IMPLEMENTATION

$$\left(\frac{\partial}{\partial t}\right)_\theta \overline{\rho^{*\theta}}^{\theta} = -\left[\overline{\nabla_\theta \cdot \left(\rho^{*\theta}\,\hat{\mathbf{v}}^{\theta}\right)}^{\theta} - \frac{\partial}{\partial \theta}\overline{\rho^{*\theta}\,\dot{\theta}}^{\theta}\right] \qquad (1)$$

$$\left(\frac{\partial}{\partial t}\right)_q \overline{\rho^{*q}}^{q} = -\left[\overline{\nabla_q \cdot \left(\rho^{*q}\,\hat{\mathbf{v}}^{q}\right)}^{q} - \frac{\partial}{\partial q}\overline{\rho^{*q}\,\dot{q}}^{q}\right] \qquad (2)$$

- Implementation is especially simple if the host model uses an isentropic vertical coordinate because (1) is one of the model's prognostic equations.
- With an isentropic vertical coordinate, $\hat{q}^{\theta} \equiv \overline{\rho^{*\theta}\,q}^{\theta} / \overline{\rho^{*\theta}}^{\theta}$ should be the prognostic variable for the moisture field rather than $\overline{\rho^{*q}}^{q}$. We can follow the following steps:
  - Diagnose $\overline{\rho^{*q}}^{q}$ from $\hat{q}^{\theta}(\theta)$ using

$$\overline{\rho^{*q}}^{q} = \overline{\rho^{*\theta}}^{\theta}\,\frac{\partial \theta}{\partial \hat{q}^{\theta}} \qquad (3)$$

  - Predict $\overline{\rho^{*q}}^{q}$ using (2)
  - Diagnose updated $\frac{\partial \hat{q}^{\theta}}{\partial \theta}$ from updated $\overline{\rho^{*q}}^{q}$ and $\overline{\rho^{*\theta}}^{\theta}$ using (3)
  - Vertically integrate updated $\frac{\partial \hat{q}^{\theta}}{\partial \theta}$ with respect to q to obtain updated $\hat{q}^{\theta}(\theta)$