z-coordinate (with anelastic approximation)

$$\frac{\partial \overline{\Psi}}{\partial t} = -\left[\nabla \cdot \left(\overline{\mathbf{v}}\,\overline{\Psi}\right) + \frac{\partial}{\rho \partial z}\left(\rho\,\overline{w}\,\overline{\Psi}\right)\right] + PS(\overline{\Psi})$$

$$PS(\overline{\Psi}) = \overline{RS(\Psi)} - \left[\overline{\nabla \cdot (\mathbf{v}\,\Psi)} - \nabla \cdot \left(\overline{\mathbf{v}}\,\overline{\Psi}\right)\right] - \frac{\partial}{\rho \partial z}\rho\left[\overline{w\Psi} - \overline{w}\,\overline{\Psi}\right]$$

Isentropic vertical coordinate

$$\frac{\partial}{\partial t}\left(\overline{\rho^*}\hat{\Psi}\right) = -\left[\nabla \cdot \left(\overline{\rho^*}\hat{\mathbf{v}}\hat{\Psi}\right) + \frac{\partial}{\partial \theta}\left(\overline{\rho^*}\,\hat{\dot{\theta}}\,\hat{\Psi}\right)\right] + \overline{\rho^*}\,PS(\hat{\Psi}) \qquad \dot{\theta} = RS(\theta)\,.$$

$$PS(\hat{\Psi}) = \overline{\rho^*\,RS(\Psi)} - \left[\overline{\nabla \cdot (\rho^*\,\mathbf{v}\Psi)} - \nabla \cdot \left(\overline{\rho^*}\hat{\mathbf{v}}\hat{\Psi}\right)\right] - \frac{\partial}{\partial \theta}\left[\overline{\rho^*\dot{\theta}\,\Psi} - \overline{\rho^*}\,\hat{\dot{\theta}}\,\hat{\Psi}\right]$$

$(\overline{\ })$: average with respect to horizontal area along a θ surface,

ρ^* : pseudo-density in the θ coordinate system

$\hat{\Psi}$: mass-weighted area average of Ψ given by $\overline{\rho^*\Psi} \,/\, \overline{\rho^*}$

Parameterization of subgrid-scale processes is now needed for both transport and source terms.