Sensitivity Analysis in Atmospheric Data Assimilation

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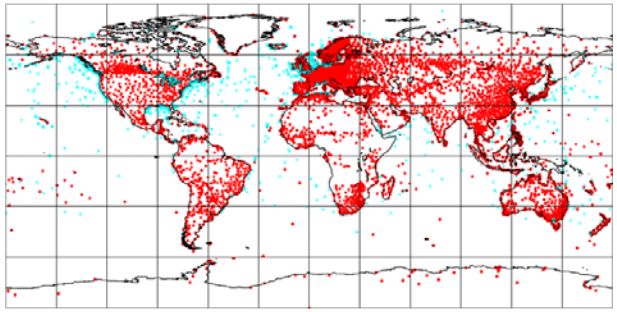
The Observing System

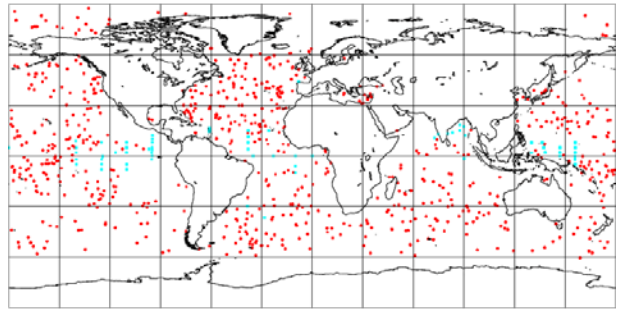
09–15 UTC

5 September 2003

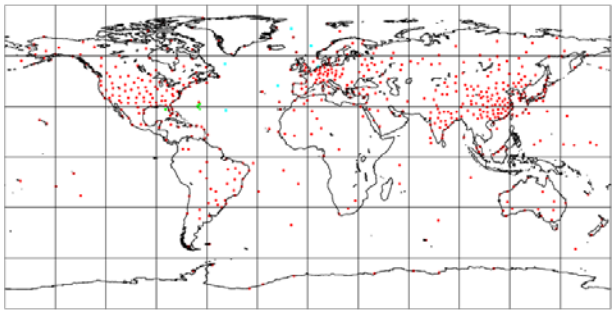
Synops and Ships



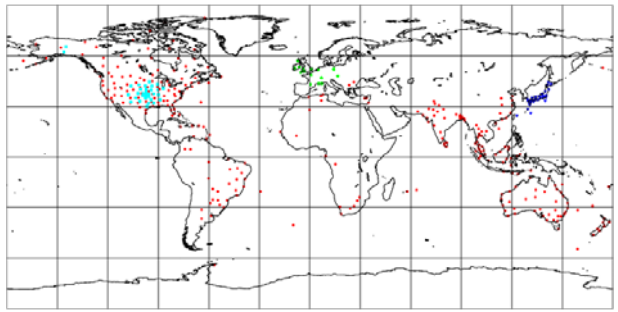
Buoys



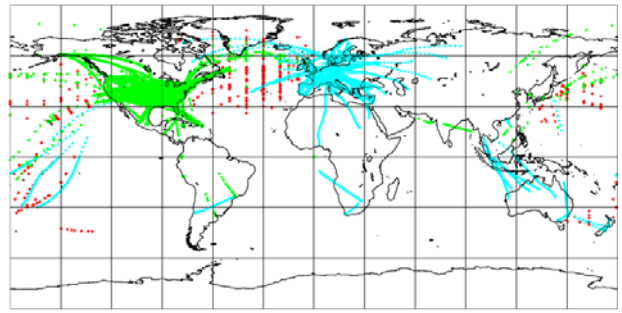
Radiosondes



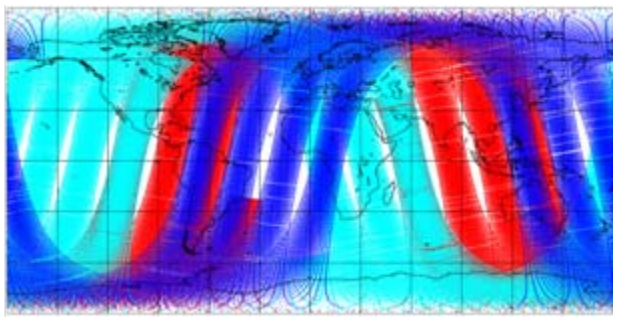
Pilots and Profilers



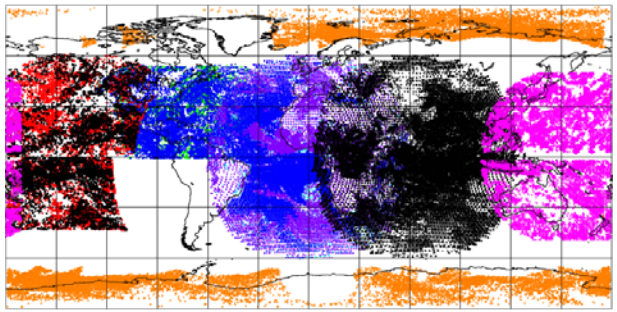
Aircraft



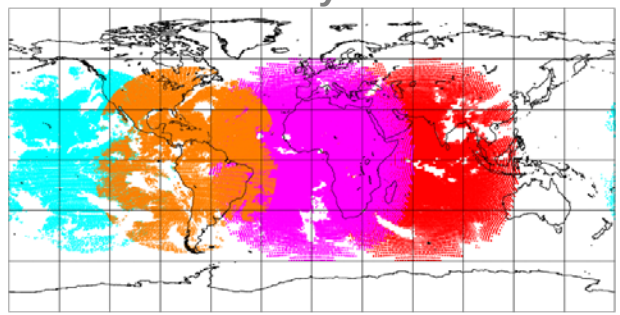
ATOVS



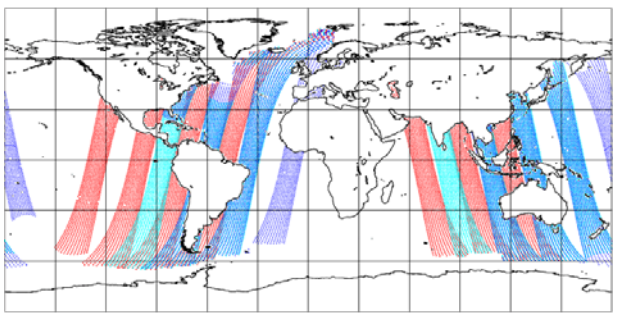
Satwinds



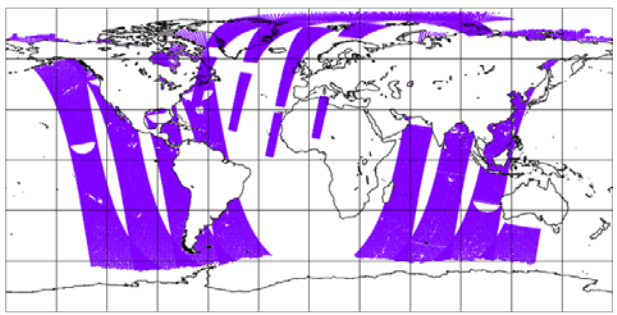
Geostationary Radiances



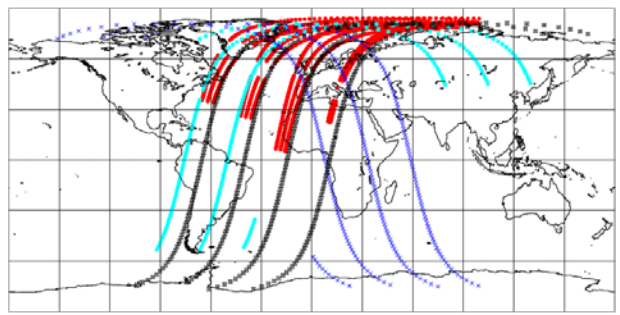
SSM/I



Scatterometer



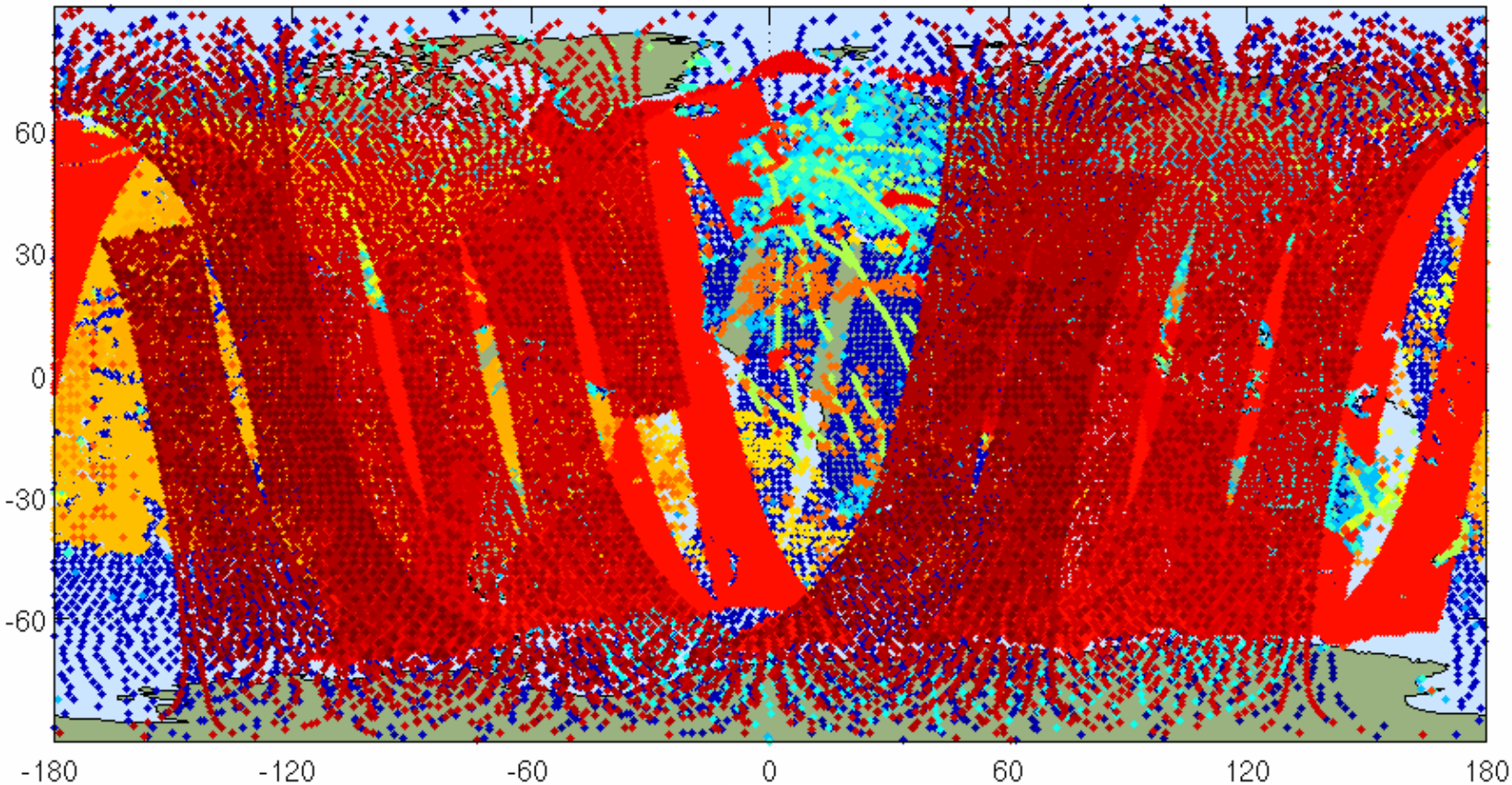
Ozone



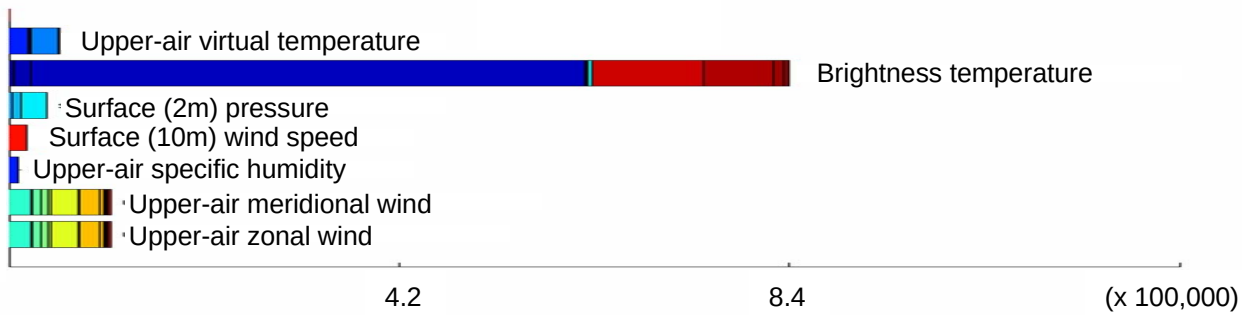
The observing system

FVGSI 16-Jan-2003 00UTC

All data: 1,178,200 observations



Data Types



Status of satellite data usage in operational NWP

- Satellite observations account for ~90% of all data assimilated in operational data assimilation/forecast systems

- These data account for << 1% of the *available* observations from lower Earth-orbiting systems

- The gap between the number of available and assimilated satellite observations is likely to widen significantly over the next decade

⇒ Intelligent strategies for data selection and usage must be developed

⇒ Flexible and efficient tools for ascertaining the "importance" of observations are required...

Adjoint sensitivity analysis

…Estimate the response of model output to possible perturbations of model input

1. Consider a model: $\mathbf{x}_\mathrm{f} = M(\mathbf{x}_\mathrm{a})$

   and a (differentiable) scalar forecast measure: $J = J(\mathbf{x}_\mathrm{f})$

2. Determine $\partial J / \partial x_{\mathrm{a}_i}$ such that:

$$\delta J = \sum_i \frac{\partial J}{\partial x_{\mathrm{a}_i}} x'_{\mathrm{a}_i} \quad \text{approximates} \quad \Delta J = J(\mathbf{x} + \mathbf{x}') - J(\mathbf{x})$$

3. The solution is: $$\frac{\partial J}{\partial x_{\mathrm{a}_i}} = \sum_j \frac{\partial x_{\mathrm{f}_j}}{\partial x_{\mathrm{a}_i}} \frac{\partial J}{\partial x_{\mathrm{f}_j}}$$

or $$\frac{\partial J}{\partial \mathbf{x}_\mathrm{a}} = \mathbf{M}^\mathrm{T} \frac{\partial J}{\partial \mathbf{x}_\mathrm{f}}$$

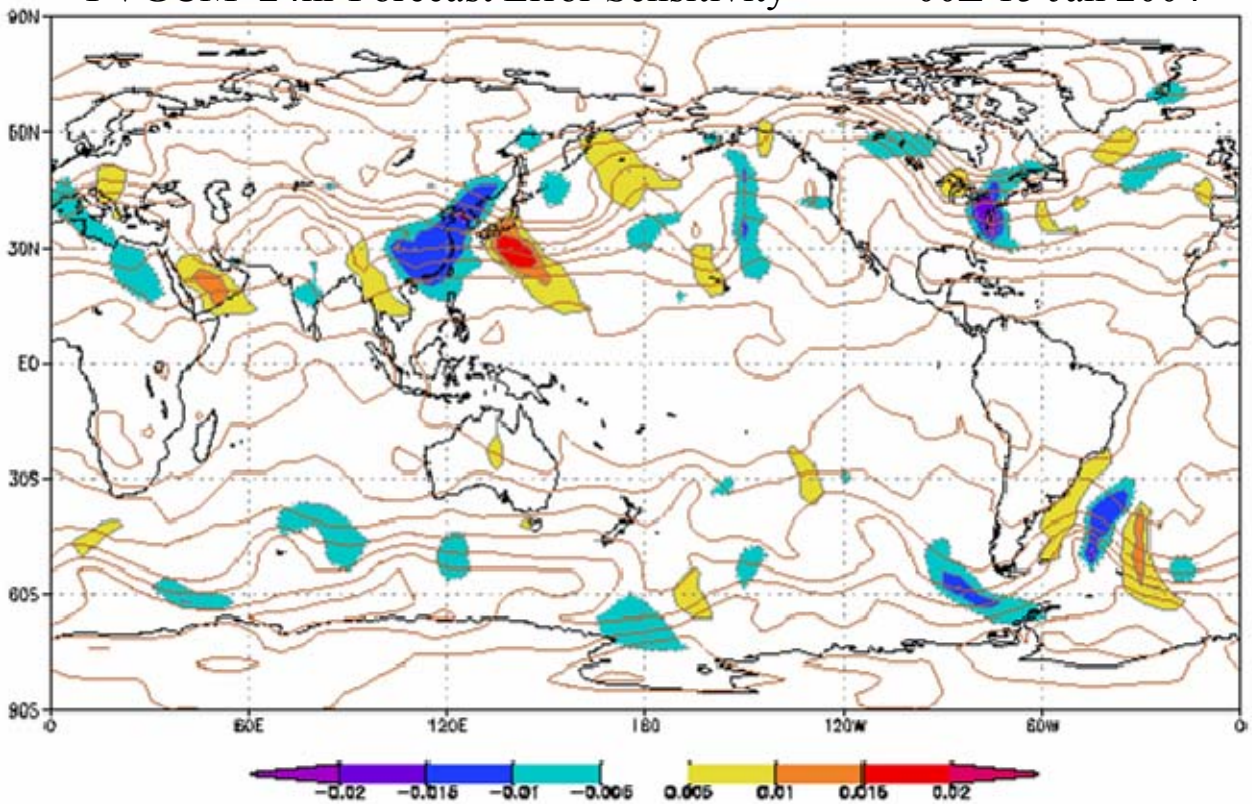
Sensitivity of J w.r.t the analysis

Adjoint model (transpose of $\mathbf{M}$*)*

Example adjoint sensitivity calculation

FVGCM adjoint model with simple physics

FVGCM 24hr Forecast Error Sensitivity 00Z 15 Jan 2004



$$J = \frac{1}{2}\mathbf{e}_{24}^{\mathrm{T}}\,\mathbf{E}\,\mathbf{e}_{24}$$

$$\frac{\partial J}{\partial \theta_{\mathrm{a}}}$$ (shaded)

θ_{a} basic state (contour)

Applications: predictability, system monitoring, parameter estimation, adaptive observing…

Data assimilation adjoint theory (*Baker and Daley, 2000*)

Analysis problem:

$$\mathbf{x}_a = \mathbf{x}_b + \mathbf{K}\left(\mathbf{y} - H(\mathbf{x}_b)\right)$$

where $\mathbf{K} = \mathbf{P}_b\mathbf{H}^T(\mathbf{H}\mathbf{P}_b\mathbf{H}^T + \mathbf{R})^{-1}$

Linear analysis problem:

$$\mathbf{x}_a = \mathbf{x}_b + \mathbf{K}(\mathbf{y} - \mathbf{H}\mathbf{x}_b) = \mathbf{K}\mathbf{y} + (\mathbf{I} - \mathbf{K}\mathbf{H})\mathbf{x}_b$$

Sensitivity of the analysis to the observations:

$$\frac{\partial \mathbf{x}_a}{\partial \mathbf{y}} = \mathbf{K}^T$$

where $\mathbf{K}^T = (\mathbf{H}\mathbf{P}_b\mathbf{H}^T + \mathbf{R})^{-1}\mathbf{H}\mathbf{P}_b$

See also Le Dimet et al. 1995, Fourrie et al. 2002, Cardinali 2004(?)

Now, relate the sensitivity of the forecast measure to the observations:

We have the sensitivity of the forecast measure to the analysis:

$$\frac{\partial J}{\partial \mathbf{x}_{\mathrm{a}}} = \mathbf{M}^{\mathrm{T}} \frac{\partial J}{\partial \mathbf{x}_{\mathrm{f}}} \qquad (1)$$

…and the sensitivity of the analysis to the observations:

$$\frac{\partial \mathbf{x}_{\mathrm{a}}}{\partial \mathbf{y}} = \mathbf{K}^{\mathrm{T}} \qquad (2)$$

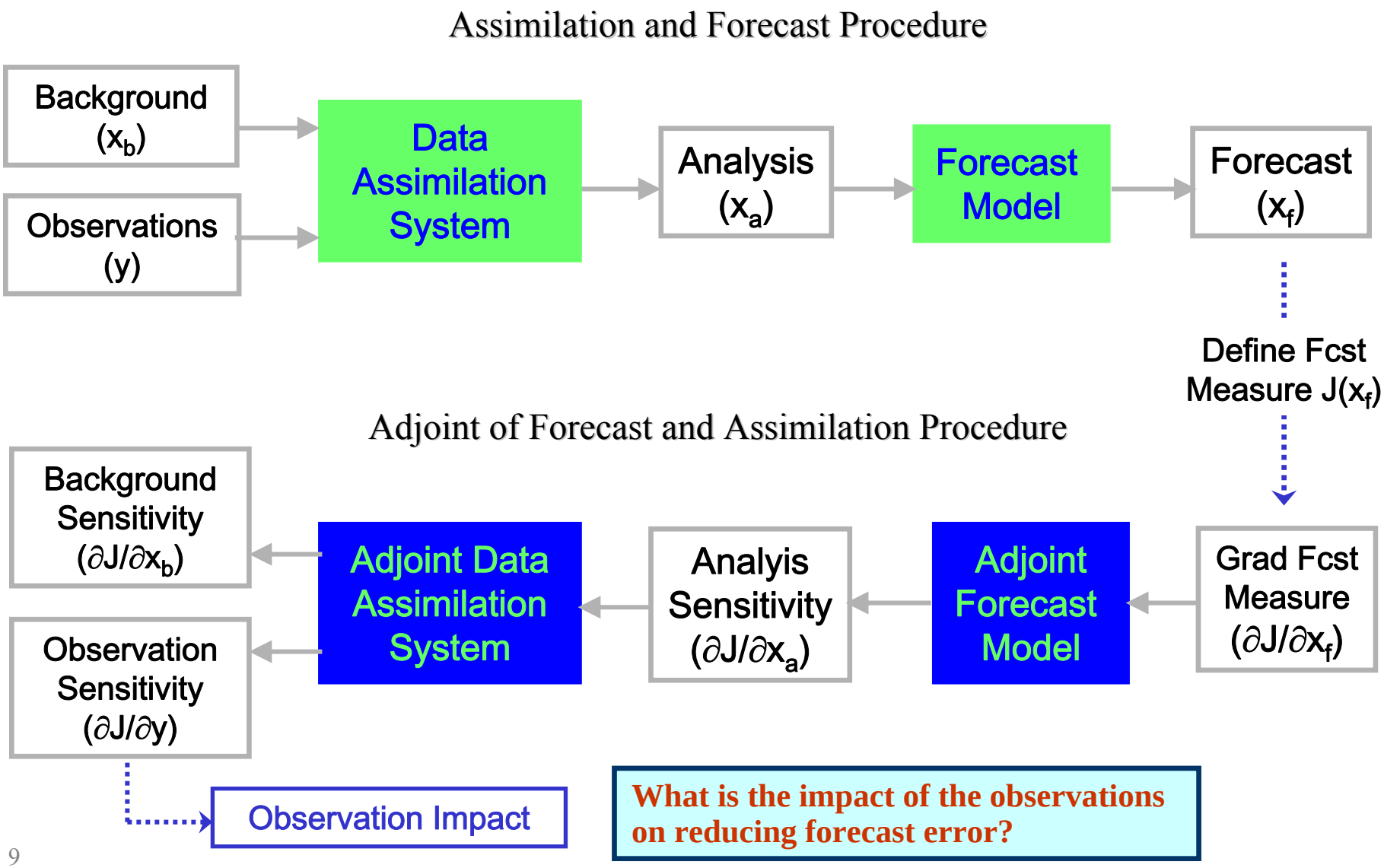
Combine (1) and (2) to get the sensitivity of the forecast measure to the observations:

$$\frac{\partial J}{\partial \mathbf{y}} = \frac{\partial \mathbf{x}_{\mathrm{a}}}{\partial \mathbf{y}} \frac{\partial J}{\partial \mathbf{x}_{\mathrm{a}}}$$

or $$\frac{\partial J}{\partial \mathbf{y}} = \mathbf{K}^{\mathrm{T}} \mathbf{M}^{\mathrm{T}} \frac{\partial J}{\partial \mathbf{x}_{\mathrm{f}}}$$

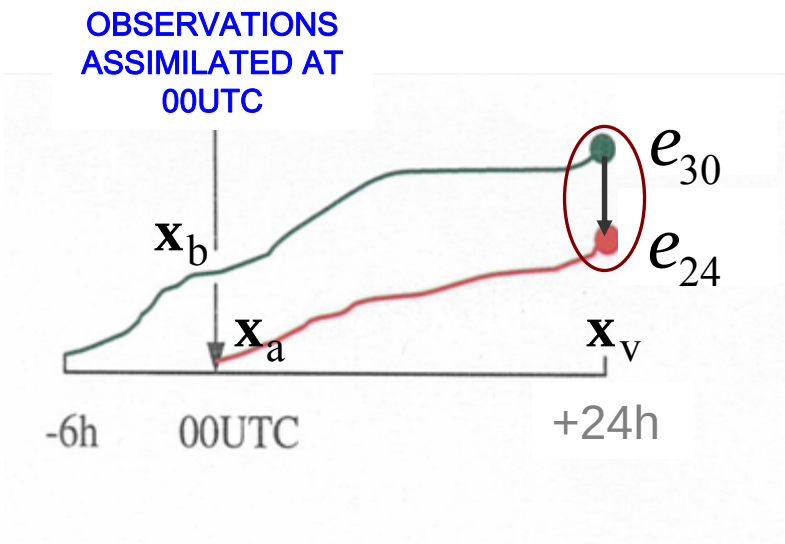
(observation space) *(model space)*

"End-to-End" Sensitivity in a Data Assimilation-Forecast System



Impact of observations on forecast error

Langland and Baker (2004)



The forecast error difference, $e_{24} - e_{30} = \Delta e_{24}^{30}$, is due entirely to the assimilation of observations at 00UTC

$\Rightarrow$ We seek an estimate of Δe_{24}^{30} in terms of sensitivity gradients in observation space…

Observation impact equation *(Langland and Baker, 2004)*

1. Define the energy-weighted scalar error, e.g.:

$$e_{24} = (\mathbf{x}_{24} - \mathbf{x}_{\mathrm{v}})^{\mathrm{T}} \mathbf{E} (\mathbf{x}_{24} - \mathbf{x}_{\mathrm{v}})$$

and forecast measure: $J_{24} = \frac{1}{2} e_{24}$

2. Express Δe_{24}^{30} in terms of gradients at verification time (*exact!*):

$$\Delta e_{24}^{30} = \left\langle (\mathbf{x}_{24} - \mathbf{x}_{30}), \frac{\partial J_{24}}{\partial \mathbf{x}_{24}} + \frac{\partial J_{30}}{\partial \mathbf{x}_{30}} \right\rangle$$

(model space)

3. Apply $\mathbf{M}^{\mathrm{T}}$ then $\mathbf{K}^{\mathrm{T}}$ to obtain the approximate expression at initial time:

$$\Delta e_{24}^{30} \approx \delta e_{24}^{30} = \left\langle (\mathbf{y} - \mathbf{H}\mathbf{x}_{\mathrm{b}}), \mathbf{K}^{\mathrm{T}} \left[\frac{\partial J_{24}}{\partial \mathbf{x}_{\mathrm{a}}} + \frac{\partial J_{30}}{\partial \mathbf{x}_{\mathrm{b}}} \right] \right\rangle$$

(observation space)

Results from *Langland and Baker, 2004*

Diagnosis of observation impact on short-range forecast errors in the Navy's operational DAS/forecast system

The analysis procedure:

NAVDAS: NRL Atmospheric Variational Data Assimilation System (3D, 1° lat-lon, 60 levels)

$\Rightarrow$ Adjoint excludes moist observations, ozone

The forecast model:

NOGAPS: Navy Operational Global Atmospheric Prediction System (T79L30)

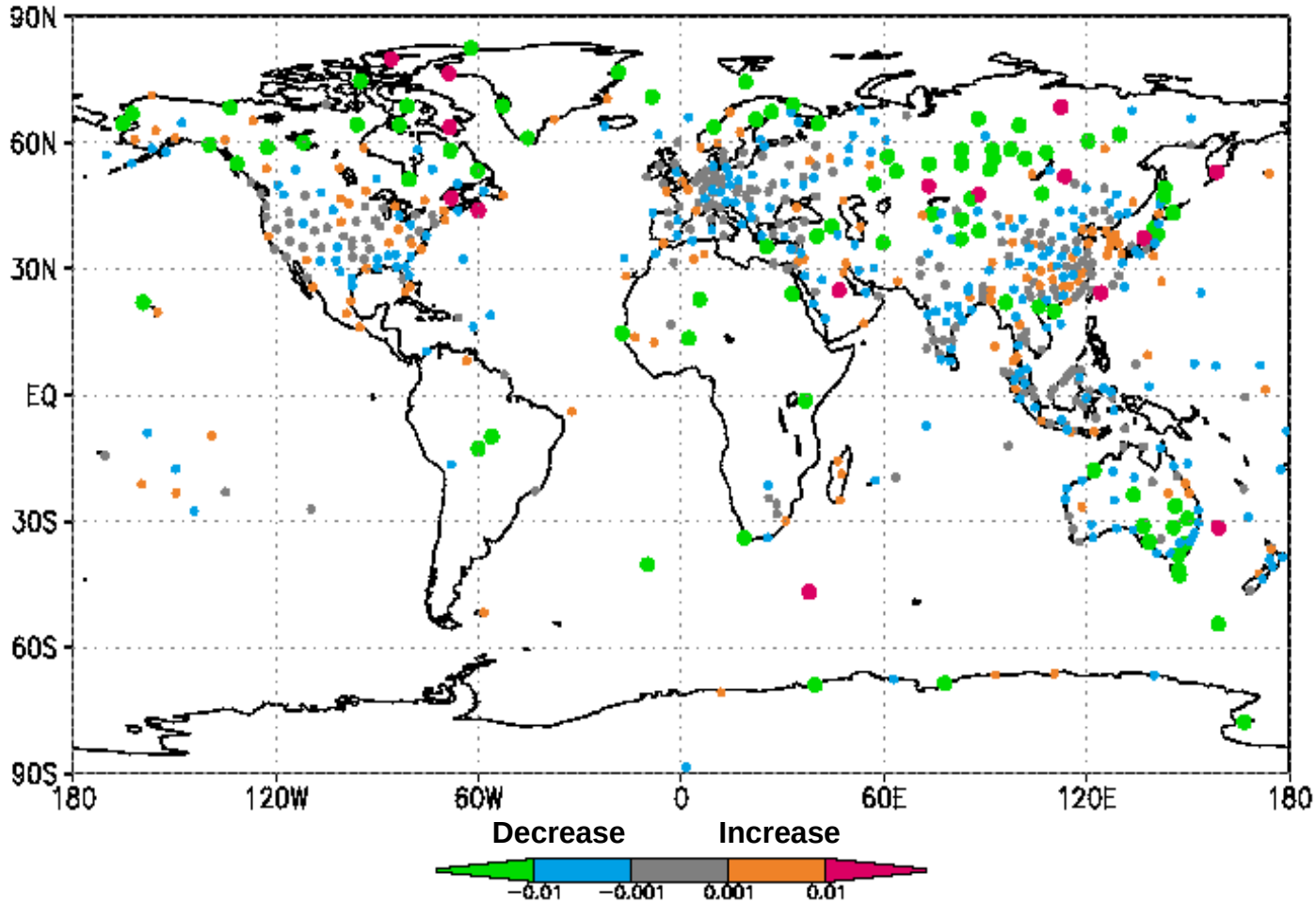
$\Rightarrow$ Adjoint excludes moist physics

Langland and Baker, 2004

Observation impact on $e_{24} - e_{30}$

00UTC 10 Dec 2002

Rawinsonde Profiles



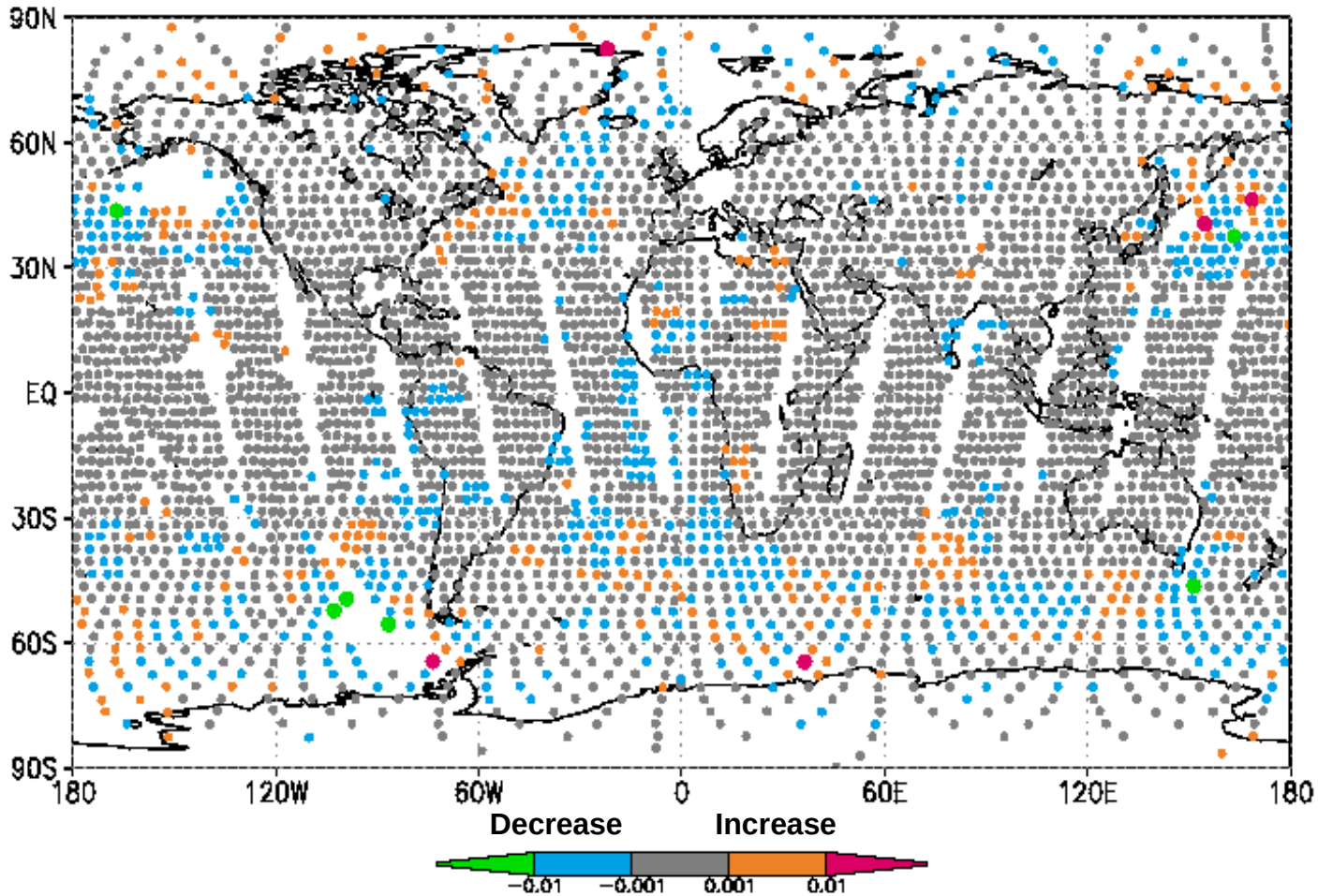
Total Impact = -2.05 J/kg

Langland and Baker, 2004

Observation impact on $e_{24} - e_{30}$

00UTC 10 Dec 2002

ATOVS Temperature Retrievals



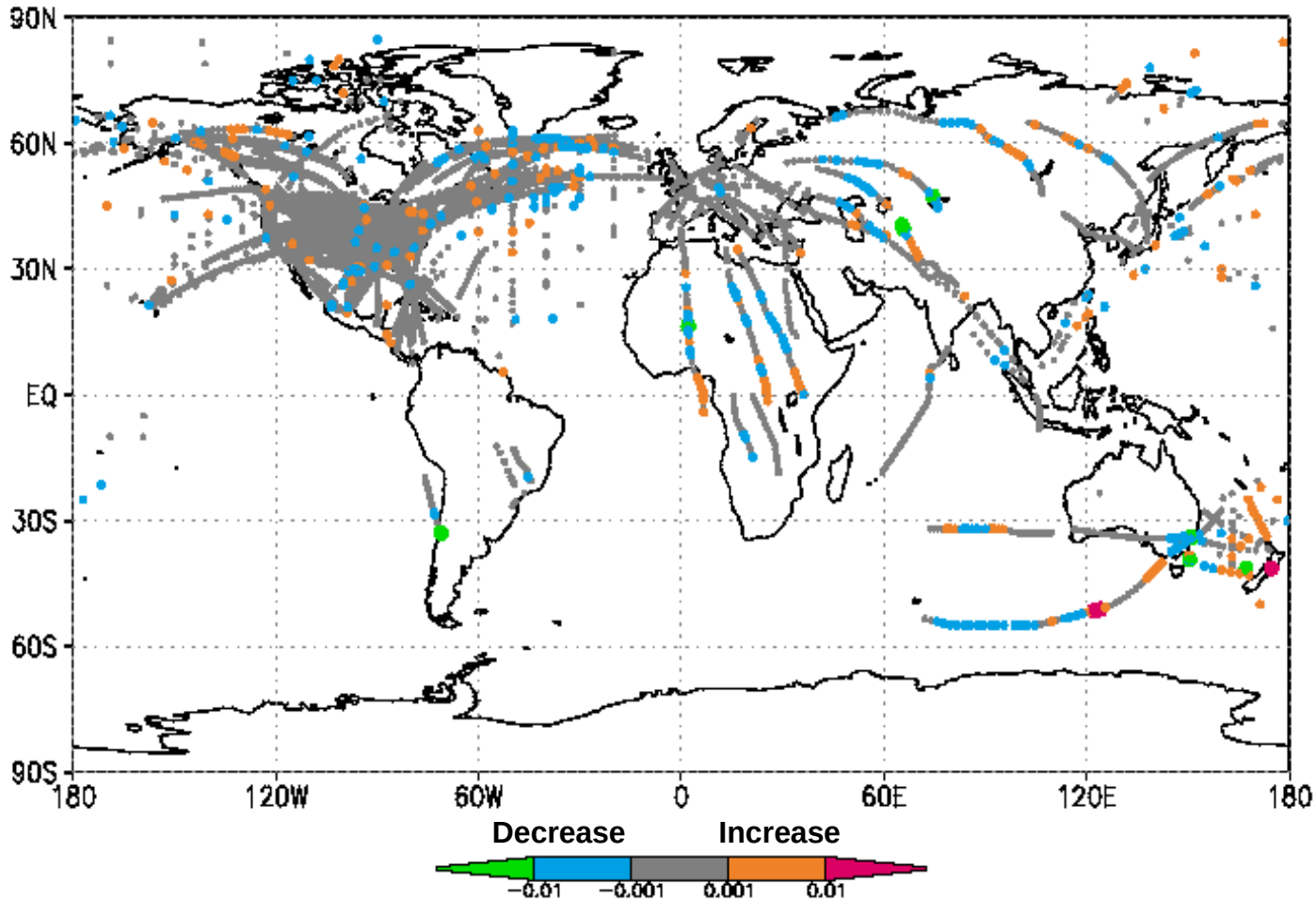
Total Impact = -1.06 J/kg

Langland and Baker, 2004

Observation impact on $e_{24} - e_{30}$

00UTC 10 Dec 2002

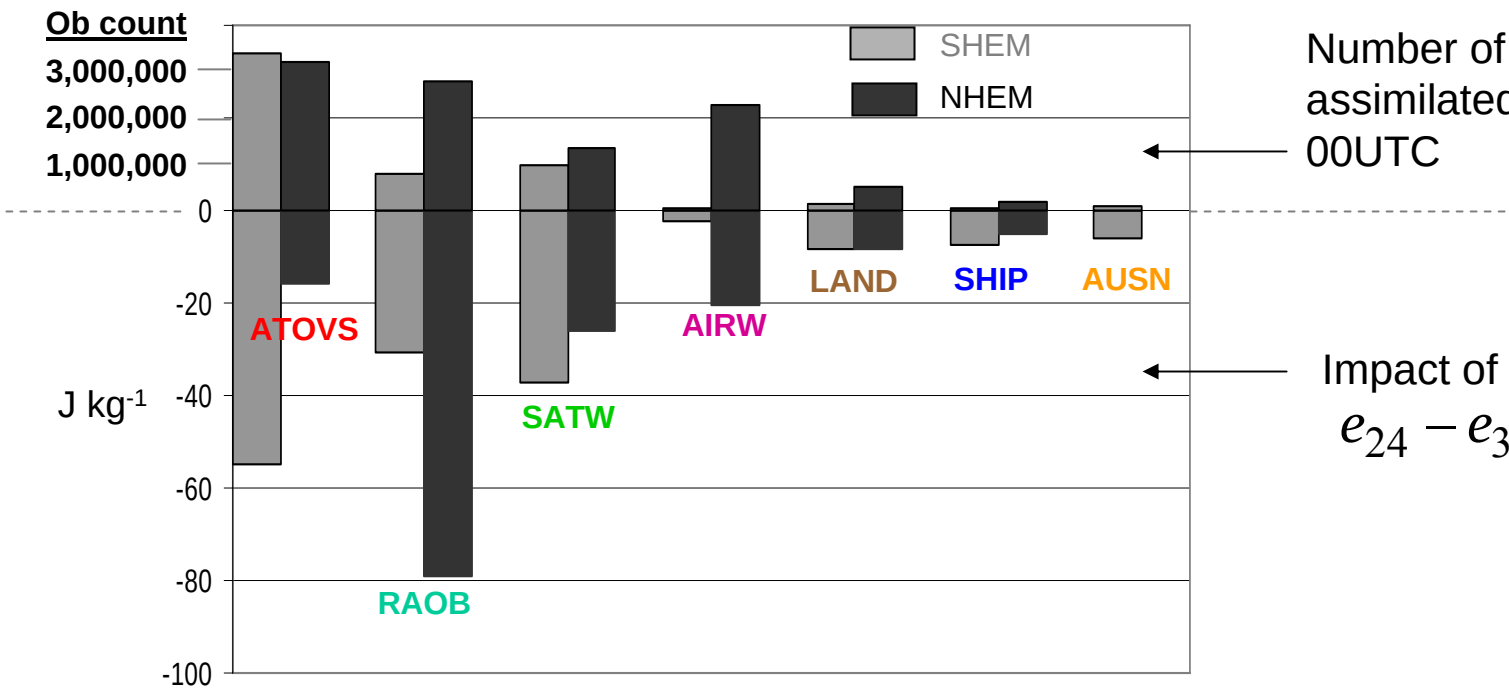
Commercial Aircraft Observations



Total Impact = -0.36 J/kg

Langland and Baker, 2004

Observation impact – by hemisphere



Number of data assimilated at 00UTC

Impact of data on $e_{24} - e_{30}$

June and December 2002

Observation impact – cloud cover correlation

…large impact of observations in cloudy regions….both positive and negative...

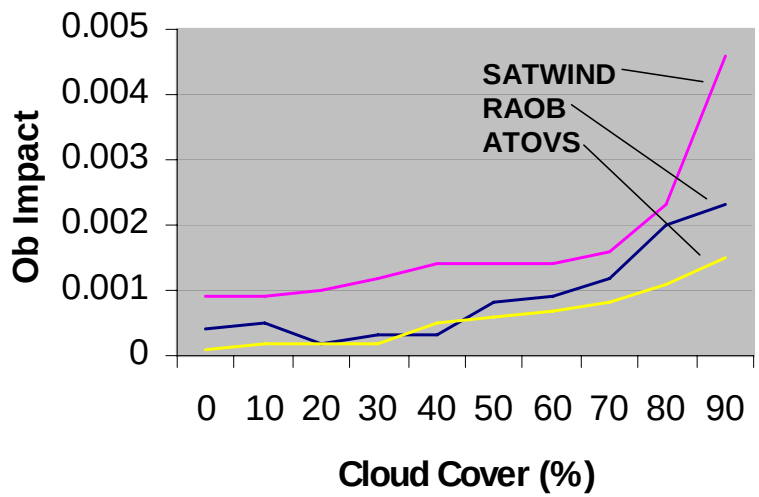


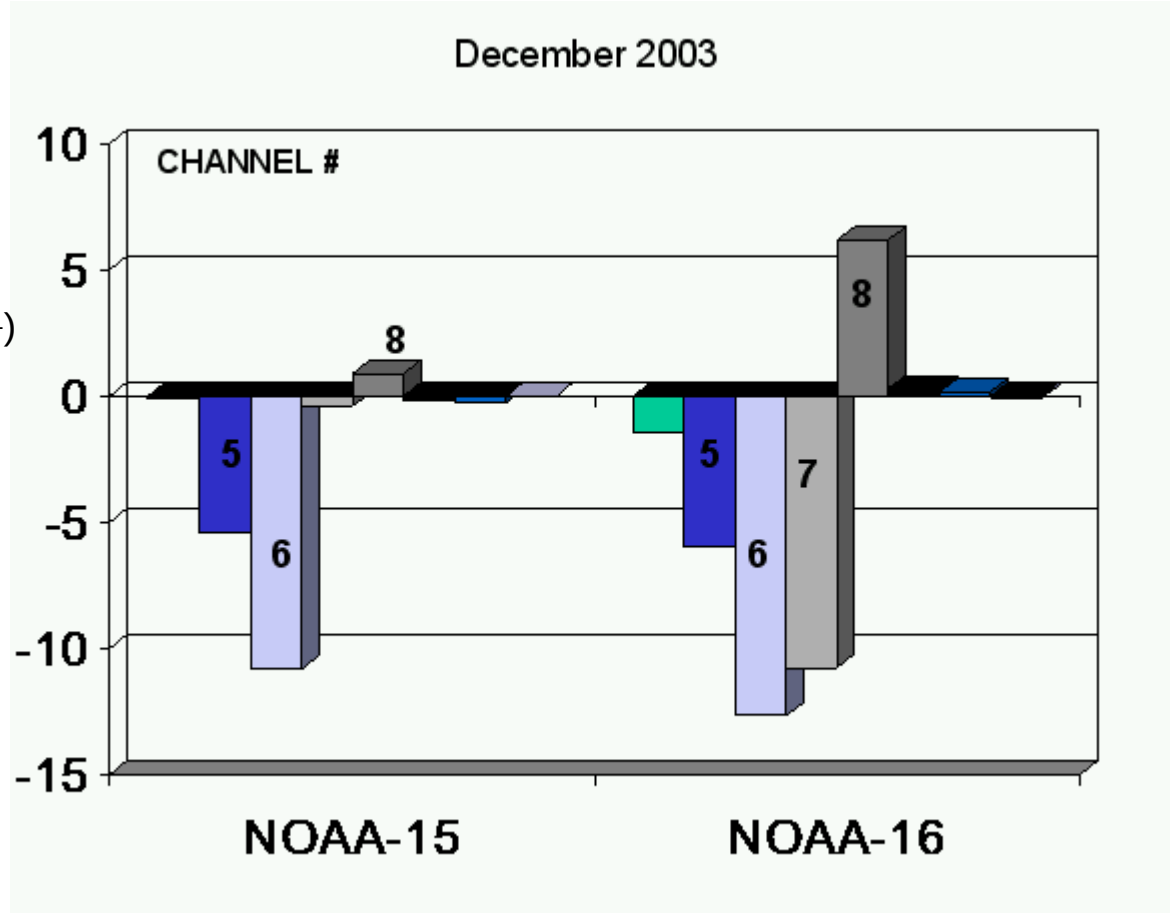
Fig. 4: Observation impact (average magnitude per observation, in J kg^{-1}) as a function of model-diagnosed cloud-cover. The "impact" in this figure includes both improvements and degradations of 72h global forecast error. Based on results from 29 June – 28 July 2002.

Observation impact – assimilation system tuning

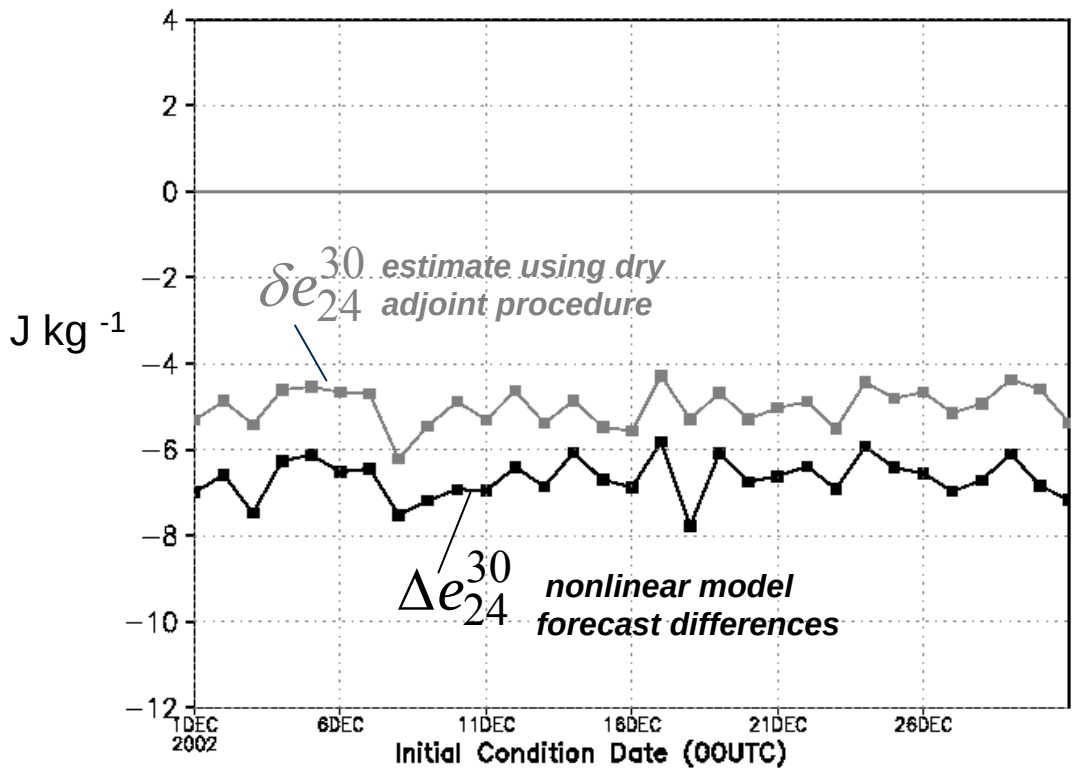
Error Reduction (J kg^{-1})

$$\delta e_{42}^{48}$$

Global Domain



Accuracy of observation impact estimate



December 2002

δe_{24}^{30} explains 75% of Δe_{24}^{30}

Remaining 25 %:

- moist observations?
- adjoint model physics?
- nonlinearity?

Nonlinear analysis problems: GSI

…Consider the incremental variational problem solved in the NCEP grid-point statistical interpolation (GSI) scheme

Minimize: $L(\delta\mathbf{x}) = L(\delta\mathbf{x}) + L_{q1} + L_{q2}$

$$L(\delta\mathbf{x}) = \delta\mathbf{x}^{\mathrm{T}}\mathbf{B}^{-1}\delta\mathbf{x} + (H(\delta\mathbf{x}) - \mathbf{d})^{\mathrm{T}}\mathbf{R}^{-1}(H(\delta\mathbf{x}) - \mathbf{d})$$

$$L_{q1} = \begin{cases} 0 & q \geq 0 \\ \rho_1^{\mathrm{T}} q^2 & q < 0 \end{cases}$$

…penalty for negative humidity

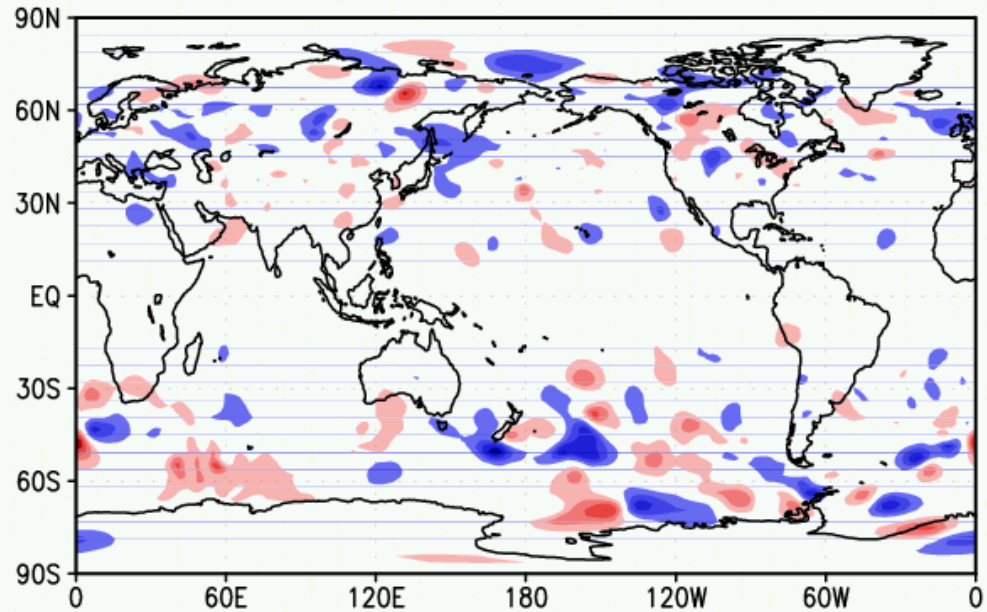
$$L_{q2} = \begin{cases} 0 & q \leq q_{sat} \\ \rho_2^{\mathrm{T}} (q - q_{sat})^2 & q > q_{sat} \end{cases}$$

…penalty for super saturation

Development of adjoint GSI requires development of tangent linear algorithm as first step…

Tangent linear behavior of the GSI algorithm

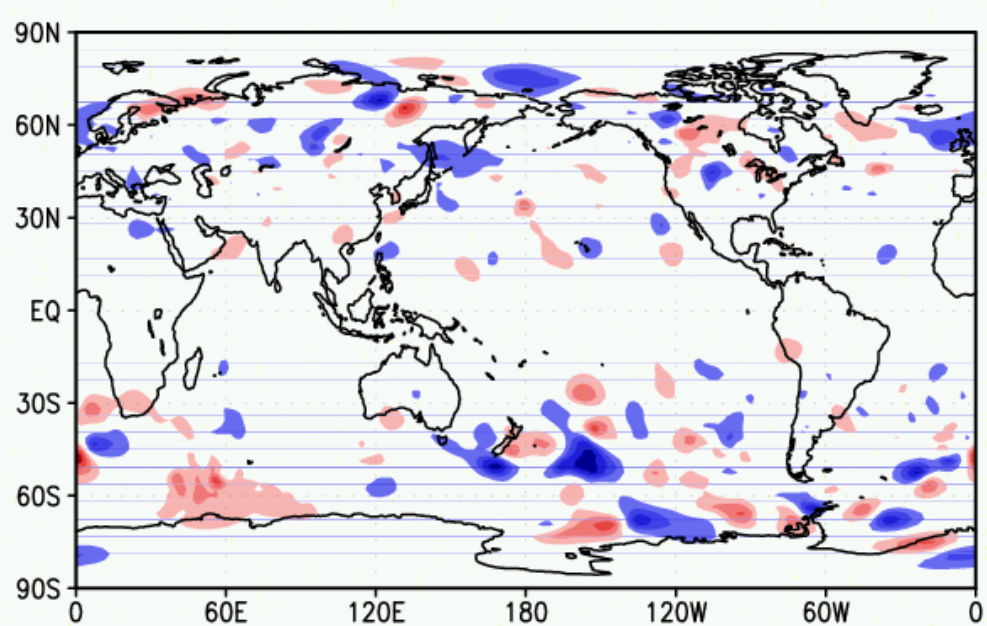
GSI TLM



Perturb: all u,v (O-F x 0.1)
Response: T_v (level 10)

100 iterations

Δ GSI



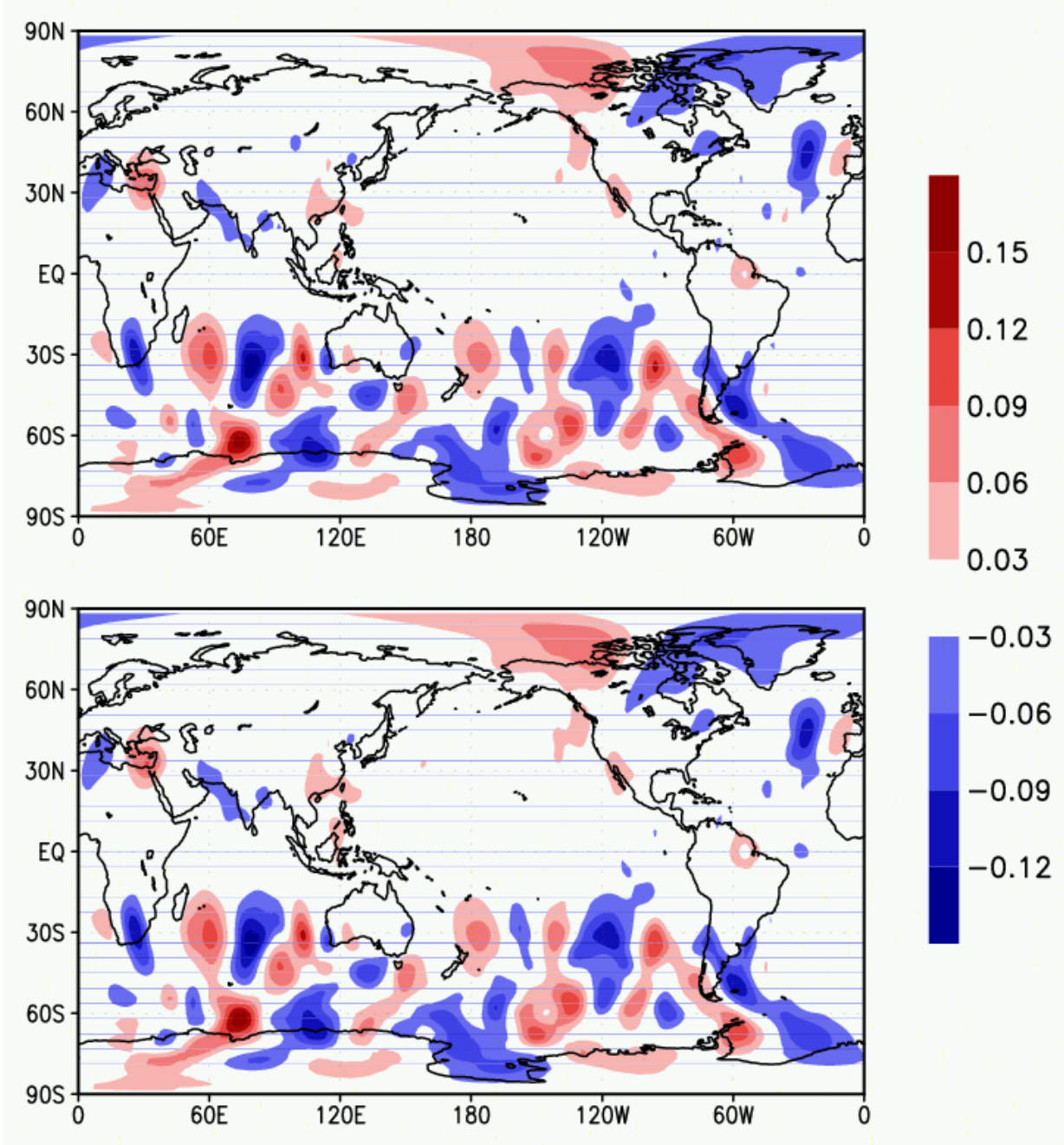
Tangent linear behavior of the GSI algorithm

GSI TLM

Perturb: all Rad (O-F x 0.1)
Response: v (level 20)

100 iterations

Δ GSI



Tangent linear behavior of the GSI algorithm

$$\text{Ratio} = \frac{rms\ (\text{TLM})}{rms\ (\Delta\ \text{GSI})}$$

Pert = all u,v (O-F x 0.01)

level=15, outerloop=1

q-penalty ON

iter	10	20	30	40	50	60	70	80	90	100
u	1.00100	0.99871	0.99675	0.80302	1.01344	0.88238	1.00910	4.42178	1.48489	0.95633
v	1.00018	0.99847	0.99741	0.80169	0.99440	0.90217	1.00523	4.30072	1.47929	0.95833
tv	1.00520	0.99886	0.99898	0.65986	0.92776	0.50994	0.98996	8.00009	2.88145	0.80390
q	1.03733	0.97618	0.94475	0.58107	0.46036	0.14779	0.88259	10.64569	9.31290	0.38858
oz	1.03846	0.92288	0.82636	0.56650	0.20387	4.59494e-2	0.15890	10.81975	12.45908	0.27507
sf	1.00513	0.99938	0.99722	0.71599	0.96000	0.73430	1.00613	5.95872	1.98929	0.92879
vp	0.99732	0.99763	0.99504	0.84753	1.08547	1.36717	1.03021	3.72594	1.47548	1.06970
lps	1.00118	0.99854	0.99963	0.79686	0.94238	0.67737	0.98866	6.72151	1.90837	0.92701
sst	1.02812	0.99209	0.98327	0.65130	0.92299	0.52509	1.02049	8.39484	3.26917	0.80348

q-penalty OFF

iter	10	20	30	40	50	60	70	80	90	100
u								0.99935		1.00134
v								1.00159		1.00277
tv								1.00280		0.99893
q								1.04508		0.84970
oz								1.48250		0.20984
sf								1.00016		0.99899
vp								0.99455		0.98998
lps								1.00049		0.99428
sst								1.00370		0.99429

Tangent linear behavior of the GSI algorithm

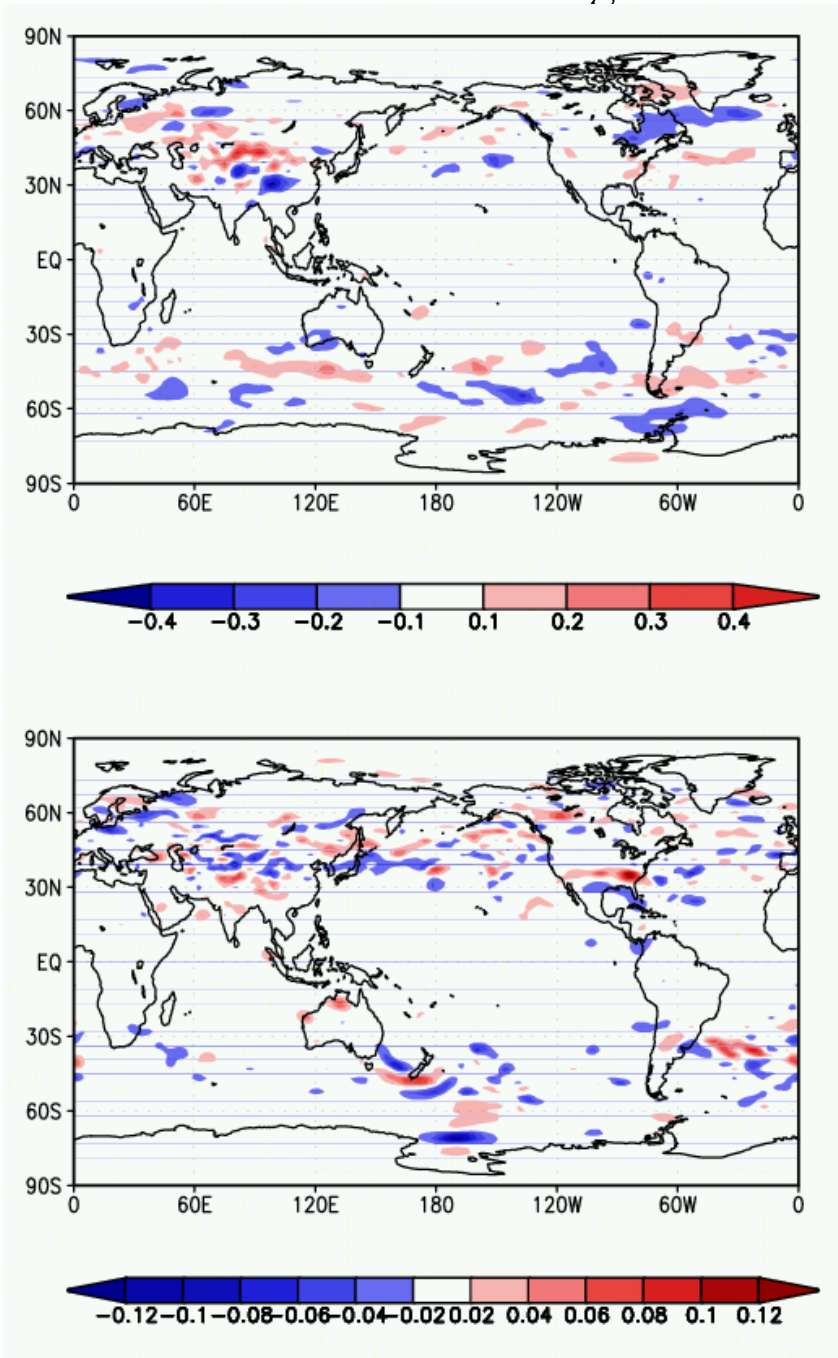
GSI TLM

q-penalty ON

Perturb: all u,v (O-F x 0.01)
Response: u (level 15)

80 iterations

Δ GSI



Tangent linear behavior of the GSI algorithm

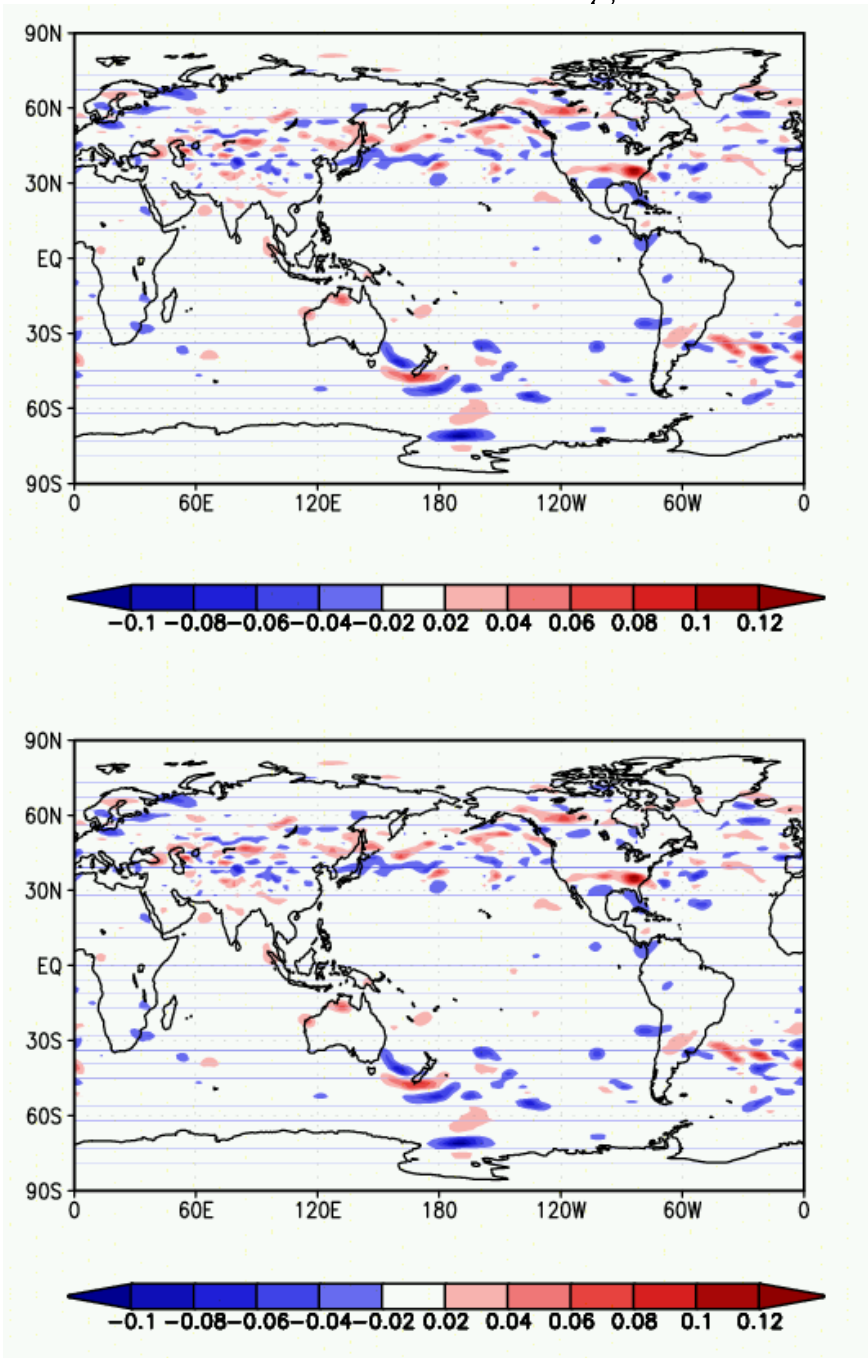
GSI TLM

q-penalty OFF

Perturb: all u,v (O-F x 0.01)
Response: u (level 15)

80 iterations

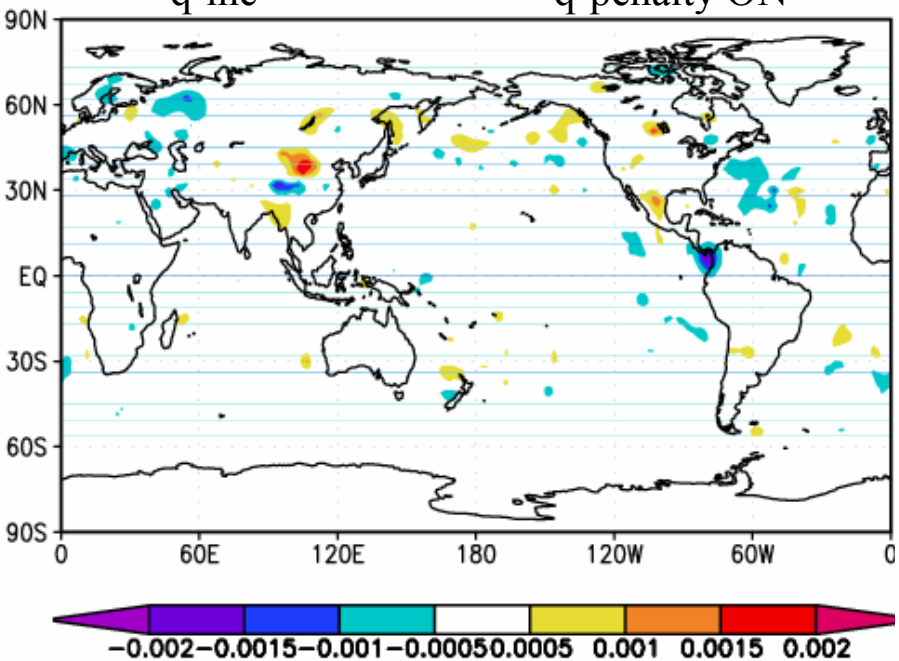
Δ GSI



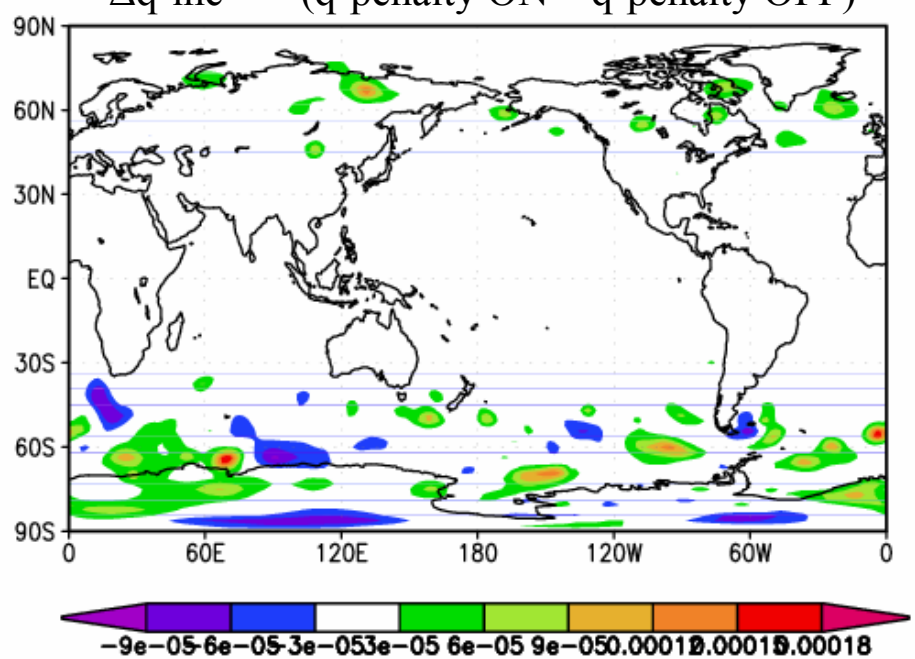
Effect of q-penalty on GSI increments

Specific humidity, 00Z 19Jun2004, level=10, outerloop=1, iteration=100

q-inc q-penalty ON



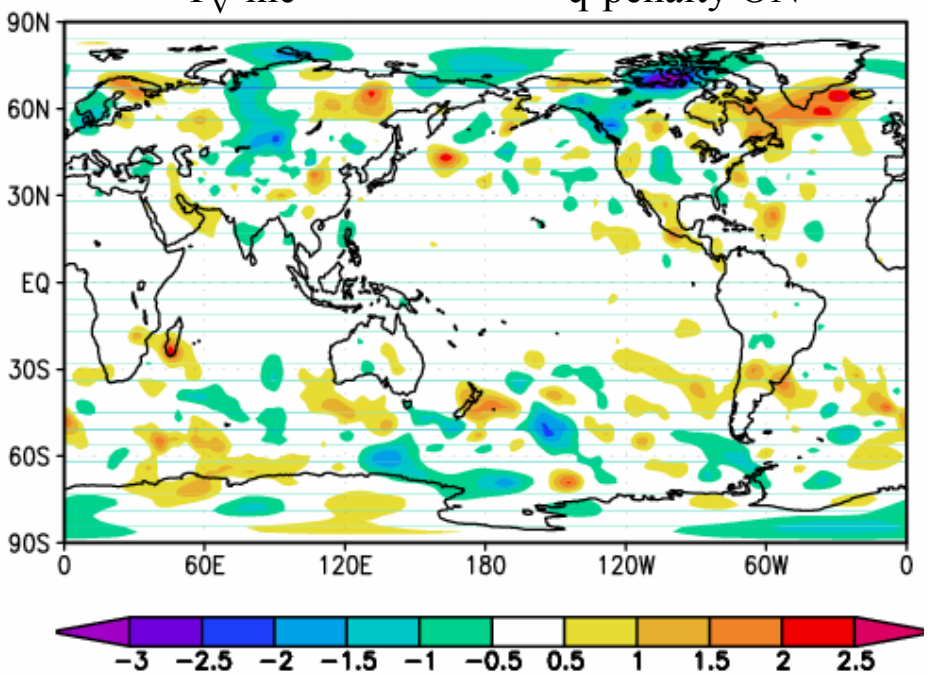
Δq-inc (q-penalty ON – q-penalty OFF)



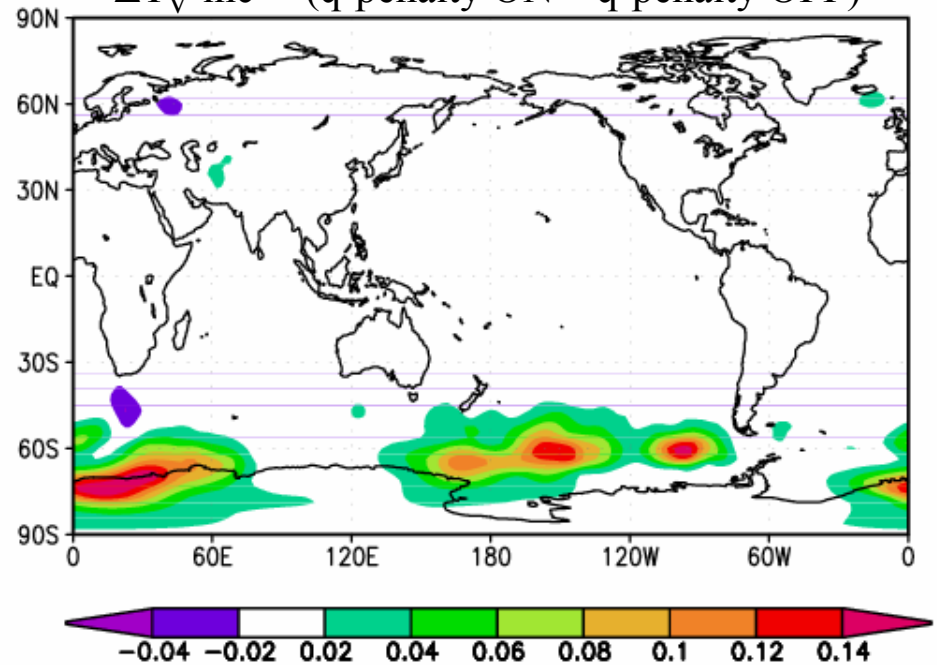
Effect of q-penalty on GSI increments

Virtual Temperature, 00Z 19Jun2004, level=10, outerloop=1, iteration=100

T_V-inc q-penalty ON



ΔT_V-inc (q-penalty ON – q-penalty OFF)



Status of GSI adjoint development

- Tangent linear version of GSI (autumn release) has been developed and tested extensively for a range of perturbation types...major step toward generation of adjoint.

- Tangent linear system is well behaved (representative of GSI) overall:
  - good to excellent results for perturbed observations of u, v, T, q, radiance
  - results improve, especially for q, when penalties for negative moisture/super saturation are removed or reduced
  - response to ozone perturbations poor (highly nonlinear), but significance is unclear

- Update to March release of GSI underway:
  - variational QC
  - begin consolidation of nonlinear and tangent codes

- Adjoint testing planned for early summer

General conclusions…so far…

- Adjoint forms of a model and assimilation system allow efficient estimation of analysis and/or observation sensitivity (impact)

  - determined with respect to observational data, background fields or assimilation parameters, all computed simultaneously
  - permits arbitrary aggregation of sensitivities, e.g., by data type, channel, location, etc.
  - information obtained should compensate pain of initial development

- Sensitivity information should be useful for designing intelligent data selection strategies, and possibly guide future observing system design

- Provide unique insight into model/assimilation system behavior and design

- Complement, but not replace, traditional techniques for estimating observation impact (OSEs, OSSEs)

1st order cience questions…

- Trade-offs between assimilating fewer observations with greater individual sensitivity, or more observations with less individual sensitivity

- How best to use observations in cloudy regions

- Whether/how sensitivity information can be utilized effectively for system tuning, e.g., observation error, bias estimation parameters,…

- Under what circumstances the required assumptions and simplifications render results unusable (simplified physics, linearity)

- Whether extremely nonlinear "switches" in forward systems are either necessary or appropriate