Misrepresentation of Model Performance by RMSE: From Mathematical Proof to Case Demonstration

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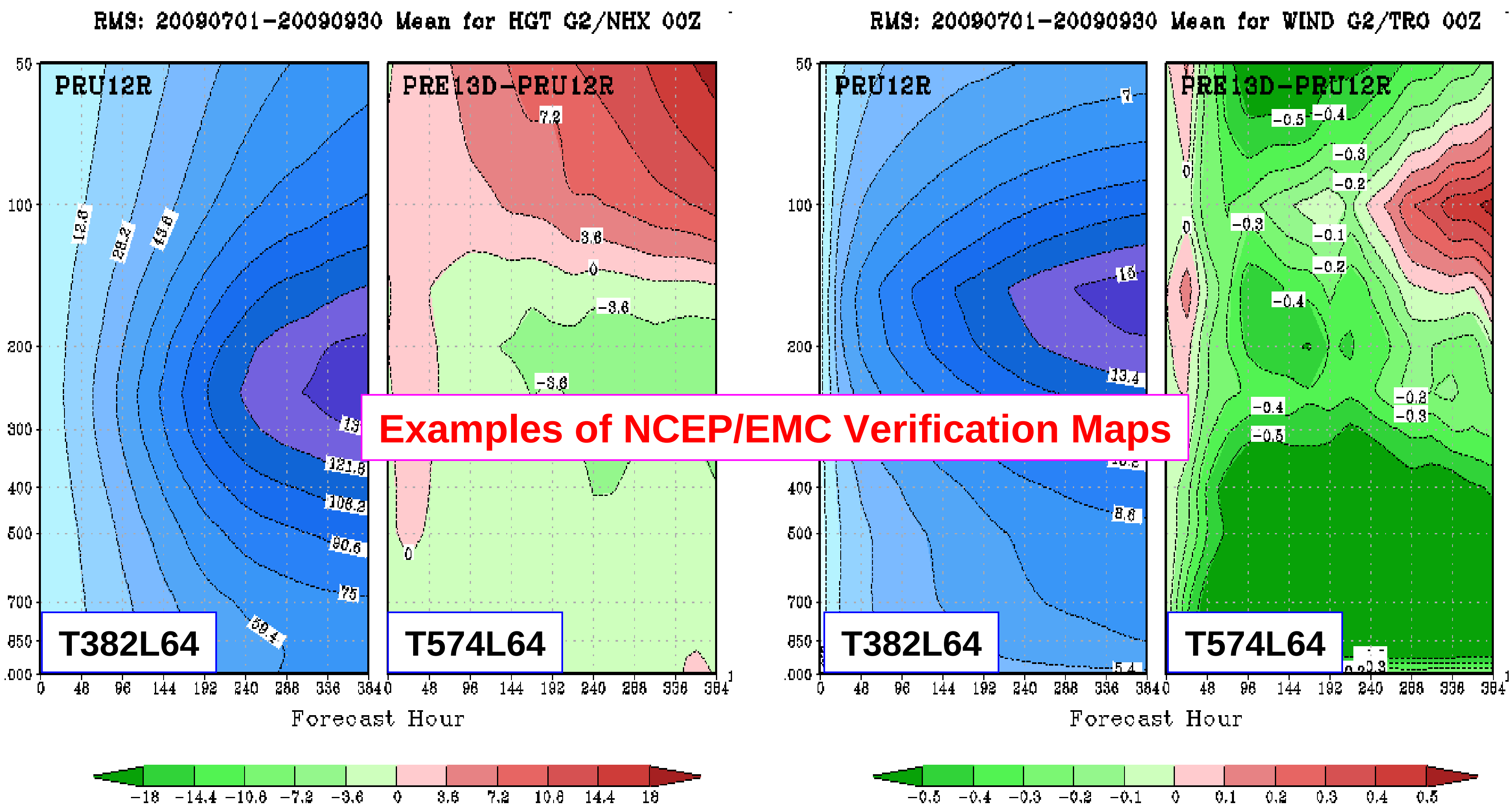
RMSE Has long been used as a performance metric for model evaluation. In this presentation I will show mathematically that RMSE can at times misrepresent model performances. A normalized RMSE is proposed, however, the normalization is not always effective.



Root-Mean Squared Error (E)

$$E = \sqrt{\frac{1}{n}\sum_{n=1}^{N}\left(F_n - A_n\right)^2}$$

Where, **F** is forecast, **A** is either analysis or observation, **N** is the total number of points in a temporal or spatial domain, or a spatial-temporal combined space.

$$E^2 = \frac{1}{n}\sum_{n=1}^{N}\left[\left(F_n - \overline{F}\right) - \left(A_n - \overline{A}\right) + \left(\overline{F} - \overline{A}\right)\right]^2$$

$$= \frac{1}{n}\sum_{n=1}^{N}\left(F_n - \overline{F}\right)^2 + \frac{1}{n}\sum_{n=1}^{N}\left(A_n - \overline{A}\right)^2 + \left(\overline{F} - \overline{A}\right)^2$$

$$+ \cancel{\frac{2}{n}\cdot\sum_{n=1}^{N}\left[\left(F_n - \overline{F}\right) - \left(A_n - \overline{A}\right)\right]\cdot\left(\overline{F} - \overline{A}\right)}$$

$$- \frac{2}{n}\cdot\sum_{n=1}^{N}\left(F_n - \overline{F}\right)\cdot\left(A_n - \overline{A}\right)$$

$$\boxed{E^2 = \sigma_f^2 + \sigma_a^2 - 2\sigma_f\sigma_a R + \left(\overline{F} - \overline{A}\right)^2}$$

Mean squared error

where

$$\sigma_f^2 = \frac{1}{n}\sum_{n=1}^{N}\left(F_n - \overline{F}\right)^2 \qquad \sigma_a^2 = \frac{1}{n}\sum_{n=1}^{N}\left(A_n - \overline{A}\right)^2$$

Variances of forecast & analysis

$$R = \frac{1}{n}\cdot\sum_{n=1}^{N}\left(F_n - \overline{F}\right)\cdot\left(A_n - \overline{A}\right)\Big/\sigma_f\sigma_a$$

anomalous pattern correlation

$$E^2 = E_p^2 + E_m^2$$

Mean Squared Error: MSE

$$E_m^2 = \left(\overline{F} - \overline{A}\right)^2$$

MSE by Mean Difference

$$E_p^2 = \sigma_f^2 + \sigma_a^2 - 2\sigma_f \sigma_a R$$

MSE by Pattern Variation

Discussion:

- Total MSE can be decomposed into two parts: the error due to differences in the mean and the error due to differences in pattern variation, which depends on standard deviation over the domain in question and anomalous pattern correlation to observation/analysis.

- If a forecast has a larger mean bias than the other, its MSE can still be smaller if it has much smaller error in pattern variation, and vice versa.

- If two forecasts are verified against different analyses/observations, differences in analysis variance and mean complicate the interpretation of forecast MSE.

- **Model performance evaluation should include both** E_m^2 **and** E_p^2

The following pages present characteristics of E_p^2, and the concept of normalized MSE.

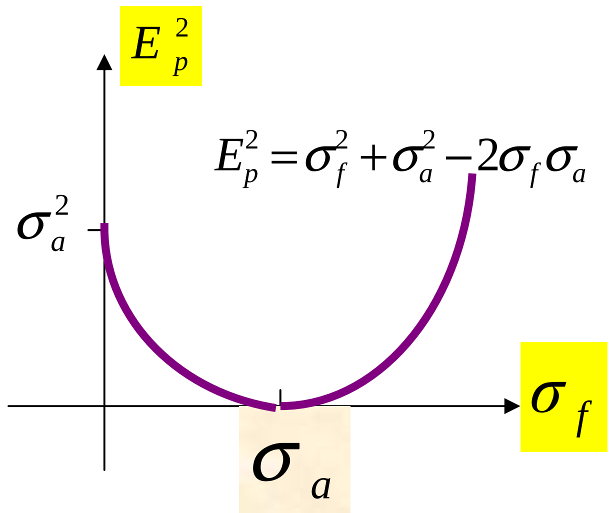
A: Given the same mean difference, will a forecast with smaller variance always give smaller MSE? The answer is no.

$$\frac{\partial E_p^2}{\partial \sigma_f} = 2\sigma_f - 2\sigma_a R = 0 \qquad \Rightarrow E_p^2 \rightarrow \min \quad \text{if } \sigma_f = \sigma_a R$$

Case 1) R =1, perfect pattern correlation $E_p^2(\min) = 0 \quad \text{when } \sigma_f = \sigma_a$

One can see that if a forecast having either too large or too small a variance away from the analysis variance , its error of pattern variation increases.

$$R = 1 \quad \Rightarrow \quad E_p^2 = (\sigma_f - \sigma_a)^2$$

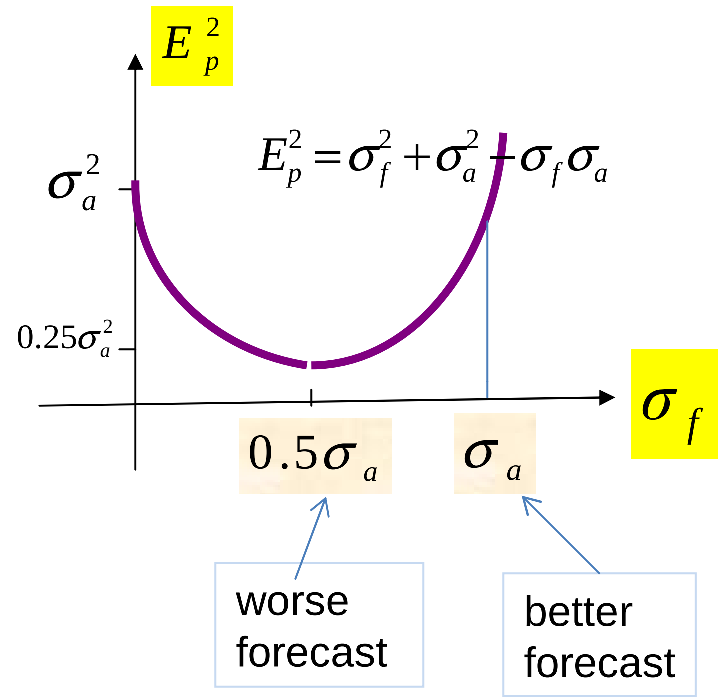


If R=1, E_p^2 does not award smooth forecasts that have smaller variances . It is not biased.

Case 2) **R =0.5, imperfect pattern correlation**

$$E_p^2(\min) = 0 \quad \text{when} \;\; \sigma_f = 0.5\sigma_a$$

In this case, if one forecast has a better variance ($\sigma_f \to \sigma_a$) than the other ($\sigma_f \to 0.5\sigma_a$), the former will have a larger E_p^2 than the latter. Good forecasts are actually penalized.

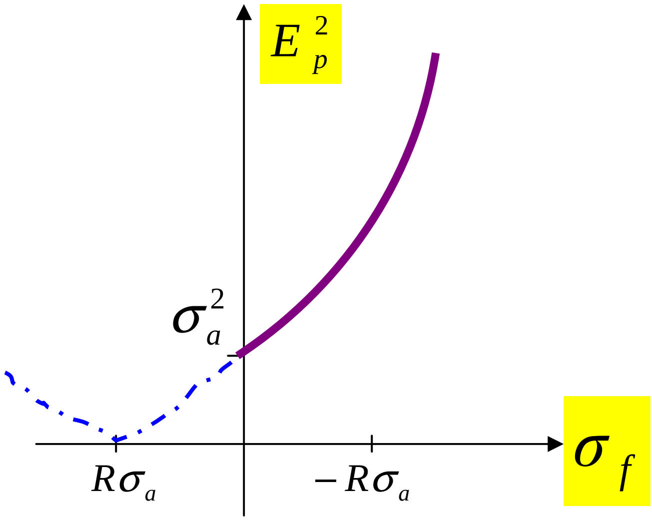


In general, if 0 < R < 1, E_p^2 awards smooth forecasts which have smaller variances close to $R\sigma_a$.

Case 3) **For cases where** $R \leq 0$ **,**

$$E_p^2(\min) = \sigma_a^2 \quad \text{when} \quad \sigma_f = 0$$

E_p^2 **Increase monotonically with** σ_f



In this case, E_p^2 **always awards smoother forecasts that have smaller variances .**

B: Will MSE normalized by analysis variance be unbiased?

$$E^2/\sigma_a^2 = E_p^2/\sigma_a^2 + E_m^2/\sigma_a^2$$

$$E_p^2/\sigma_a^2 = 1 - 2R\lambda + \lambda^2 \qquad \lambda = \sigma_f/\sigma_a$$

Assume $E_m^2 = 0$

λ \ R	−1.0	−0.8	−0.6	−0.4	−0.2	0.0	0.2	0.4	0.6	0.8	1.0
0.0	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.2	1.44	1.36	1.28	1.20	1.12	1.04	0.96	0.88	0.80	0.72	0.64
0.4	1.96	1.80	1.64	1.48	1.32	1.16	1.00	0.84	0.68	0.52	0.36
0.6	2.56	2.32	2.08	1.84	1.60	1.36	1.12	0.88	0.64	0.40	0.16
0.8	3.24	2.92	2.60	2.28	1.96	1.64	1.32	1.00	0.68	0.36	0.04
1.0	**4.00**	**3.60**	**3.20**	**2.80**	**2.40**	**2.00**	**1.60**	**1.20**	**0.80**	**0.40**	**0.00**
1.2	4.84	4.36	3.88	3.40	2.92	2.44	1.96	1.48	1.00	0.52	0.04
1.4	5.76	5.20	4.64	4.08	3.52	2.96	2.40	1.84	1.28	0.72	0.16
1.6	6.76	6.12	5.48	4.84	4.20	3.56	2.92	2.28	1.64	1.00	0.36
1.8	7.84	7.12	6.40	5.68	4.96	4.24	3.52	2.80	2.08	1.36	0.64
2.0	9.00	8.20	7.40	6.60	5.80	5.00	4.20	3.40	2.60	1.80	1.00

$\lambda \to 1$

$E_p^2/\sigma_a^2 \downarrow$

R_c

$\lambda \to 1$

$E_p^2/\sigma_a^2 \downarrow$

Ideally, for a given correlation R, the normalized error should always decrease as the ratio of forecast variance to analysis variance reaches to one from both sides. In the above table only when R is close to one (highly corrected patterns) does this feature exist. For most other cases, especially when R is negative, the normalized error decreases as the variance ratio decrease from two to zero. In other words, **the normalized error still favors smoother forecasts that have a variance smaller than the analysis variance (the truth).**

C: Mean-Squared-Error Skill Score (Murphy, MWR, 1988, p2419)

$$E^2/\sigma_a^2 = E_p^2/\sigma_a^2 + E_m^2/\sigma_a^2$$

$$E_p^2/\sigma_a^2 = 1 - 2R\lambda + \lambda^2 \qquad \lambda = \sigma_f/\sigma_a$$

$$MSESS = 1 - E^2/\sigma_a^2 = 2\lambda R - \lambda^2 - E_m^2/\sigma_a^2$$

Assume $E_m^2 = 0$

λ \ R	– 1.0	– 0.8	– 0.6	– 0.4	– 0.2	0.0	0.2	0.4	0.6	0.8	1.0
0.0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	**0.00**
0.2	– 0.44	– 0.36	– 0.28	– 0.20	– 0.12	– 0.04	0.04	0.12	0.20	0.28	**0.36**
0.4	– 0.96	– 0.80	– 0.64	– 0.48	– 0.32	– 0.16	0.00	0.16	0.32	0.48	**0.64**
0.6	– 1.56	– 1.32	– 1.08	– 0.84	– 0.60	– 0.36	– 0.12	0.12	0.36	0.60	**0.84**
0.8	– 2.24	– 1.92	– 1.60	– 1.28	– 0.96	– 0.64	– 0.32	0.00	0.32	0.64	**0.96**
1.0	**– 3.00**	**– 2.60**	**– 2.20**	**– 1.80**	**– 1.40**	**– 1.00**	**– 0.60**	**– 0.20**	**0.20**	**0.60**	**1.00**
1.2	– 3.84	– 3.36	– 2.88	– 2.40	– 1.92	– 1.44	– 0.96	– 0.48	0.00	–0.48	**0.96**
1.4	– 4.76	– 4.20	– 3.64	– 3.08	– 2.52	– 1.96	– 1.40	– 0.84	– 0.28	0.28	**0.84**
1.6	– 5.76	– 5.12	– 4.48	– 3.84	– 3.20	– 2.56	– 1.92	– 1.28	– 0.64	0.00	**0.64**
1.8	– 6.84	– 6.12	– 5.40	– 4.68	– 3.96	– 3.24	– 2.52	– 1.80	– 1.08	– 0.36	**0.36**
2.0	– 8.00	– 7.20	– 6.40	– 5.60	– 4.80	– 4.00	– 3.20	– 2.40	– 1.60	– 0.80	**0.00**

$\lambda \to 1$

$MSESS \uparrow$

R_c

$\lambda \to 1$

$MSESS \uparrow$

The best case is MSESS=1 when R=1 and Lambda=1. For most cases, especially when R is negative, MSESS decreases monotonically with Lambda. Therefore, MSESS **still favors smoother forecasts that have a variance smaller than the analysis variance.**

Summary I

Conventional RMSE can be decomposed into Error of Mean Difference (Em) and Error of Patter Variation (Ep)
Ep is unbiased and can be used as an objective measure of model performance *only if* the anomalous pattern correlation R between forecasts and analysis is one (or very close to one)
If R <1, Ep is biased and favors smoother forecasts that have smaller variances.
Ep normalized by analysis variance is still biased and favors forecasts with smaller variance if anomalous pattern correlation is not perfect. An ideal normalization method is yet to be found.

A complete model verification should include Anomalous Pattern Correlation, Ratio of Forecast Variance to Analysis Variance, Error of Mean Difference, and Error of Pattern Variation. At NCEP EMC, only RMSE has been used as a metric to verify tropical vector wind. RMSE can at times be misleading, especially when different forecasts are verified against different analyses, *and/or* the anomalous pattern correlation between forecast and analysis is low.

$$E^2 = E_p^2 + E_m^2$$

$$E_p^2 = \sigma_f^2 + \sigma_a^2 - 2\sigma_f \sigma_a R$$

$$E_m^2 = \left(\overline{F} - \overline{A}\right)^2$$

Vector Wind Stats

So far the deviations are for scalar variables. For vector wind, the corresponding stats are defined in the following way.

Define $\vec{V}_f = u_f \vec{i} + v_f \vec{j} \qquad \vec{V}_a = u_a \vec{i} + v_a \vec{j}$

Then MSE:

$$E^2 = \frac{1}{n}\sum_{n=1}^{N}\left(\vec{V}_{fn} - \vec{V}_{an}\right)^2 = \frac{1}{n}\sum_{n=1}^{N}\left(\vec{V}_{fn} - \vec{V}_{an}\right)\bullet\left(\vec{V}_{fn} - \vec{V}_{an}\right)$$

$$= \frac{1}{n}\sum_{n=1}^{N}\left(u_{fn}^2 + v_{fn}^2\right) + \frac{1}{n}\sum_{n=1}^{N}\left(u_{an}^2 + v_{an}^2\right) - \frac{2}{n}\sum_{n=1}^{N}\left(u_{fn}u_{an} + v_{fn}v_{an}\right)$$

$$= \mathrm{A} + \mathrm{B} - 2\mathrm{C}$$

where $A = \frac{1}{n}\sum_{n=1}^{N}\left(u_{fn}^2 + v_{fn}^2\right) \quad B = \frac{1}{n}\sum_{n=1}^{N}\left(u_{an}^2 + v_{an}^2\right) \quad C = \frac{1}{n}\sum_{n=1}^{N}\left(u_{fn}u_{an} + v_{fn}v_{an}\right)$

A, B, and C are partial sums in NCEP EMC VSDB database

Anomalous Pattern Correlation:

$$R = \frac{\frac{1}{n}\sum_{n=1}^{N}\left(\vec{V}_{fn} - \bar{\vec{V}}_{fn}\right)\bullet\left(\vec{V}_{an} - \bar{\vec{V}}_{an}\right)}{\sqrt{\frac{1}{n}\sum_{n=1}^{N}\left(\vec{V}_{fn} - \bar{\vec{V}}_{fn}\right)^2 \cdot \frac{1}{n}\sum_{n=1}^{N}\left(\vec{V}_{an} - \bar{\vec{V}}_{an}\right)^2}} = \frac{\sum_{n=1}^{N}\left[\left(u_{fn} - \bar{u}_f\right)\left(u_{an} - \bar{u}_a\right) + \left(v_{fn} - \bar{v}_f\right)\left(v_{an} - \bar{v}_a\right)\right]}{\sqrt{\sum_{n=1}^{N}\left[\left(u_{fn} - \bar{u}_f\right)^2 + \left(v_{fn} - \bar{v}_f\right)^2\right] \cdot \sum_{n=1}^{N}\left[\left(u_{an} - \bar{u}_a\right)^2 + \left(v_{an} - \bar{v}_a\right)^2\right]}}$$

Vector Wind Stats

$$
\begin{aligned}
E^2 &= \frac{1}{n}\sum_{n=1}^{N}\left[\left(\vec{V}_{fn}-\overline{\vec{V}}_f\right)-\left(\vec{V}_{an}-\overline{\vec{V}}_a\right)+\left(\overline{\vec{V}}_f-\overline{\vec{V}}_a\right)\right]^2 \\
&= \frac{1}{n}\sum_{n=1}^{N}\left(\vec{V}_{fn}-\overline{\vec{V}}_f\right)^2+\frac{1}{n}\sum_{n=1}^{N}\left(\vec{V}_{an}-\overline{\vec{V}}_a\right)^2+\left(\overline{\vec{V}}_f-\overline{\vec{V}}_a\right)^2 \\
&\quad -\frac{2}{n}\sum_{n=1}^{N}\left(\vec{V}_{fn}-\overline{\vec{V}}_f\right)\bullet\left(\vec{V}_{an}-\overline{\vec{V}}_a\right) \\
&\quad \cancel{+2\left(\overline{\vec{V}}_f-\overline{\vec{V}}_a\right)\bullet\left\{\frac{1}{n}\sum_{n=1}^{N}\left(\vec{V}_{fn}-\overline{\vec{V}}_f\right)-\frac{1}{n}\sum_{n=1}^{N}\left(\vec{V}_{an}-\overline{\vec{V}}_a\right)\right\}} \\
&= \sigma_f^2+\sigma_a^2-2\sigma_f\sigma_a R+\left(\bar{u}_f-\bar{u}_a\right)^2+\left(\bar{v}_f-\bar{v}_a\right)^2 \\
&= E_p^2+E_m^2
\end{aligned}
$$

where

$$E_m^2=\left(\overline{\vec{V}}_f-\overline{\vec{V}}_a\right)^2=\left(\bar{u}_f-\bar{u}_a\right)^2+\left(\bar{v}_f-\bar{v}_a\right)^2$$

MSE by Mean Difference

$$E_p^2=\sigma_f^2+\sigma_a^2-2\sigma_f\sigma_a R$$

MSE by Pattern Variation

$$\sigma_f^2=\frac{1}{n}\sum_{n=1}^{N}\left(\vec{V}_{fn}-\overline{\vec{V}}_f\right)^2=\frac{1}{n}\sum_{n=1}^{N}\left[\left(u_{fn}-\bar{u}_f\right)^2+\left(v_{fn}-\bar{v}_f\right)^2\right]$$

Variance of forecast

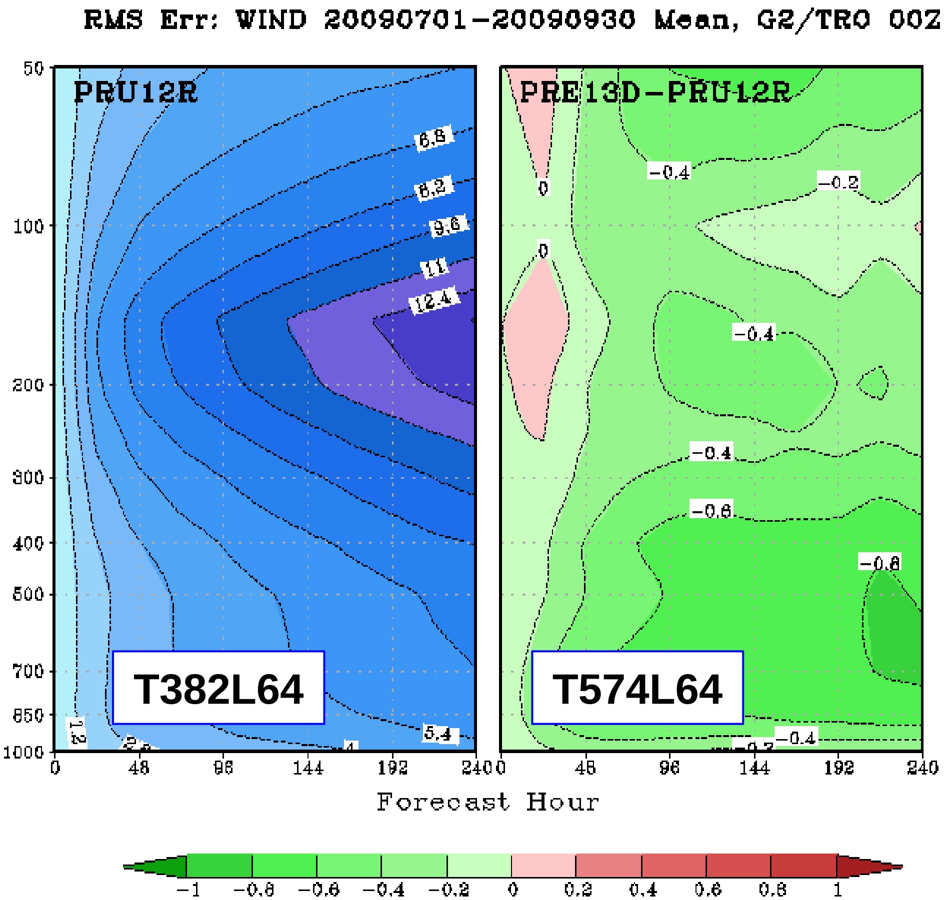
$$\sigma_a^2=\frac{1}{n}\sum_{n=1}^{N}\left(\vec{V}_{an}-\overline{\vec{V}}_a\right)^2=\frac{1}{n}\sum_{n=1}^{N}\left[\left(u_{an}-\bar{u}_a\right)^2+\left(v_{an}-\bar{v}_a\right)^2\right]$$

Variance of analysis

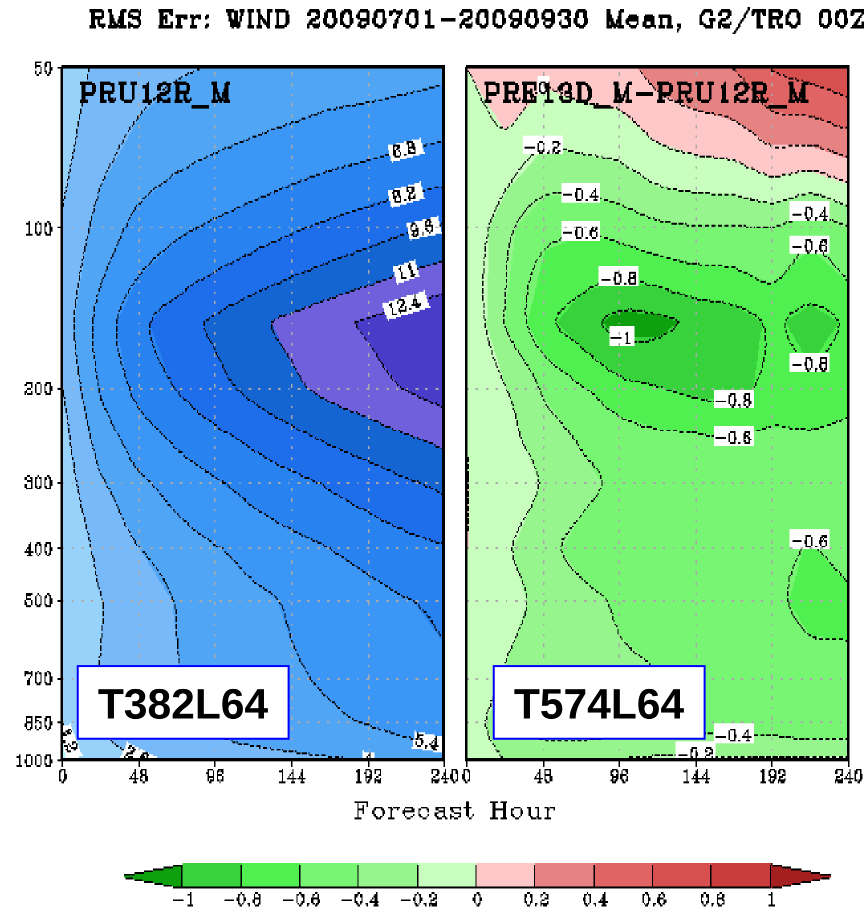
$$E = \sqrt{\frac{1}{n}\sum_{n=1}^{N}\left(F_n - A_n\right)^2}$$

Case Demonstration:

Impact of Analyses on Tropical Vector Wind RMSE

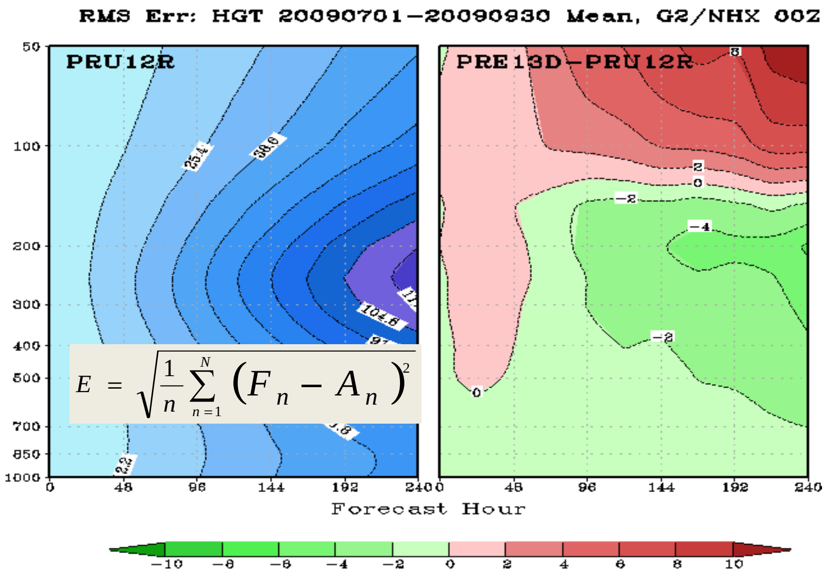


Each experiment is verified against its **own analysis.**

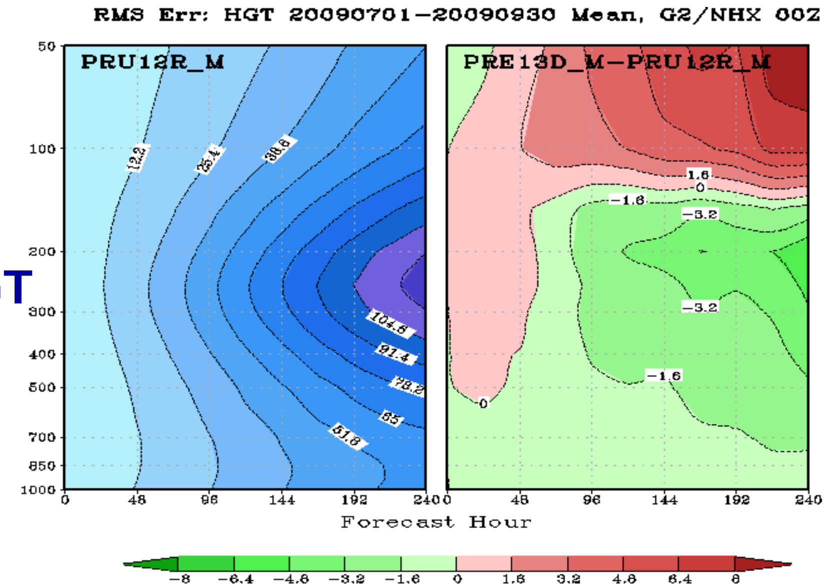


Both experiments are verified against the **same analysis**, which is the mean of the two experiments.

Impact of Analyses on RMSE



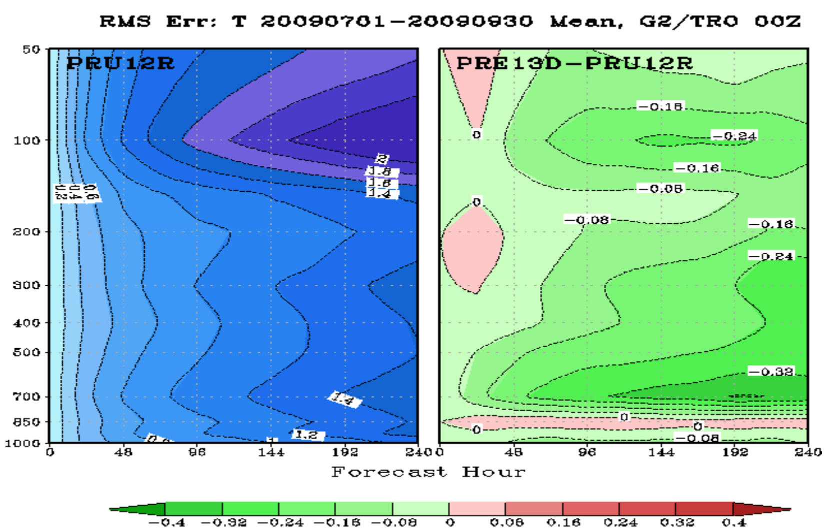
$$E = \sqrt{\frac{1}{n}\sum_{n=1}^{N}\left(F_n - A_n\right)^2}$$



NH HGT

own analysis

same analysis



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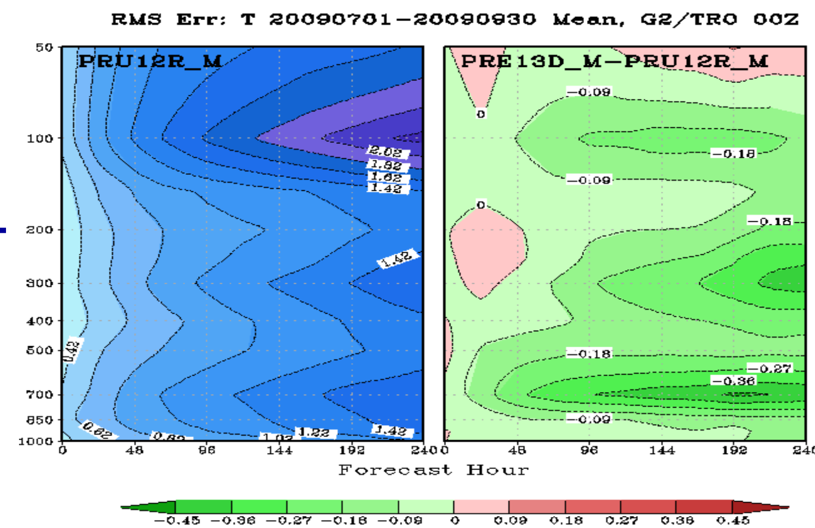
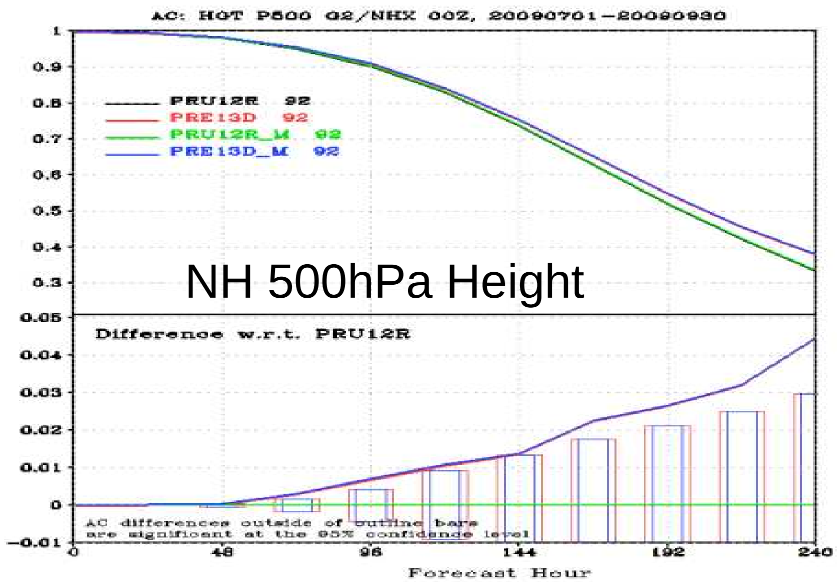
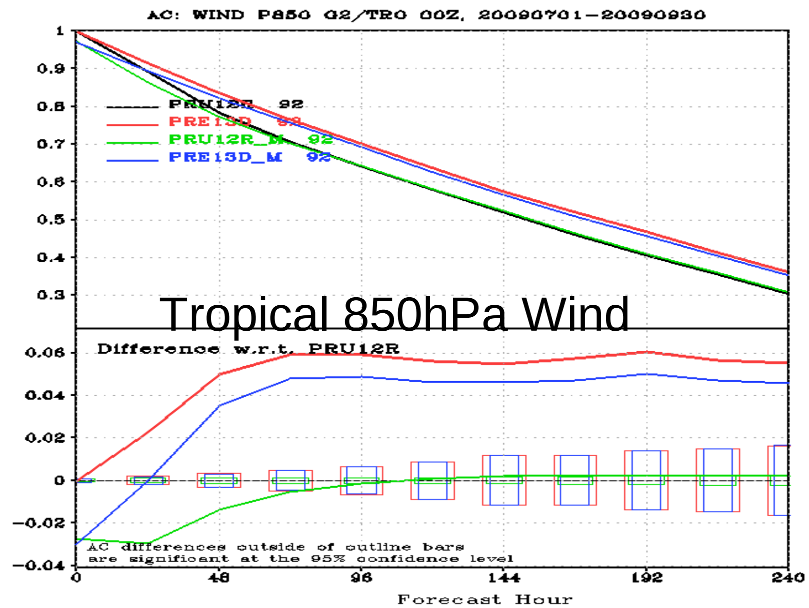
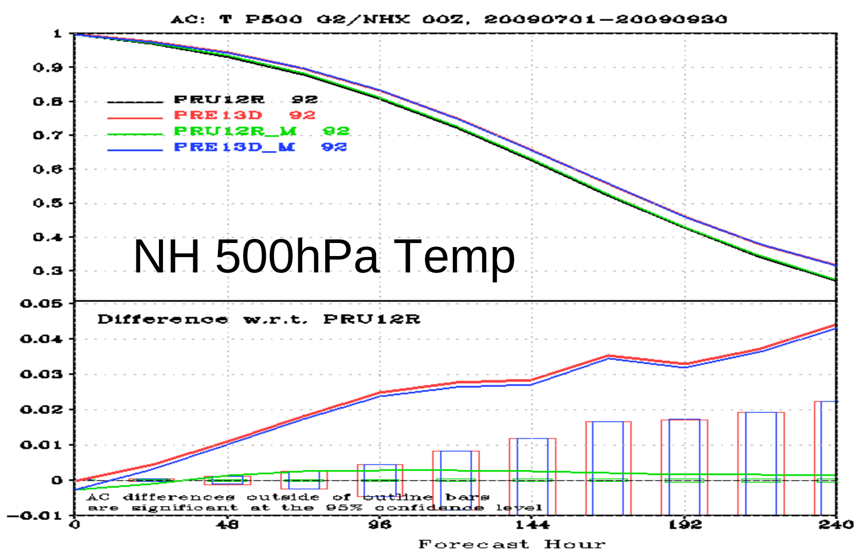


Impact of Analyses on Anomaly Correlation







Using different analysis has little impact on anomaly correlation for all variables except for winds at initial forecast time

Summary II

Using different analysis has significant impact on the RMSE of winds. Its impact on the RMSE of height and temperature is smaller.

Using different analysis has negligible impact on Anomaly Correlation, except for winds at initial time.

***Recommendation*: the same analysis should be used for verification when comparing different models and/or different experiments.**

In the next few slides the same analysis is used for verification.

Case Demonstration: Decomposing MSE of Scalar Variables

The following five components will be examined. All forecasts are verified against the same analysis, i.e., the mean of the two experiments pru12r and pre13d.

$$E^2 = E_p^2 + E_m^2$$

Total MSE

$$E_m^2 = \left(\overline{F} - \overline{A}\right)^2$$

MSE by Mean Difference

$$E_p^2 = \sigma_f^2 + \sigma_a^2 - 2\sigma_f \sigma_a R$$

MSE by Pattern Variation

$$\lambda = \frac{\sigma_f}{\sigma_a}$$

Ratio of Standard Deviation: Fcst/Anal

$$MSESS = 1 - E^2 / \sigma_a^2 = 2\lambda R - \lambda^2 - E_m^2 / \sigma_a^2$$

Murphy's Mean-Squared Error Skill Score

$$R = \frac{1}{n} \cdot \sum_{n=1}^{N} \left(F_n - \overline{F}\right) \cdot \left(A_n - \overline{A}\right) \Big/ \sigma_f \sigma_a$$

Anomalous Pattern Correlation

$$\sigma_f^2 = \frac{1}{n} \sum_{n=1}^{N} \left(F_n - \overline{F}\right)^2 \qquad \sigma_a^2 = \frac{1}{n} \sum_{n=1}^{N} \left(A_n - \overline{A}\right)^2$$

Case Demonstration: Decomposing RMSE of Vector Wind

The following five components will be examined. All forecasts are verified against the same analysis, i.e., the mean of the two experiments pru12r and pre13d.

$$E^2 = E_p^2 + E_m^2$$

Total MSE

$$E_m^2 = \left(\overline{u}_f - \overline{u}_a\right)^2 + \left(\overline{v}_f - \overline{v}_a\right)^2$$

MSE by Mean Difference

$$E_p^2 = \sigma_f^2 + \sigma_a^2 - 2\sigma_f \sigma_a R$$

MSE by Pattern Variation

$$\lambda = \frac{\sigma_f}{\sigma_a}$$

Ratio of Standard Deviation: Fcst/Anal

$$MSESS = 1 - E^2/\sigma_a^2 = 2\lambda R - \lambda^2 - E_m^2/\sigma_a^2$$

Murphy's Mean-Squared Error Skill Score

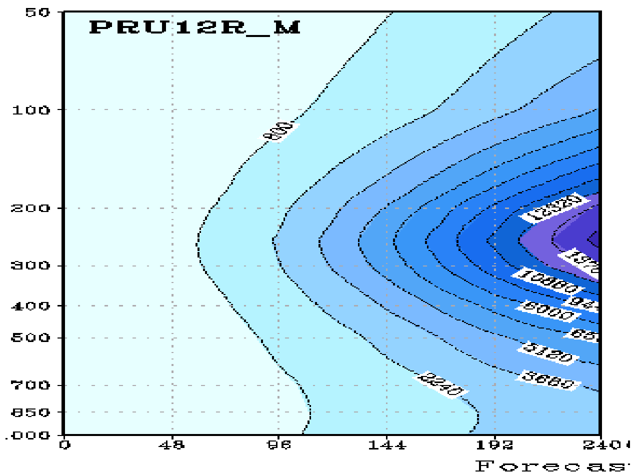
$$R = \frac{\frac{1}{n}\sum_{n=1}^{N}\left[\left(u_{fn} - \overline{u}_f\right)\left(u_{an} - \overline{u}_a\right) + \left(v_{fn} - \overline{v}_f\right)\left(v_{an} - \overline{v}_a\right)\right]}{\sigma_f \cdot \sigma_a}$$

Anomalous Pattern Correlation

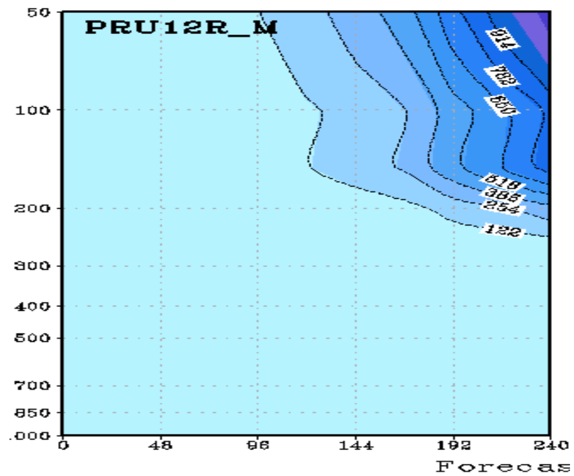
$$\sigma_f^2 = \frac{1}{n}\sum_{n=1}^{N}\left[\left(u_{fn} - \overline{u}_f\right)^2 + \left(v_{fn} - \overline{v}_f\right)^2\right] \qquad \sigma_a^2 = \frac{1}{n}\sum_{n=1}^{N}\left[\left(u_{an} - \overline{u}_a\right)^2 + \left(v_{an} - \overline{v}_a\right)^2\right]$$

Decomposing NH HGT RMSE^2, T382L64 GFS, 200907-200909

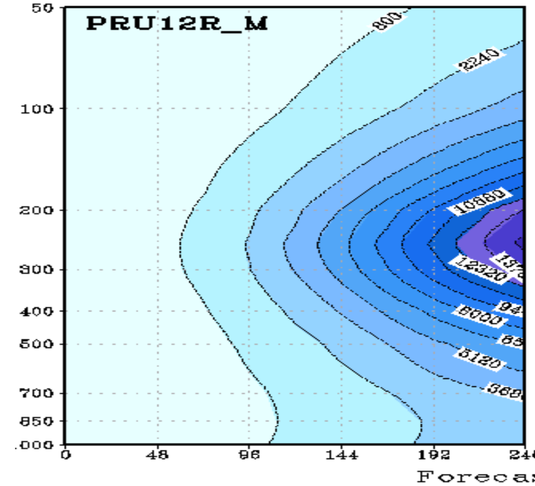
Total MSE



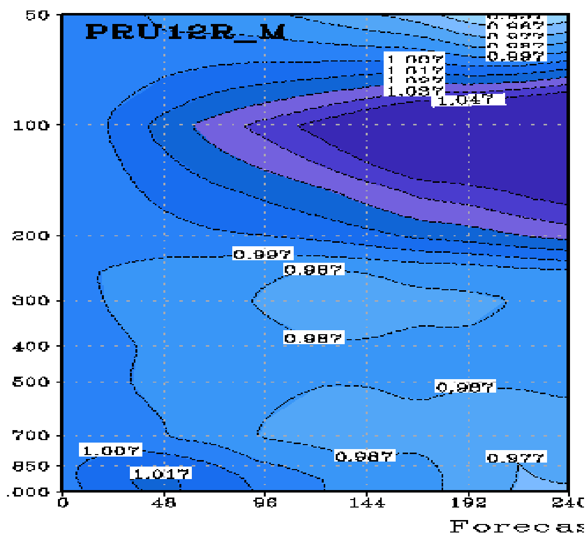
MSE by Mean Difference



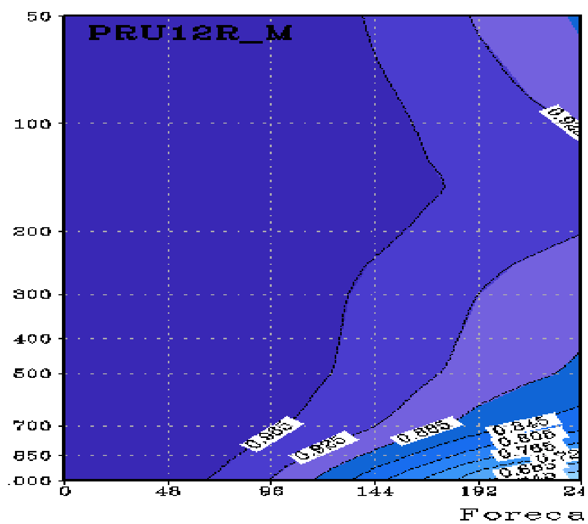
MSE by Pattern Variation



Ratio of Standard Deviation



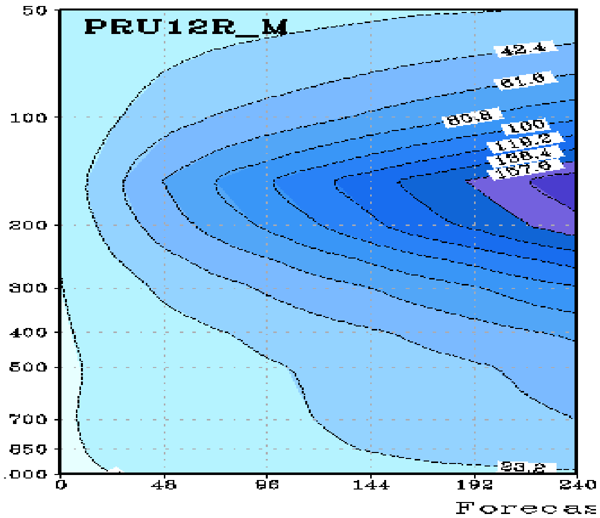
Anomalous Pattern Correlation



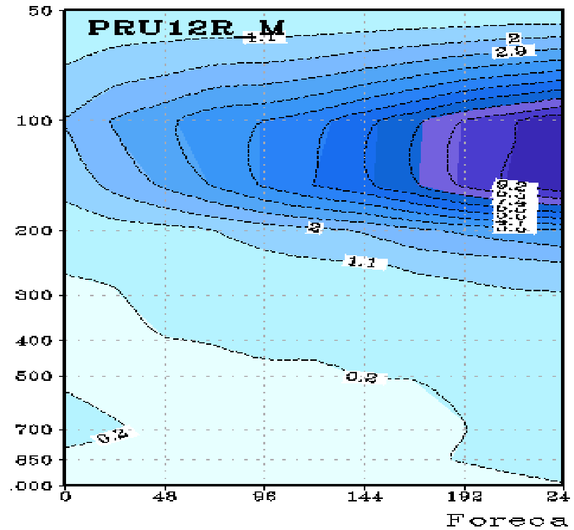
- Total RMSE is primarily composed of EMD in the lower stratosphere and EPV in the troposphere.
- HGT generally has high anomalous pattern correlation.
- The forecast variance is lower than that of analysis in the lower troposphere and stratosphere, and larger near the tropopause.
- Forecast variance near tropopause increases with forecast lead time .

Decomposing Tropical Vector Wind RMSE^2, T382L64 GFS, 200907-200909

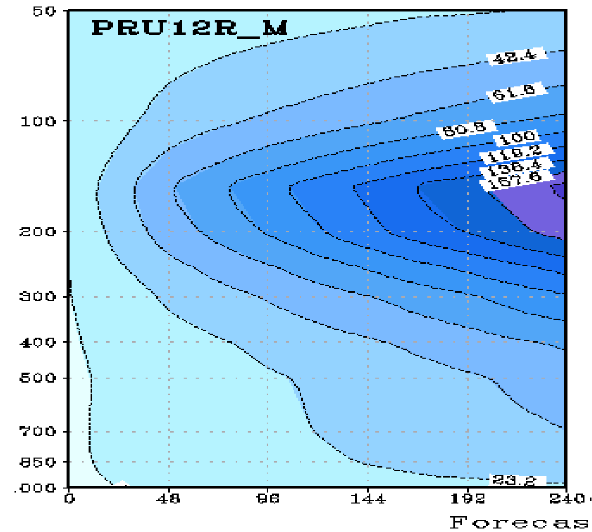
Total MSE



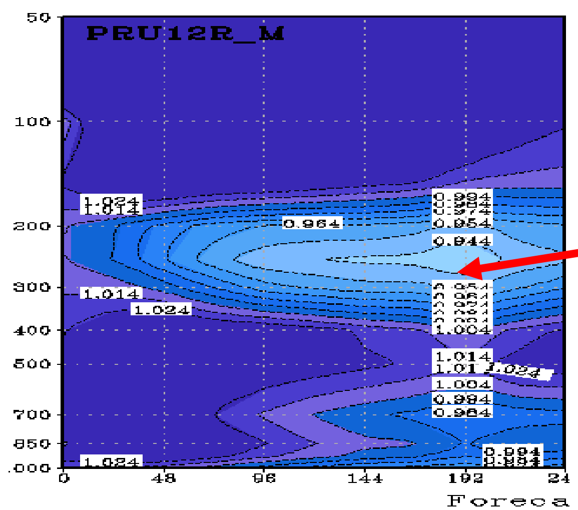
MSE by Mean Difference



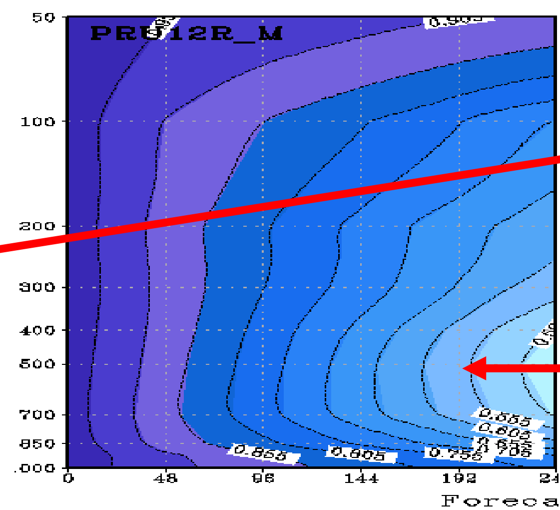
MSE by Pattern Variation



Ratio of Standard Deviation



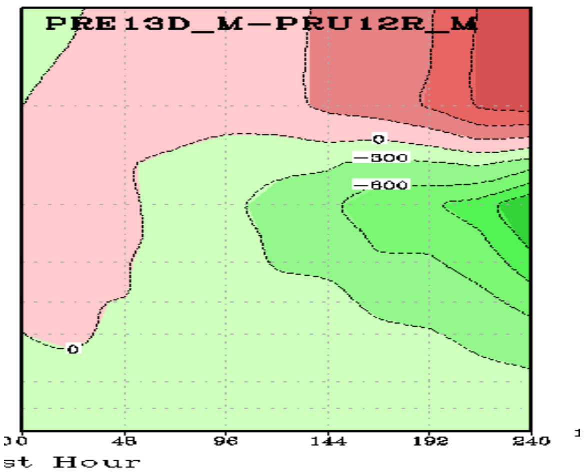
Anomalous Pattern Correlation



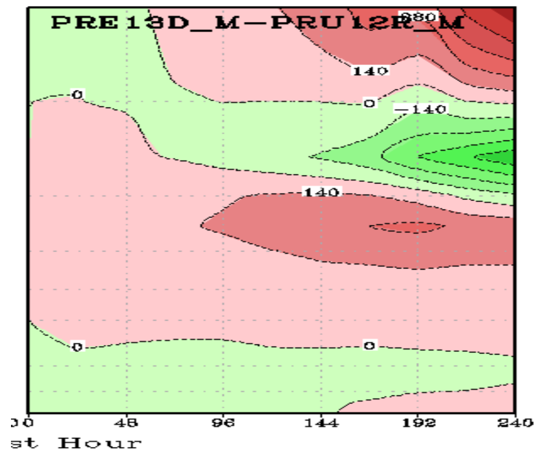
- For tropical Wind, both EMD and EPV are concentrated near the tropopause , and increase with forecast lead time.
- T382 GFS is not able to maintain wind variance near the tropopause, and has stronger variance everywhere else.
- Wind anomalous pattern correlation is much poorer than that of HGT, and faints quickly with forecast lead time, especially in the lower troposphere.

Decomposing NH HGT RMSE^2, Comparing T574 to T382, 200907-200909

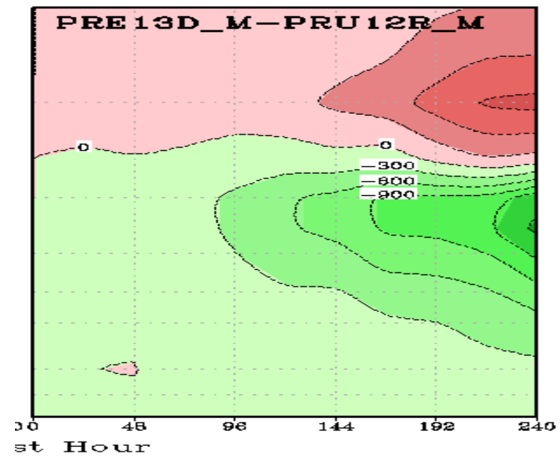
Total MSE



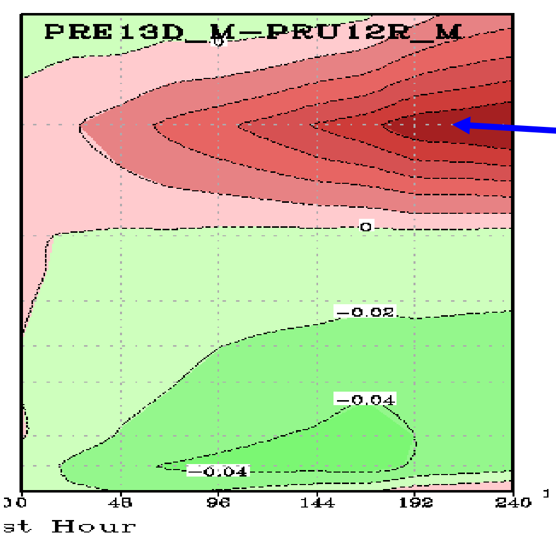
MSE by Mean Difference



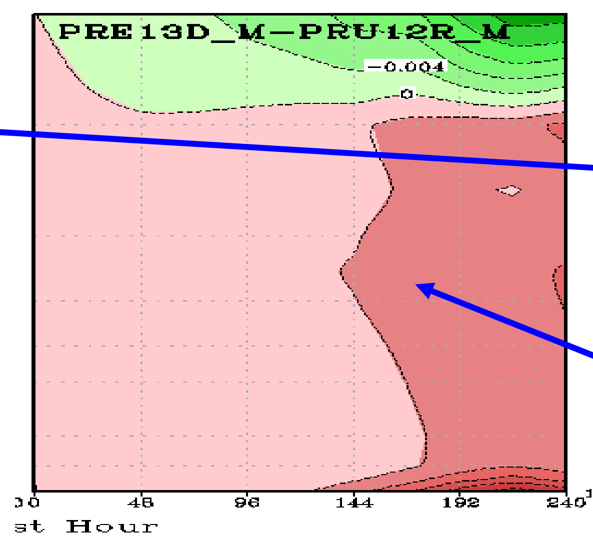
MSE by Pattern Variation



Ratio of Standard Deviation



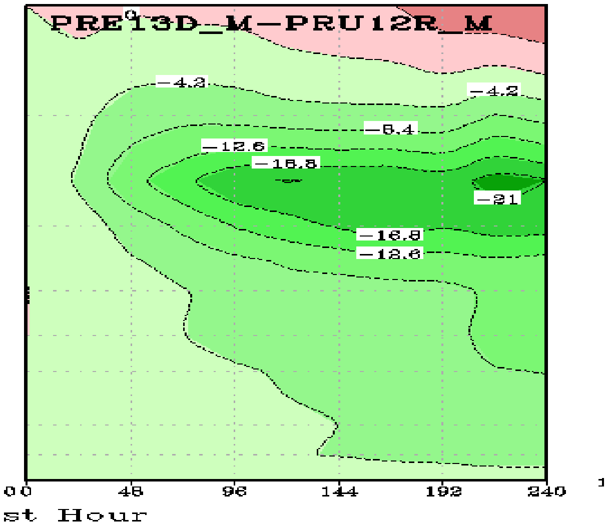
Anomalous Pattern Correlation



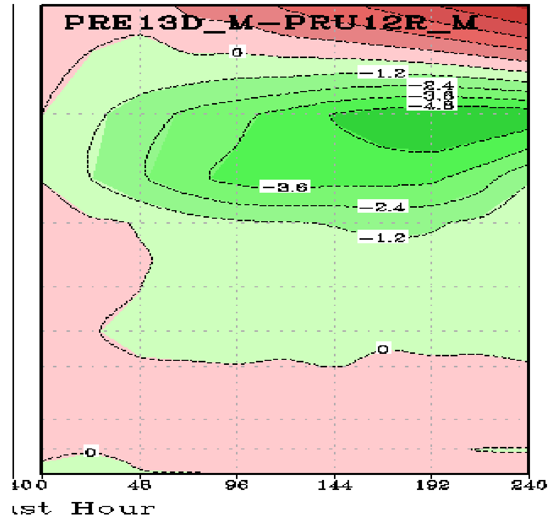
- The reduction of total HGT RMSE in the troposphere comes from EPV reduction. Both EMD and EPV increased in the lower stratosphere.
- Compare to T382, T574 has larger forecast variance near the tropospause, and smaller variance in the lower troposphere.
- Compare to T382, T574 has better HGT AC in the troposphere and worse AC in the lower stratosphere.

Decomposing Tropical Vector Wind RMSE^2, Compare T574 with T382, 200907-200909

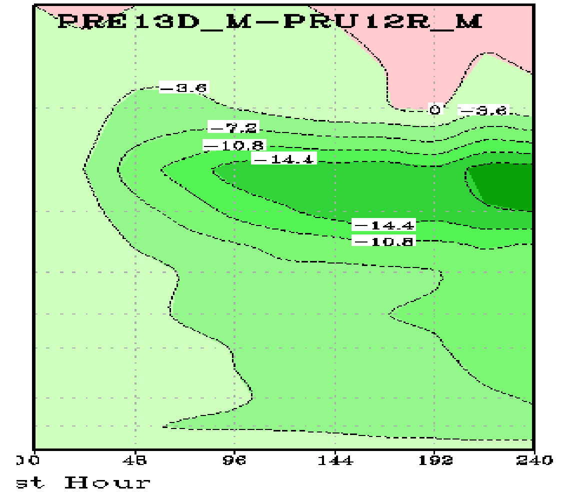
Total MSE



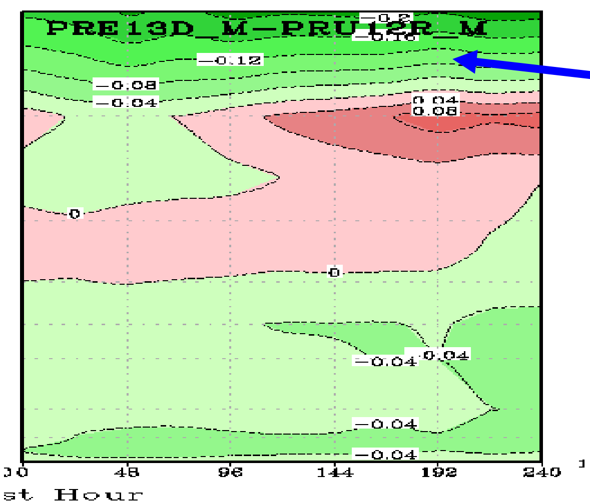
MSE by Mean Difference



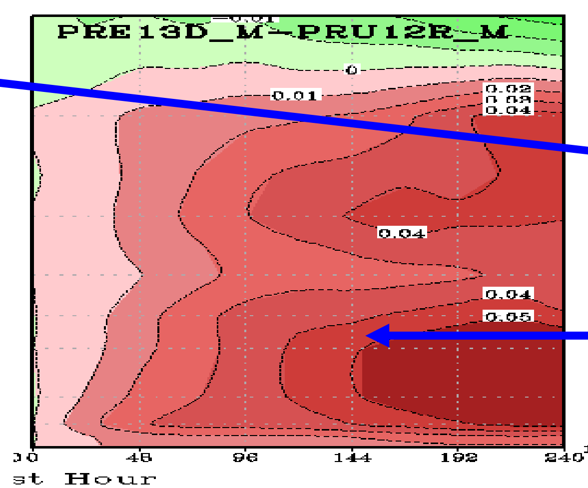
MSE by Pattern Variation



Ratio of Standard Deviation



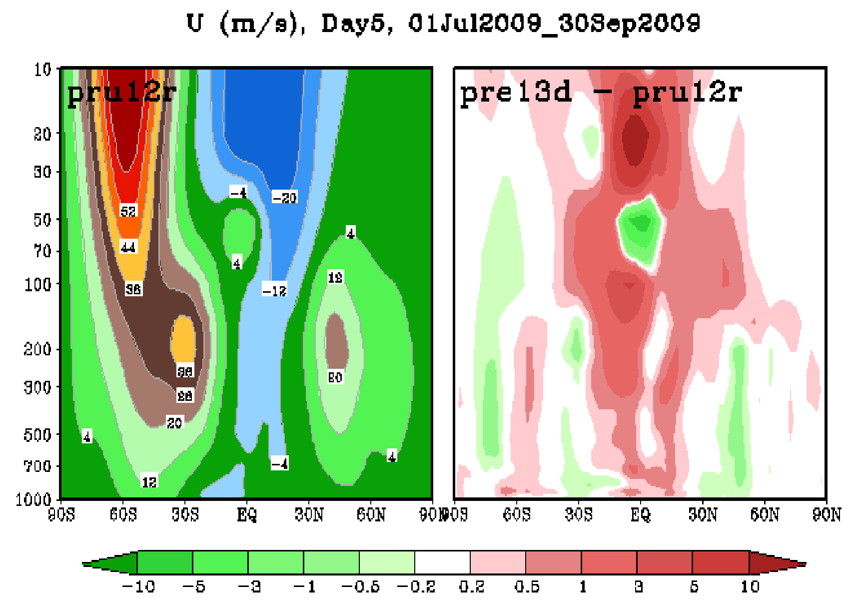
Anomalous Pattern Correlation



- Compared to T382, T574 has smaller RMSE in the troposphere, coming from reduction in both EMD and EPV. In the lower stratosphere, EMD increased.
- Compare to T382, T574 has much weaker wind variance in the lower stratosphere.
- T574 has better anomalous pattern correlation in the troposphere. Therefore, the reduction in EPV near the tropopause is credible, and the wind variance is also stronger.

Compared to T382 GFS, T574 GFS has better forecast skills in the troposphere.

T574 reduced tropical wind variance in the lower stratosphere. Mean tropical wind in the lower stratosphere is also weaker.



Summary

RMSE/MSE can be at times misleading. Its fairness as a performance metric depends on the goodness of mean difference, standard deviation, and pattern correlation.

If pattern correlation is low, RMSE tends to award forecasts with smoother fields. The implication is that RMSE should not be used for extended NWP forecasts and seasonal forecasts either.

The same analysis should be used for verification when comparing different models and/or different experiments. The impact of analysis is on anomaly correlation than on RMSE, and less on height than on winds.

At NCEP/EMC, RMSE has been almost exclusively used to measure the performance of tropical wind. A more comprehensive verification should at least include MSE, MSE by Mean Difference, Anomalous Pattern Correlation, and Ratio of Forecast Variance to Analysis Variance.

MSE should be used instead of RMSE or standard deviation, the summation of the latter is hard to interpret in math terms.