Model Uncertainty in Ensemble Forecasting: Stochastic Physics and Truncation Error

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Joao Teixeira and Carolyn Reynolds

During the first six months of this project we explored the time-step sensitivity of two non-linear atmospheric models and showed that solutions with small but different time-steps will decouple from each other after a certain finite amount of simulation time. The decoupling time depends on the time-step in a logarithmic way (due to the exponential growth of perturbations), which means that the decoupling time can be predicted. The logarithmic nature of decoupling time also implies that however small the time-step may be, there is always a finite point in time after which the numerical solution diverges, which means that for chaotic systems uniform numerical convergence can only be guaranteed for a finite amount of time.

The logarithmic relation is present in simulations using two models of different levels of complexity: the Lorenz-63 equations and a quasi-geostrophic (QG) model. In the Lorenz equations, for regimes that are not fully chaotic, the sensitivity to the time-step is more complex and different time-steps can lead to different model climates or even different regimes. These results suggest that the statistics of climate models may be affected by the time-step and grid-size.

Using a QG model we show that truncation errors caused by different time-steps evolve in a substantially different manner when compared to initial condition errors. Initial condition errors grow exponentially as expected while truncation errors show an initially rapid growth followed by a plateau (slow error growth) and then exponential error growth until saturation.

We propose a simple analytic model of truncation error growth that considers the relative error growth rates in the stable and unstable directions. This simple model suggests that time-step truncation error in non-linear models is a combination of two errors: (i) a stable error with an initially rapid growth caused by the fact that each model simulation moves rapidly toward its own attractor, away from the attractor of the control, exponentially decaying towards a saturation value that is a function of the time-step; and (ii) an unstable error with a basically exponential growth similar to initial condition error.

The project has followed closely the work plan for the first six months of the first year, which implied to conduct experiments assessing the impact of the truncation error on a hierarchy of atmospheric models: (a) Lorenz-63 equations and (b) Quasi-Geostrophic model (time-step sensitivity).