TRAPEZOIDSAL AND EQUIVALENT TRAPEZOIDAL SCHEMES

Trapezoidal scheme (TZ) :

$$h_i^{n+1} - h_i^n = -\frac{H\Delta t}{2\Delta x}\left(u_{i+1/2}^{n+1} + u_{i+1/2}^n - u_{i-1/2}^{n+1} - u_{i-1/2}^n\right)$$

$$u_{i-1/2}^{n+1} - u_{i-1/2}^n = -\frac{g\Delta t}{2\Delta x}\left(h_i^{n+1} + h_i^n - h_{i-1}^{n+1} - h_{i-1}^n\right)$$

$$\lambda = \left(1 - 2C_T\right) \pm \sqrt{4\,C_T\left(-1 + C_T\right)} \qquad C_T \equiv \frac{\mu^{*2}}{1+\mu^{*2}} \qquad \mu^{*2} \equiv \mu^2\left(\sin\frac{k\Delta x}{2}\right)^2$$

Aloways λ is complex and $|\lambda| = 1$

Equivalent trapezoidal scheme (ETZ):

A multi-point forward-backward scheme that shares the same λ with the TZ scheme.

$$\lambda = (1 - 2C) \pm \sqrt{4C(-1 + C)} \qquad C = \mu^{*2}\left(a_0 + 2\sum_{j'=1}^{J} a_{j'}\cos(kj'\Delta x)\right)^2$$

$$C = C_T \text{ requires } \frac{1}{2\sqrt{1+\mu^{*2}}} = \frac{a_0}{2} + \sum_{j'=1}^{J} a_{j'}\cos\left(kj'\Delta x\right)$$

Thus the coefficients “a” for ETZ are the Fourier coefficients of the function

$$F(\xi) \equiv \frac{1}{2\sqrt{1+\mu^2\sin^2(\xi/2)}}\,, \quad \text{where } \xi = k\Delta x.$$