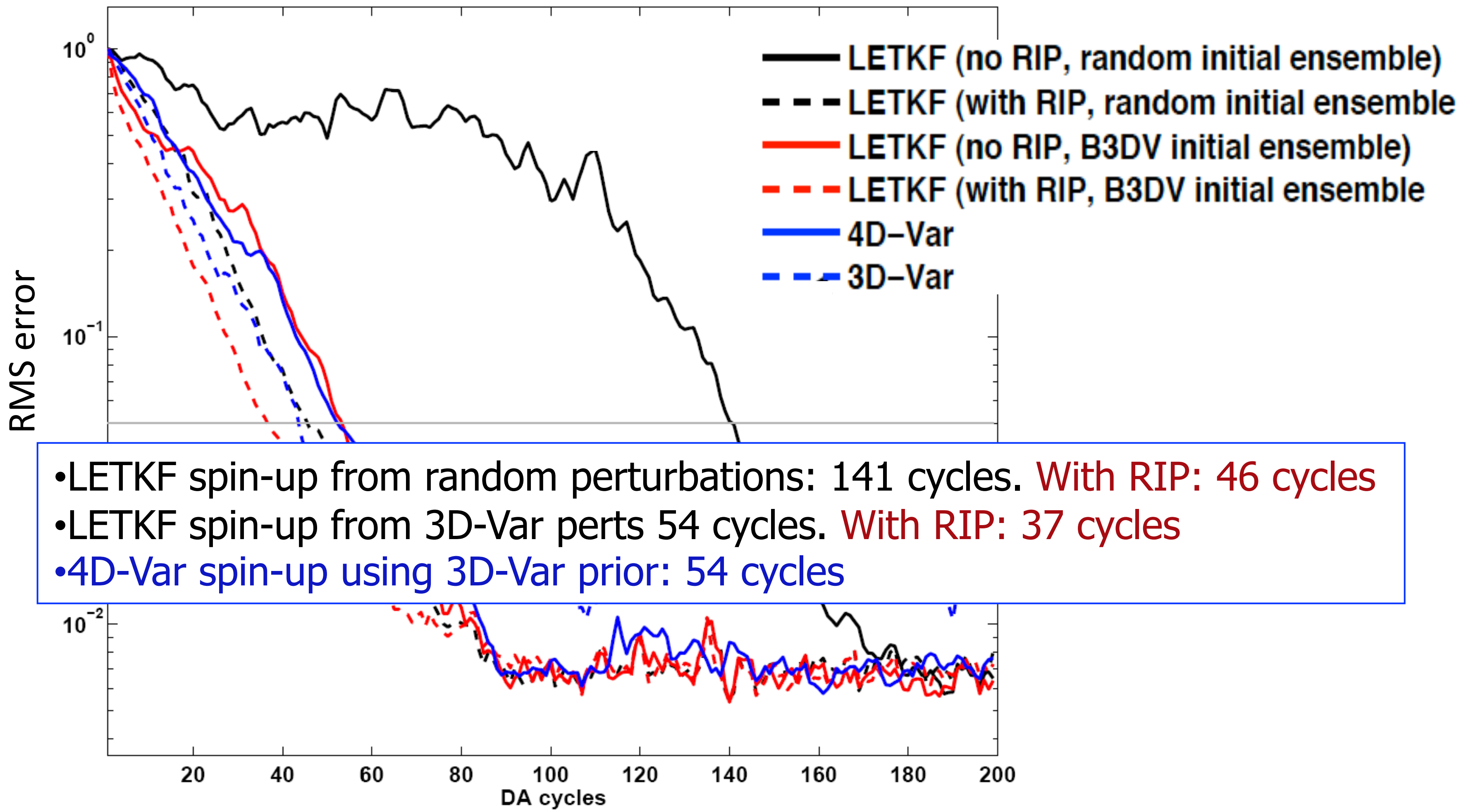
Analysis error of potential vorticity of a QG model



RIP-LETKF with the QG model

(Kalnay and Yang, 2009)

Analysis error of potential vorticity of a QG model



- LETKF spin-up from random perturbations: 141 cycles. With RIP: 46 cycles
- LETKF spin-up from 3D-Var perts 54 cycles. With RIP: 37 cycles
- 4D-Var spin-up using 3D-Var prior: 54 cycles