REAL SOURCE vs. PARAMETERIZED SOURCE

(in z-ccordinate with the anelastic approximation)

Local and instantaneous governing equation for Ψ:

$$\frac{\partial \Psi}{\partial t} = -\nabla \cdot (\mathbf{v}\Psi) - \frac{\partial}{\rho \partial z}(\rho w \Psi) \quad + \quad \underset{\text{real source}}{RS(\Psi)} \qquad (1)$$

Model's governing equation for $\overline{\Psi}$:

$$\frac{\partial \overline{\Psi}}{\partial t} = -\nabla \cdot (\overline{\mathbf{v}}\,\overline{\Psi}) - \frac{\partial}{\rho \partial z}(\rho \overline{w}\,\overline{\Psi}) \quad + \quad \underset{\text{parameterized source}}{PS(\overline{\Psi})} \qquad (2)$$

$\overline{}$: running horizontal-area average

For (2) to be consistent with (1),

$$\text{Required } PS(\overline{\Psi}) = \overline{RS(\Psi)} - \left[\overline{\nabla \cdot (\mathbf{v}\Psi)} - \nabla \cdot (\overline{\mathbf{v}}\,\overline{\Psi})\right] - \frac{\partial}{\rho \partial z}\rho\left[\overline{w\Psi} - \overline{w}\,\overline{\Psi}\right]$$

Thus, $-\frac{\partial}{\rho \partial z}\rho\left[\overline{w\Psi}\right]$ *is artificially split into* $-\frac{\partial}{\rho \partial z}\rho\left[\overline{w}\,\overline{\Psi}\right]$ *and* $-\frac{\partial}{\rho \partial z}\rho\left[\overline{w\Psi} - \overline{w}\,\overline{\Psi}\right]$