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LFM 1988: A DOCUMENTATION

PATRICIA M. PAULEY
UNIVERSITY OF WISCONSIN

AND

DENNIS G. DEAVEN
DEVELOPMENT DIVISION

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This paper is a description of the Limited Area Fine Mesh (LFM) model as it existed in the National Meteorological Center's job suite in August 1988. The LFM computer codes have not been modified since the Nested Grid Model (NGM) became operational in March 1985 at which time the NGM became the primary numerical model for regional guidance. There are no modifications or improvements planned for the current version of the LFM because the Technique Development Laboratory's Model Output Statistics (MOS) are based upon the current "frozen" version of the model. Since 1985 the LFM has provided the early first-look at regional guidance to field forecasters and is integrated twice daily out to 48 hours with a start time of about one hour and twenty minutes after data cutoff time.

MODEL COORDINATE SYSTEM

The LFM is a grid point model calculated on a limited domain covering North America and portions of the adjacent oceans. The horizontal grid is a square mesh superimposed on a polar-stereographic map projection true at 60 degrees north latitude. Distance on the projection at latitude is related to earth distance through a map factor, m, given by

$$m(\phi) = \frac{1 + \sin 60^\circ}{1 + \sin \phi} .$$

The horizontal grid length is 190.5 km at 60 North with the domain extending 53 grid points in the x-direction and 45 in the y-direction.

The vertical coordinate used by the LFM is a sigma-p terrain-following coordinate defined separately in each of three vertical domains. The general form for sigma is given by

$$\sigma_i = \frac{p_i - p_T}{p_B - p_T} ,$$

where Pi is the pressure at a point within the ith vertical domain, and Pt and Pb are the pressures at the top and bottom, respectively, of the ith vertical domain. Thus, sigma takes on a value of 1 at the bottom and 0 at the top of a given domain. The lowest of the three vertical domains is the boundary-layer domain. It contains a single sigma-layer which is held at a constant pressure thickness of 50 mb. The tropospheric domain occupies the remainder of the troposphere and is divided into three layers of equal pressure thickness extending from the top of the boundary layer to a material surface tropopause. The remaining

domain is the stratospheric domain which is also divided into three equal layers. The "top" of the model is a material surface located at 50 mb.

MODEL EQUATIONS

Within this polar stereographic, sigma-p framework, the LFM is a hydrostatic, primitive equation model based on the six-layer primitive equation model described in Shuman and Hovermale (1968). Predicted quantities include the u and v components of wind, potential temperature, precipitable water, and the pressure thickness of the sigma layers.

The u and v wind components are oriented in the x and y directions defined for the polar stereographic grid. These components are converted from earth oriented west-east, south-north components by

$$u = -u_s \sin\lambda - v_s \cos\lambda$$

$$v = u_s \cos\lambda - v_s \cos\lambda \ ,$$

where the s subscript refers to earth oriented wind components in spherical coordinates and lamda represents longitude. In the LFM, the momentum equations are formulated as

$$\frac{\partial}{\partial t}\left[\frac{u}{m}\right] = (\zeta+f)\frac{v}{m} - \frac{\partial \phi}{\partial x} - c_p\theta\frac{\partial \Pi}{\partial x} - \dot{\sigma}\frac{\partial}{\partial \sigma}\left[\frac{u}{m}\right] - \frac{\partial}{\partial x}\left[\frac{u^2 + v^2}{2}\right]$$

$$\frac{\partial}{\partial t}\left[\frac{v}{m}\right] = -(\zeta+f)\frac{u}{m} - \frac{\partial \phi}{\partial y} - c_p\theta\frac{\partial \Pi}{\partial y} - \dot{\sigma}\frac{\partial}{\partial \sigma}\left[\frac{v}{m}\right] - \frac{\partial}{\partial y}\left[\frac{u^2 + v^2}{2}\right],$$

where ζ is the vertical component of relative vorticity, defined as

$$\zeta = m^2\left[\frac{\partial}{\partial x}\left[\frac{v}{m}\right] - \frac{\partial}{\partial y}\left[\frac{u}{m}\right]\right] \ ,$$

f is the Coriolis parameter, ϕ is the geopotential gz, and Cp is the specific heat of dry air at constant pressure, Π is the Exner function, defined by

$$\Pi \equiv \left[\frac{p}{1000\ mb}\right]^{\kappa} \ ,$$

κ is R/Cp, R is the gas constant for dry air, and $\dot{\sigma}$ is the vertical motion in sigma coordinates

$$\dot{\sigma} \equiv \frac{d\sigma}{dt} \ .$$

Frictional effects are parameterized and added later in the forecast sequence.

Potential temperature is predicted isentropically and subsequently modified by parameterization of various diabatic effects. The isentropic thermodynamic equation is given by

$$\frac{\partial \theta}{\partial t} = -m^2 \left[\frac{u}{m} \frac{\partial \theta}{\partial x} + \frac{v}{m} \frac{\partial \theta}{\partial y} \right] - \dot{\sigma} \frac{\partial \theta}{\partial \sigma} .$$

The moisture field is defined in terms of precipitable water W, which is given by

$$W = \frac{1}{g} \int_{\sigma_1}^{\sigma_2} q \frac{\partial p}{\partial \sigma} \, d\sigma ,$$

where q is specific humidity. The moisture tendency is calculated from

$$\frac{\partial W}{\partial t} = -m^2 \left[\frac{\partial}{\partial x} \left[\frac{uW}{m} \right] + \frac{\partial}{\partial y} \left[\frac{vW}{m} \right] \right] - \frac{\partial}{\partial \sigma} (\dot{\sigma} W) ,$$

and modified to reflect the addition of moisture at the lower boundary by sea surface moisture flux and the removal of moisture by precipitation. Moisture is allowed only in the lowest three model layers, with saturation occurring at a seasonally varying value of relative humidity ranging from 90 percent to 96 percent.

The remaining predictive variable is the pressure thickness of sigma layers, $\partial p / \partial \sigma$. This quantity is determined from the continuity equation,

$$\frac{\partial}{\partial t} \left[\frac{\partial p}{\partial \sigma} \right] = -m^2 \left[\frac{\partial}{\partial x} \left[\frac{u}{m} \frac{\partial p}{\partial \sigma} \right] + \frac{\partial}{\partial y} \left[\frac{v}{m} \frac{\partial p}{\partial \sigma} \right] \right] - \frac{\partial}{\partial \sigma} \left[\dot{\sigma} \frac{\partial p}{\partial \sigma} \right] .$$

Two additional quantities must be determined diagnostically in order to close this set of equations, namely ϕ and $\dot{\sigma}$. The geopotential ϕ is calculated in mid layers by integrating the hydrostatic equation given by

$$\frac{\partial \Phi}{\partial \sigma} = -c_p \theta \frac{\partial \Pi}{\partial \sigma} ,$$

which can be rewritten as

$$\frac{\partial \Phi}{\partial \Pi} = -c_p \theta .$$

where Π is the mass weighted Exner Function as documented in TPB #267. The vertical motion $\dot{\sigma}$ is found on sigma

levels by differentiating $\frac{\partial}{\partial \sigma}\left[\frac{\partial p}{\partial \sigma}\right]$ with respect to σ, noting that $\partial p/\partial \sigma$ within each vertical domain is constant with respect to σ.

This yields

$$\frac{\partial p}{\partial \sigma}\frac{\partial^2 \dot{\sigma}}{\partial \sigma^2} = -m^2\left[\frac{\partial}{\partial x}\left[\frac{\partial p}{\partial \sigma}\frac{\partial}{\partial \sigma}\left[\frac{u}{m}\right]\right] + \frac{\partial}{\partial y}\left[\frac{\partial p}{\partial \sigma}\frac{\partial}{\partial \sigma}\left[\frac{v}{m}\right]\right]\right] .$$

The vertical motion in the boundary layer is somewhat easier to find since the boundary layer thickness is held at a constant value of 50 mb. Thus the time derivative of $\partial p/\partial \sigma$ is equal to zero resulting in

$$\frac{\partial \dot{\sigma}}{\partial \sigma} = -m^2\left[\frac{\partial}{\partial x}\left[\frac{u}{m}\right] + \frac{\partial}{\partial y}\left[\frac{v}{m}\right]\right] .$$

The vertical motion $\dot{\sigma}$ is assumed to be zero at the surface of the earth, the tropopause, and the constant 50 mb upper boundary. The vertical motion at the bottom of the tropospheric domain is matched to that at the top of the boundary layer, assuming continuity of vertical mass transport.

PHYSICAL PARAMETERIZATION

The fields predicted by the previous closed set of equations are modified to reflect the influences of several physical processes. Most of these processes occur on a scale not resolvable by the model and so must be parameterized in terms of the previously defined grid scale model variables.

Radiative heating and cooling in the LFM are calculated once per model hour and then applied at each time step. Heating due to solar radiation by water vapor is allowed using a six-line approximation of the absorption curve. Reflection of solar radiation from the surface is accomplished using an albedo of 1.0 if the surface is snow or water and 0.9 if land, modified by cloud albedos of 1.00, 0.75, or 0.5 if clouds exist in layers 1 through 3, respectively. Clouds are defined to exist if 60 per cent of saturation is exceeded. Radiative cooling occurs in all layers above any diagnosed cloud (in all layers if no cloud is diagnosed) at the rate of 0.06 K/hr. This rate is increased by 0.1 K/hr in the boundary layer over snow cover on cloudless nights.

Sensible heating of the boundary layer occurs when the ground is warmed by non-reflected solar radiation. The

assigned albedo values imply that sensible heating is allowed only over land without snow cover and only if clouds are not detected in the boundary layer. Sea surface heat and moisture fluxes into the boundary layer are also included in the LFM using bulk formulations. The heat flux is proportional to the difference between the boundary layer temperature and the sea surface temperature, the latter of which is held constant at its initialized values. No heat flux is allowed if the boundary layer is warmer than the sea surface. A similar formula is used for the sea surface moisture flux, with the addition of moisture to the boundary layer occurring only if sea surface heat flux is also occurring, if the boundary layer is at or below 70 per cent of saturation, and if the moisture flux is directed from the sea into the atmosphere.

Grid scale precipitation is calculated depending upon the saturation surplus or deficit in each grid box. If a saturation surplus exists in a layer, then the excess is removed as precipitation, releasing a commensurate amount of latent heat in that layer. If a saturation deficit occurs, then precipitation falling into the layer is evaporated until saturation is achieved, with evaporative cooling occurring as well. The calculation progresses downward from layer 3 which is the topmost layer containing moisture. The latent heating/cooling is divided by two and applied to the temperature field at both the predicted and current time levels causing temporal smoothing of the heat release.

The subgrid scale precipitation is calculated using a moist convective adjustment. Convection is assumed to occur in each layer if the following criteria are met:

1) grid scale precipitable water is increasing with time, indicative of moisture convergence;

2) grid scale moisture is greater than 75 per cent of its saturation value

3) lifted condensation level for the layer in question is below the midpoint of the next layer above; and

4) parcel temperature is warmer by more than 0.1 K than the layer temperature of the next layer above, after an initial impulse of 1.5 K.

Convective precipitation r_c is calculated as

$$r_c = \frac{\Delta\theta \; c_p \; p_\sigma(k+1) \; \Pi(k+1)}{g \; L} \; ,$$

where $\Delta\theta$ is the difference between the parcel potential temperature (the potential temperature a parcel would attain if lifted first dry then moist adiabatically from the

midpoint of layer k to the midpoint of layer k+1) and the grid scale potential temperature, and L is the latent heat of condensation. The calculation proceeds from the boundary layer upwards with any released latent heat calculated for layer "k" assigned to layer "k+1", to simulate upward convective transport of latent heat. No evaporation is permitted during the convective parameterization process. The latent heat is applied equally to the new and current time levels. Convection is allowed to occur at each time step during the first four hours of the integration; however, any resulting precipitation and latent heat are discarded including the latent heat and precipitation resulting from large scale precipitation. This procedure eliminates large spurious values of precipitation which may occur during the first few model hours as the divergence and vertical motion are developing. Afterwards, the convective calculation is done only once per model hour on the first time step of each hour.

Dry convection eliminates any superadiabatic lapse rates that might develop. The algorithm uses a layer mass weighted average of potential temperature conserving internal and potential energy.

Surface frictional drag in the LFM affects the lowest two layers of the model and is parameterized using a bulk formulation, in which drag is proportional to the square of the wind speed and is directed opposite the wind vector. The stress profile is assumed to approach a value of zero at the top of the second layer. The drag coefficient (Cressman, 1960) varies from a value of 15x10-4 over water to 95x10-4 over high terrain.

NUMERICAL APPROXIMATIONS

The LFM uses fourth order horizontal and second order vertical centered finite differencing in a modified leapfrog (centered time and space) scheme on an unstaggered grid. The time integration scheme used is one detailed by Brown and Campana (1978) is which the time derivative is approximated by a second order centered difference using a time step of 400 seconds. Brown and Campana found that the time step could be lengthened to this value by temporally averaging the pressure gradient force in the momentum equations. The form of this average is

$$\Phi_x = \alpha(\Phi_x^{t+1} + \Phi_x^{t-1}) + (1 - 2\alpha)\Phi_x^{t} ,$$

where the superscripts t+1, t, and t-1 refer to the new current and preceding time levels, respectively, and ϕ_x is the pressure gradient. The constant α is related to the constant in the time smoother which is applied at each time

step to control the linearly amplifying computational modes. This time smoother is given by

$$A^t = b(A_*^{t+1} + A^{t-1}) + (1 - 2b)A_*^t \ ,$$

where A is any forecast variable, b is a constant, and A* is an unsmoothed value. The optimum value for α was shown by Brown and Campana to be

$$\alpha = (b^2 + 1)\ (b + 1)\ /\ 4 \ .$$

The LFM uses 0.075 for b and 0.270 for the value of α .

The fourth order centered finite differences used to approximate horizontal derivatives utilizes four consecutive grid point values indicated sequentially as i-1, i, i+1, and i+2. For an arbitrary variable A, the approximation takes the form

$$\left.\frac{\partial A}{\partial x}\right|_{i+\frac{1}{2}} \simeq A_x = \frac{27}{24}\left[\frac{A_{i+1} - A_i}{\Delta x}\right] - \frac{1}{24}\left[\frac{A_{i+2} - A_{i-1}}{\Delta x}\right] \ .$$

The subscript x notation follows the convention of Shuman and Hovermale (1968), but refers to a fourth order rather than a second order form. This yields an approximated value which is located midway between grid points i and i+1. A derivative in the y direction is approximated in an analogous fashion.

To obtain the value of a given variable midway between grid points, a second approximation, known as a "bar-x" operation using the notation of Shuman and Hovermale (1968) is formulated as

$$A_{i+\frac{1}{2}} \simeq \overline{A}^x = \frac{9}{16}(A_{i+1} + A_i) - \frac{1}{16}(A_{i+2} + A_{i-1}) \ .$$

This form is fourth order accurate and is also derived from the Taylor Series by forming a finite sum rather than a finite difference. An analogous "bar-y" operator is also used.

All calculations in the LFM occur midway between grid points in both the x and y directions, which will be referred to as a "grid box" calculation or value. Therefore, a combination of the previous finite difference forms is used. For example, a difference in the x direction yields a value centered between grid points in the x

direction. If the resulting values are then subjected to a "bar-y" operation, one obtains a grid box estimate of the x derivative. Similarly, a combination of a y difference and a bar-x yields a grid box estimate of the y derivative. A grid box estimate of a quantity itself is obtained by combining the bar-x and bar-y operators, referred to as a "bar-xy" operator. Each of these combinations of operators involve 16 grid points centered around the grid box in question. To see how these operators are used in practice, two examples will be given using the notation introduced in , and . First, the horizontal advection of an arbitrary scalar A is calculated from

$$u\frac{\partial A}{\partial x} + v\frac{\partial A}{\partial y} \simeq \bar{u}^{xy}\,\bar{A}^{y}_{x} + \bar{v}^{xy}\,\bar{A}^{x}_{y} \quad .$$

Second, the flux divergence of A is formed as the sum of its advective and divergent parts, as

$$\frac{\partial uA}{\partial x} + \frac{\partial vA}{\partial y} \simeq \bar{u}^{xy}\,\bar{A}^{y}_{x} + \bar{v}^{xy}\,\bar{A}^{x}_{y} + \bar{A}^{xy}(\bar{u}^{y}_{x} + \bar{v}^{x}_{y}) \quad .$$

After the terms in the equation of interest are calculated, it is necessary to restore the grid box values to grid points in order to predict grid point values for the next time step. This is done using a second bar-xy operation. Whereas the previous operations begin with grid point values and give their results shifted half a grid length in both the x and y directions to yield grid box values, this bar-xy operation begins with grid box values and gives its result shifted half a grid length in both the x and the y directions to yield grid point values.

This numerical scheme is used to help damp the smaller scale variations which arise for numerical, rather than physical, reasons. Leapfrog schemes have difficulty with the shortest resolvable wavelength in the system, namely twice the grid length. This shortest wavelength results from the nonlinear interaction of longer wavelengths, but is itself unable to disperse into shorter wavelengths because of the discrete nature of the calculation. This is especially a problem in leapfrog schemes because of their lack of damping at shorter wavelengths. Therefore, artificial damping is introduced both through the use of the previously defined bar operators, and through the addition of a linear diffusion term in all of the predictive equations at each time step, of the form

$$\left.\frac{\partial A}{\partial t}\right|_{dif} = \kappa_1 \nabla^2 A^{t-1} \quad ,$$

where the diffusion coefficient is set equal to 1.8×10^{8}. The LaPlacian is calculated using a second order finite difference approximation with no additional spatial smoothing.

Vertical derivatives are approximated using second order centered differences. A bar-xy operation is applied first to form grid box values, which are then used in the vertical difference calculations. Finally, a second bar-xy is used as explained above to shift back to the grid points.

Since it is a limited area model, the LFM requires lateral boundary values. The time varying boundary conditions are obtained form the previous NMC run of the global spectral model at six hour intervals. Values of the predicted quantities are interpolated to the LFM grid and applied to the five outermost rows in the horizontal domain using

$$\left.\frac{\partial A}{\partial t}\right|_{bnd} = \kappa_2 \nabla^2 (A_*^{t-1} - A_s^{t-1}) \quad ,$$

where A* is a quantity defined by the LFM, and As is a quantity defined by the spectral model. The diffusion coefficient is set equal to 1.4×10^{8} in the boundary region and is set to zero in the interior part of the domain. The spectral model values are used unmodified at the outermost grid points. This procedure is applied to all predicted variables with the exception of moisture which is held constant on the lateral boundaries during the integration.