U.S. DEPARTMENT OF COMMERCE
NATIONAL OCEANIC AND ATMOSPHERIC ADMINISTRATION
NATIONAL WEATHER SERVICE
NATIONAL METEOROLOGICAL CENTER

OFFICE NOTE 67

Detection and Correction of Errors in Height and Temperature Analyses

Frederick G. Shuman
National Meteorological Center

DECEMBER 1971

1. Introduction

Shuman and Hovermale (1968) noted that in interpolating for heights into their "sigma" coordinate system, one should not expect the ten sets of isobaric height and temperature analyses to exactly agree through any simplified version of the hydrostatic equation. Consequently, their interpolation system demands no agreement at all, and discards information on the mean temperatures given by temperature analyses. From the temperature analyses they draw information only on the lapse rates.

Desmarais and others at NMC, however, have recently examined samples of temperature soundings made from temperature and thickness analyses, and have shown that disagreements, particularly in lapse rates, go beyond what can reasonably be attributed to the difference between mean (thickness) temperatures and point temperatures. This raises the question of whether the initialization section of the prediction program should have procedures to detect and correct certain gross errors which crop up in analysis systems. To put it another way, by changing (minimally, we expect) height and/or temperature analyses so that they reasonably agree at all ten analysis levels, we should then be better able to use all of the information contained in the 19 analyses from 100 mb to 1000 mb.

This note proposes a method to bring the multilevel analyses into reasonable agreement.

2. "Error" detection.

We first write the hydrostatic equation in the following form.

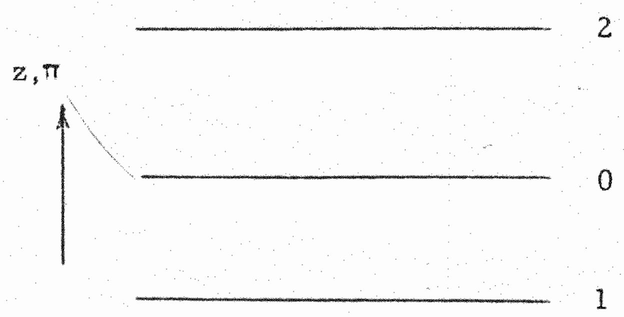
$$\frac{\partial z}{\partial \pi} = \frac{T}{\gamma} \qquad (1)$$

where $\pi = \ln \dfrac{P}{p}$

$\gamma = \dfrac{g}{R}$

$P = 1000 \text{ mb}$

Consider three adjacent analysis levels (e. g., 850, 700, 500 mb or 300, 250, 200 mb) and invent subscripts as suggested in the figure. We will say that ϵ_1 and ϵ_2 are measures of disagreement among z_0, z_1, T_0, T_1, and z_2, z_0, T_2, T_0, respectively.



Corresponding to (1), we will define ϵ_1 and ϵ_2 as follows.

$$\frac{z_2 - z_0}{\Delta_2} - \frac{T_2 + T_0}{2\gamma} = \epsilon_2 \qquad (2)$$

$$\frac{z_0 - z_1}{\Delta_1} - \frac{T_0 + T_1}{2\gamma} = \epsilon_1 \qquad (3)$$

where $\Delta_2 = \pi_2 - \pi_0$ and $\Delta_1 = \pi_0 - \pi_1$. Eliminating T_0 from (2) and (3), we get

$$\frac{\Delta_1 \; z_2 - \Delta_0 \; z_0 + \Delta_2 \; z_1}{\Delta_1 \; \Delta_2} - \frac{T_2 - T_1}{2\gamma} = \epsilon_2 - \epsilon_1 \qquad (4)$$

where $\qquad \Delta_0 = \Delta_1 + \Delta_2 = \pi_2 - \pi_1$.

Similarly, elimination of z_0 from (2) and (3) yields

$$z_2 - z_1 - \frac{\Delta_2 \, T_2 + \Delta_0 \, T_0 + \Delta_1 \, T_1}{2\gamma} = \Delta_2 \, \epsilon_2 + \Delta_1 \, \epsilon_1 \qquad (5)$$

Now, the right-hand side of (4) will be regarded as a measure of disagreement between two estimates of the temperature lapse, and the right-hand side of (5) between two estimates of the mean temperature. With (4) being the difference between ϵ_2 and ϵ_1, for (5) to correspond it should approximate the sum of ϵ_2 and ϵ_1. The weights in (5) on ϵ_2 and ϵ_1 should therefore sum to two, and we divide (5) by $\frac{1}{2}\Delta_0$ for this reason.

$$2\,\frac{z_2 - z_1}{\Delta_0} - \frac{\Delta_2 T_2 + \Delta_0 T_0 + \Delta_1 T_1}{\gamma\,\Delta_0} = 2\,\frac{\Delta_2 \epsilon_2 + \Delta_1 \epsilon_1}{\Delta_0} \tag{5a}$$

Finally, for convenience, we multiply (4) and (5a) by γ so that the right-hand sides will be direct measures of the "error in the central temperature, T_0, in the case of (5a), and in the central height, z_0, scaled to the same units as T_0 in the case of (4).

$$\gamma\,\frac{\Delta_1 z_2 - \Delta_0 z_0 + \Delta_2 z_1}{\Delta_1 \Delta_2} - \frac{T_2 - T_1}{2} = \delta_1 \tag{6}$$

z_1

$$2\gamma\,\frac{z_2 - z_0}{\Delta_0} - \frac{\Delta_2 T_2 + \Delta_0 T_0 + \Delta_1 T_1}{\Delta_0} = \delta_2 \tag{7}$$

where $\delta_1 = \gamma\,(\epsilon_2 - \epsilon_1)$

$$\delta_2 = 2\gamma\,\frac{\Delta_2 \epsilon_2 + \Delta_1 \epsilon_1}{\Delta_0}$$

3. "Error" correction.

It is proposed to use (6) and (7) in the following way. Given both temperature and height analyses at the nine levels 850, 700, 500, 400, 300, 250, 200, 150, and 100 mb, both δ_1 and δ_2 will be calculated at a given grid point at the seven levels from 700 to 150 mb, inclusive. The magnitude of the largest absolute value will be compared with a preselected criterion (2° C is suggested). If the largest absolute value, be it $|\delta_1|$ or $|\delta_2|$, is smaller than the criterion, then no change at that grid point will be made. If on the other hand it is larger, then a new z_0 will be calculated for the level if it is $|\delta_1|$ which is the largest, or a new T_0 if $|\delta_2|$ is largest. The new z_0 or T_0 will be calculated from (6) or (7), setting δ_1 or δ_2 equal to zero. Having already calculated δ_1 or δ_2, the new value, say z_0' or T_0', may be calculated with

$$z_0' = z_0 + \frac{\Delta_1 \Delta_2}{\gamma\,\Delta_0}\,\delta_1 \tag{8}$$

$$T_0' = T_0 + \delta_2 \tag{9}$$

If a new value of z_0 or T_0 is calculated at a grid point, the entire procedure will be repeated at that grid point until the largest absolute value of the set of δ_1 and δ_2 is less than the criterion.

The grid will be scanned point by point in this manner.

4. Potential problems.

There are several latent problems connected with the procedure. The analyses are done with a high degree of quality control, and therefore should be changed only with great caution. The criterion adopted should be relatively large for this reason, and also because there is no reason why the analyses should agree exactly by (6) or (7) with vanishing δ_1 and δ_2.

The built-in categorical decision based on comparison with a criterion can potentially lead to quite different changes at neighboring grid points and lead to discontinuities. This would almost certainly happen with a tight criterion. If such behavior is noted, loosening of the criterion may alleviate the problem.

Calculation of T_0' with (7) and (9) amounts to an unsmoothing of the vertical distribution of T_0, and may introduce unwanted small scale variations. If such a problem occurs, loosening of the criteria may alleviate it.

5. Concluding remarks.

The proposed system on its face appears to be a new and powerful tool to tie multi-level analyses together in a realistic way. It does, however, have several potential problems, and can only be proven by trial. If it indeed does turn out to be a good buffer between the analysis and forecast systems, it should be considered as a tool to be used in the analysis system itself.

Reference

Shuman and Hovermale, 1968: Journal of Applied Meteorology, Vol. 7, No. 4, pp. 525-547.