Northern American Ensemble Forecast System NCEP Development Oriented Verification

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NOAA/NWS/NCEP

Acknowledgements:
Binbin Zhou and Zoltan Toth EMC

NAEFS verification

- Web-site:
  - http://wwwt.emc.ncep.noaa.gov/gmb/yzhu/html/opr/naefs.html
- Reference: NCEP/NCAR 40y reanalysis (next slide)
- Variables:
  - 1000hPa, 500hPa heights, 850hPa, 2m temperature, 10m u and v
- Verified for ensemble mean:
  - PAC, RMS errors, spread, mean error (bias) and absolute error
- Verified for ensemble distribution:
  - Histogram (Talagrand)
- Verified for ensemble probabilistic forecast
  - ROC, RPSS, CRPS, BSS (Resolution and Reliability), EV
- Regions:
  - NH, SH, Tropical, Asia, Europe and Northern American
- Statistics from seasonal average

Climatological Data

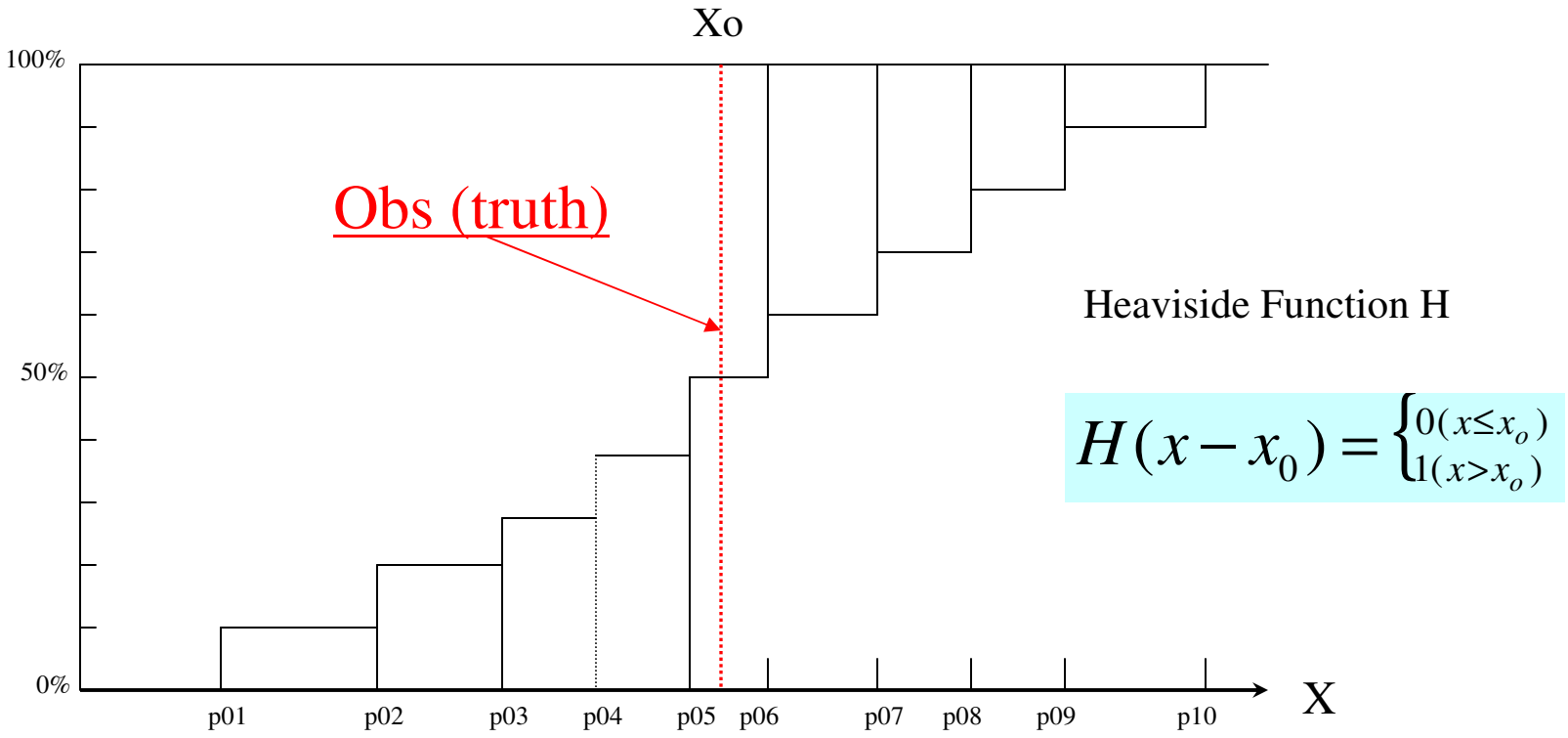
- NCEP/NCAR 40 years (1958-1997) reanalysis
- Monthly Sampling
  - For example: 40*30=1200
- Generating10 equally-a-likely, based on monthly sampling
- Projected to verify date
- All forecast skills will base on 10 equally-a-likely climatological bins
- There is a limitation of skill scores
  - Need to consider the analysis difference between reanalysis and current GFS/GSI analysis
  - Will set up on future verification scheme

Continuous Rank Probability Score

$$CRPS = \int_{-\infty}^{+\infty} [F(x) - H(x - x_0)]^2 \, dx$$

CRP Skill Score is

$$CRPSS = \frac{CRPS_c - CRPS_f}{CRPS_c}$$

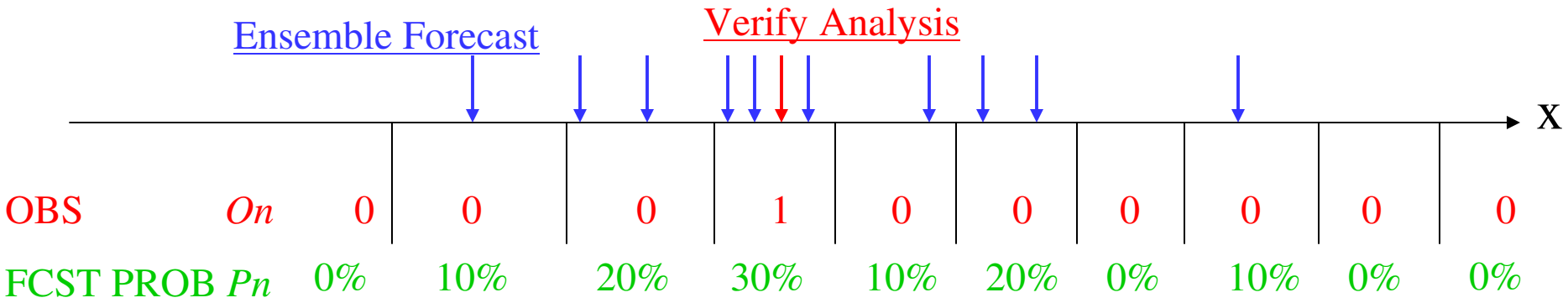


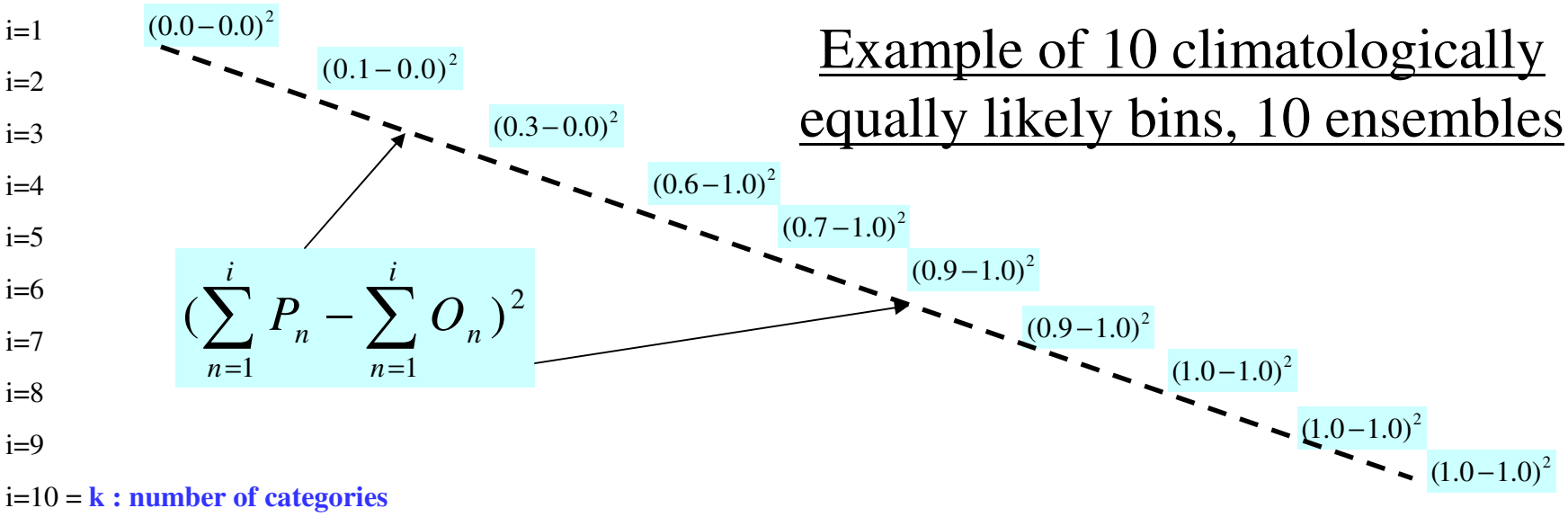
Order of 10 ensemble members (p01, p02,…,p10)

Ranked Probabilistic Score

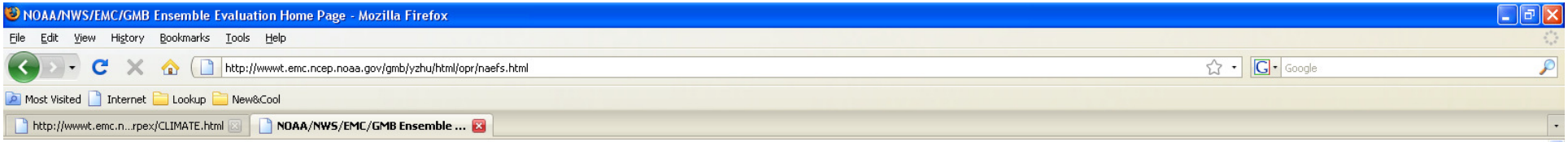
Ranked (ordered) Probability Score (RPS) is to verify multi-category probability forecasts, to measure both reliability and resolution which based on climatologically equally likely bins

$$RPS = 1 - \frac{1}{k-1}\left[\sum_{i=1}^{k}\left(\sum_{n=1}^{i} P_n - \sum_{n=1}^{i} O_n\right)^2\right] \quad \text{and} \quad RPSS = \frac{RPS_f - RPS_c}{1 - RPS_c}$$





Example of 10 climatologically equally likely bins, 10 ensembles



NAEFS Verification Home Page of

Developed by Yuejian Zhu

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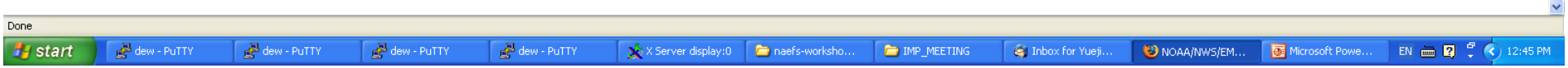
Ensemble Verification (short presentation)

Predictability Metting Presentation

Period	NCEP .vs NCEPb	CMC .vs CMCb	NCEP .vs CMC	NCEPb .vs CMCb	NAEFS
sum2008	Yes	Yes	Yes	Yes	Yes
spr2008	Yes	Yes	Yes	Yes	Yes
win0708	Yes	Yes	Yes	Yes	Yes
fal2007	Yes	Yes	Yes	Yes	Yes
sum2007	Yes	Yes	Yes	Yes	Yes
spr2007	Yes	Yes	Yes	Yes	Yes
win0607	Yes	Yes	Yes	Yes	Yes
fal2006	Yes	Yes	Yes	Yes	Yes
sum2006	Yes	Yes	Yes	Yes	Yes



Example of web-page setting:
http://wwwt.emc.ncep.noaa.gov/gmb/yzhu/html/opr/naefs.html

Global Ensemble Model Evaluation: (NCEP against NCEPb)

500 hPa Height Scores NCEP .vs NCEPb									
NH	ROC	EV	RPSS	BSS	CRP	CRPS	RMS/SPRD	ERR/ABSE	HISTOGRAM
SH	ROC	EV	RPSS	BSS	CRP	CRPS	RMS/SPRD	ERR/ABSE	HISTOGRAM
TROP	ROC	EV	RPSS	BSS	CRP	CRPS	RMS/SPRD	ERR/ABSE	HISTOGRAM

1000 hPa Height Scores (NCEP .vs NCEPb)									
NH	ROC	EV	RPSS	BSS	CRP	CRPS	RMS/SPRD	ERR/ABSE	HISTOGRAM
SH	ROC	EV	RPSS	BSS	CRP	CRPS	RMS/SPRD	ERR/ABSE	HISTOGRAM
TROP	ROC	EV	RPSS	BSS	CRP	CRPS	RMS/SPRD	ERR/ABSE	HISTOGRAM

850 hPa Temperature Scores (NCEP .vs NCEPb)									
NH	ROC	EV	RPSS	BSS	CRP	CRPS	RMS/SPRD	ERR/ABSE	HISTOGRAM
SH	ROC	EV	RPSS	BSS	CRP	CRPS	RMS/SPRD	ERR/ABSE	HISTOGRAM
TROP	ROC	EV	RPSS	BSS	CRP	CRPS	RMS/SPRD	ERR/ABSE	HISTOGRAM

2 Meters Temperature Scores (NCEP .vs NCEPb)									
NH	ROC	EV	RPSS	BSS	CRP	CRPS	RMS/SPRD	ERR/ABSE	HISTOGRAM
SH	ROC	EV	RPSS	BSS	CRP	CRPS	RMS/SPRD	ERR/ABSE	HISTOGRAM
TROP	ROC	EV	RPSS	BSS	CRP	CRPS	RMS/SPRD	ERR/ABSE	HISTOGRAM

ISSUES ADDRESSED

- Long-term performance measurement
  - Deterministic
  - Ensemble mean forecast
  - Probabilistic forecast
- Effect of bias-correction
  - Different variables
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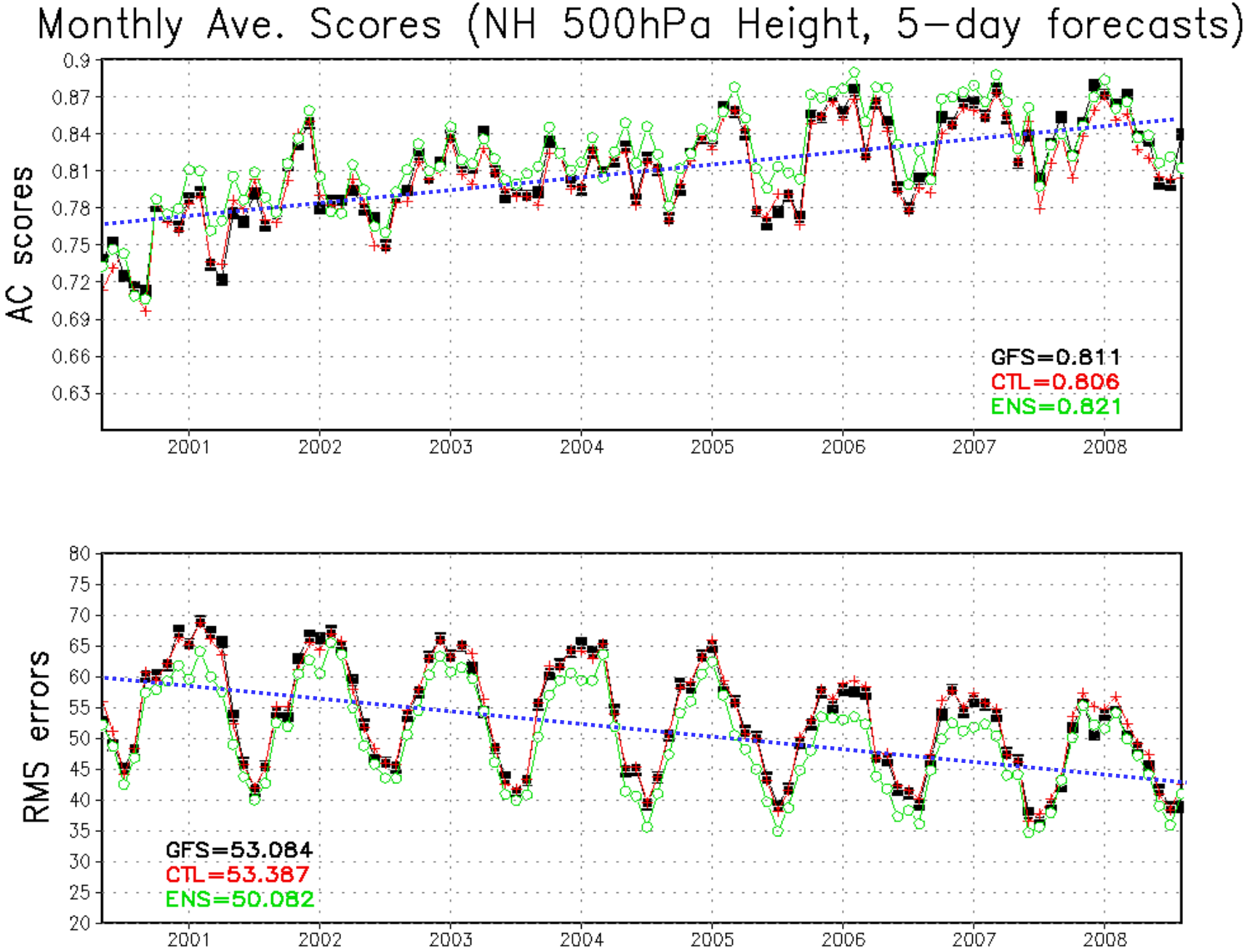
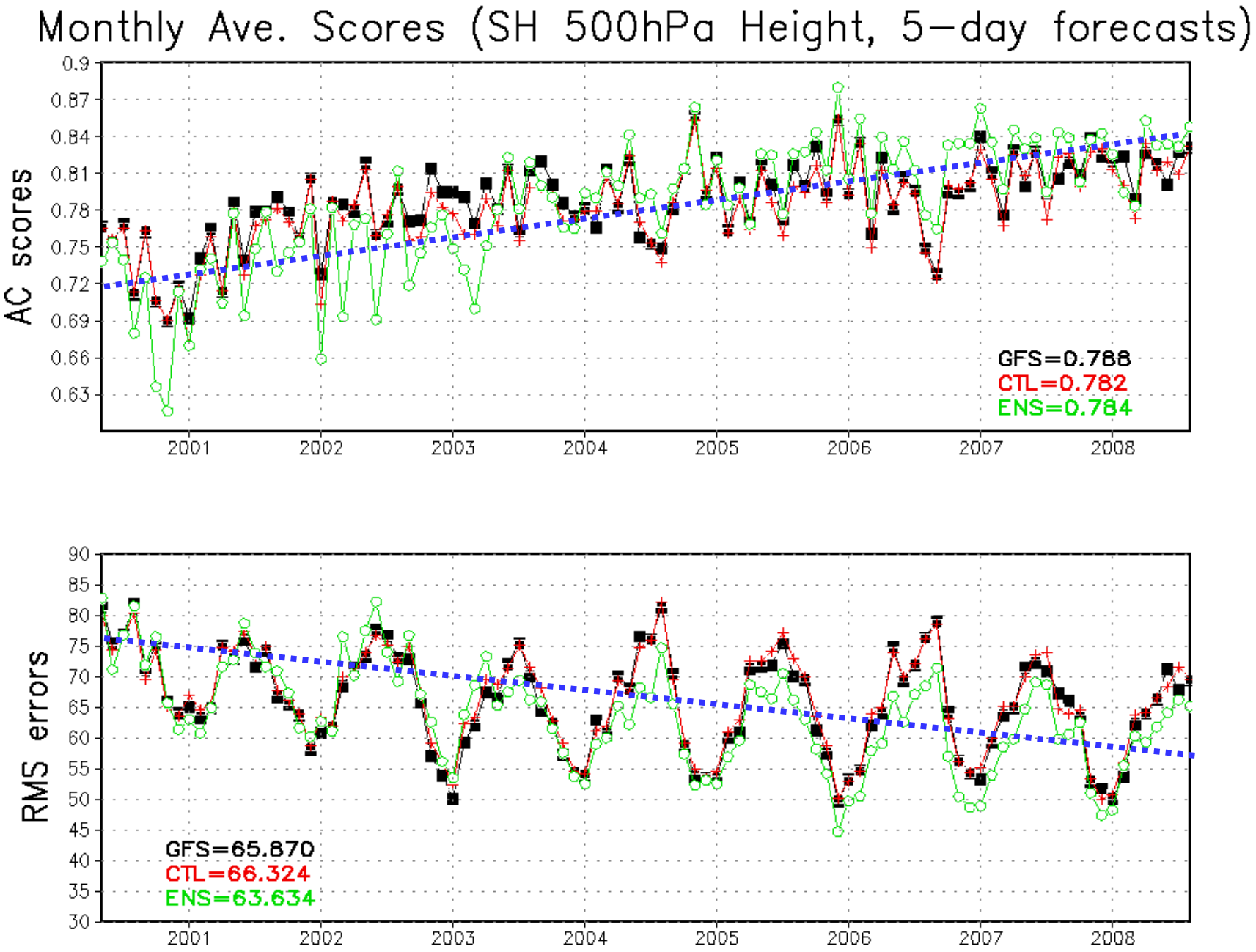
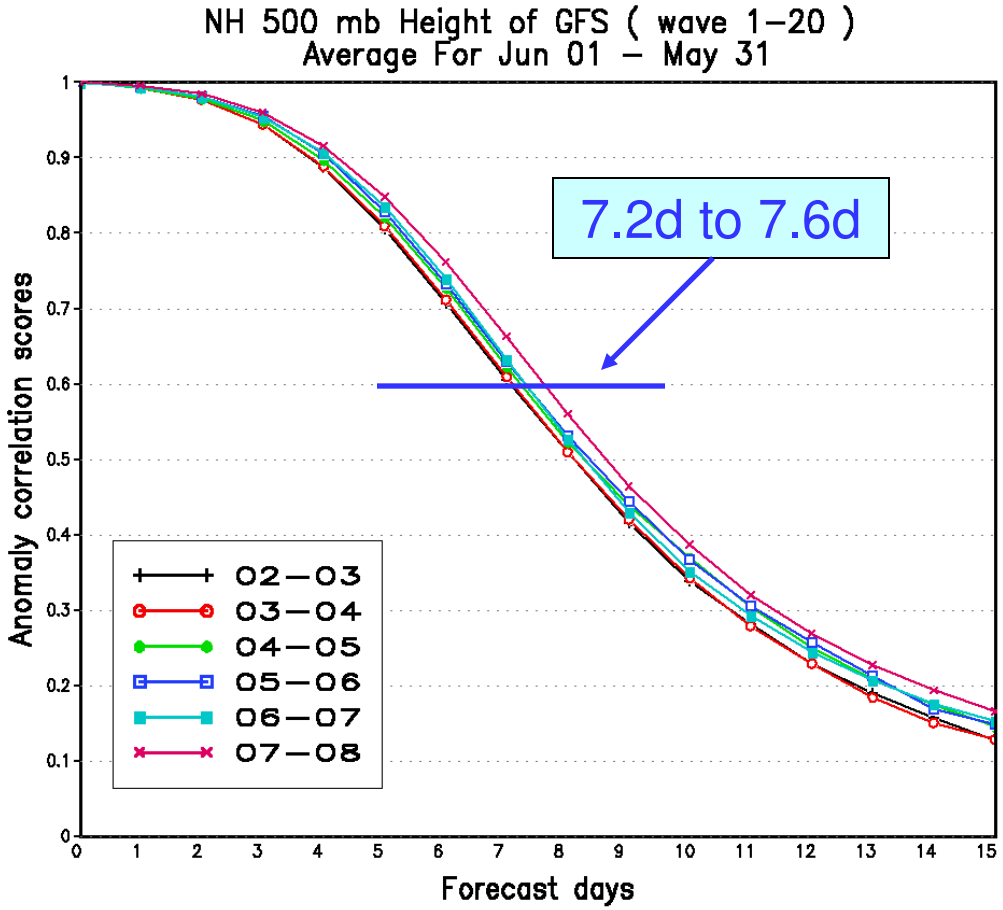
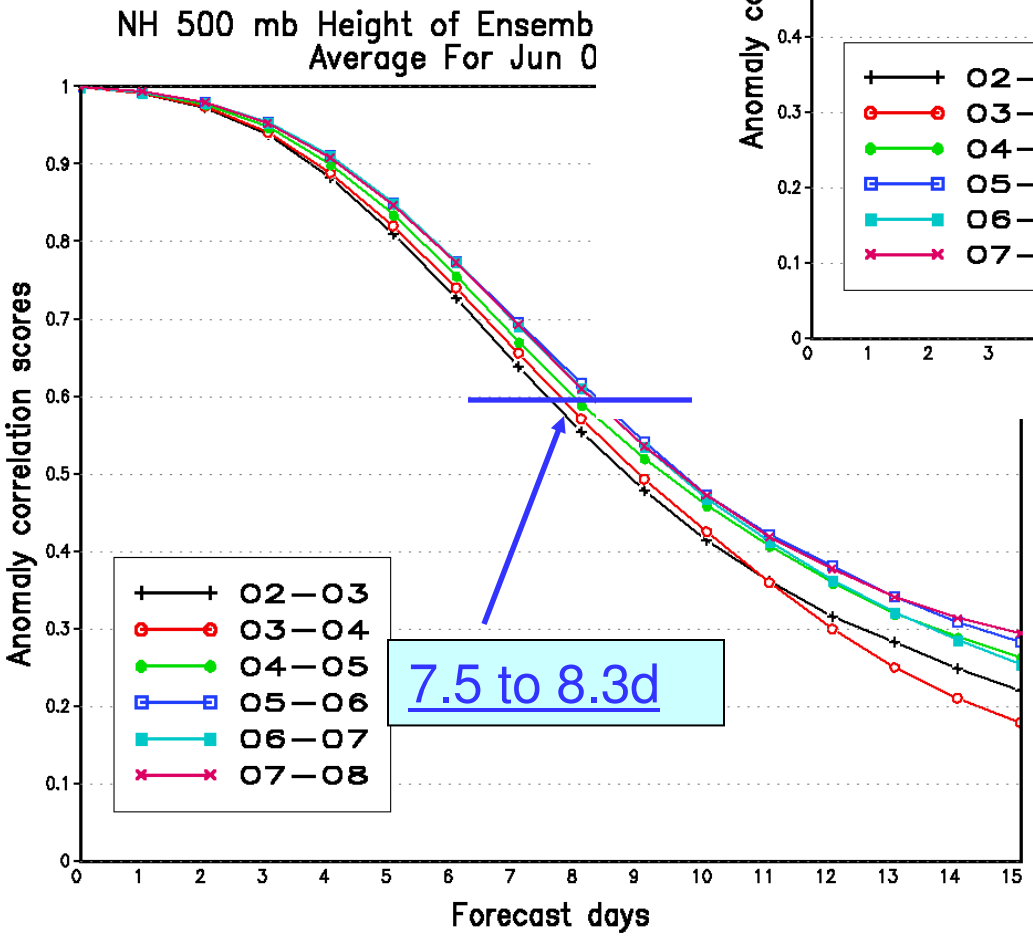
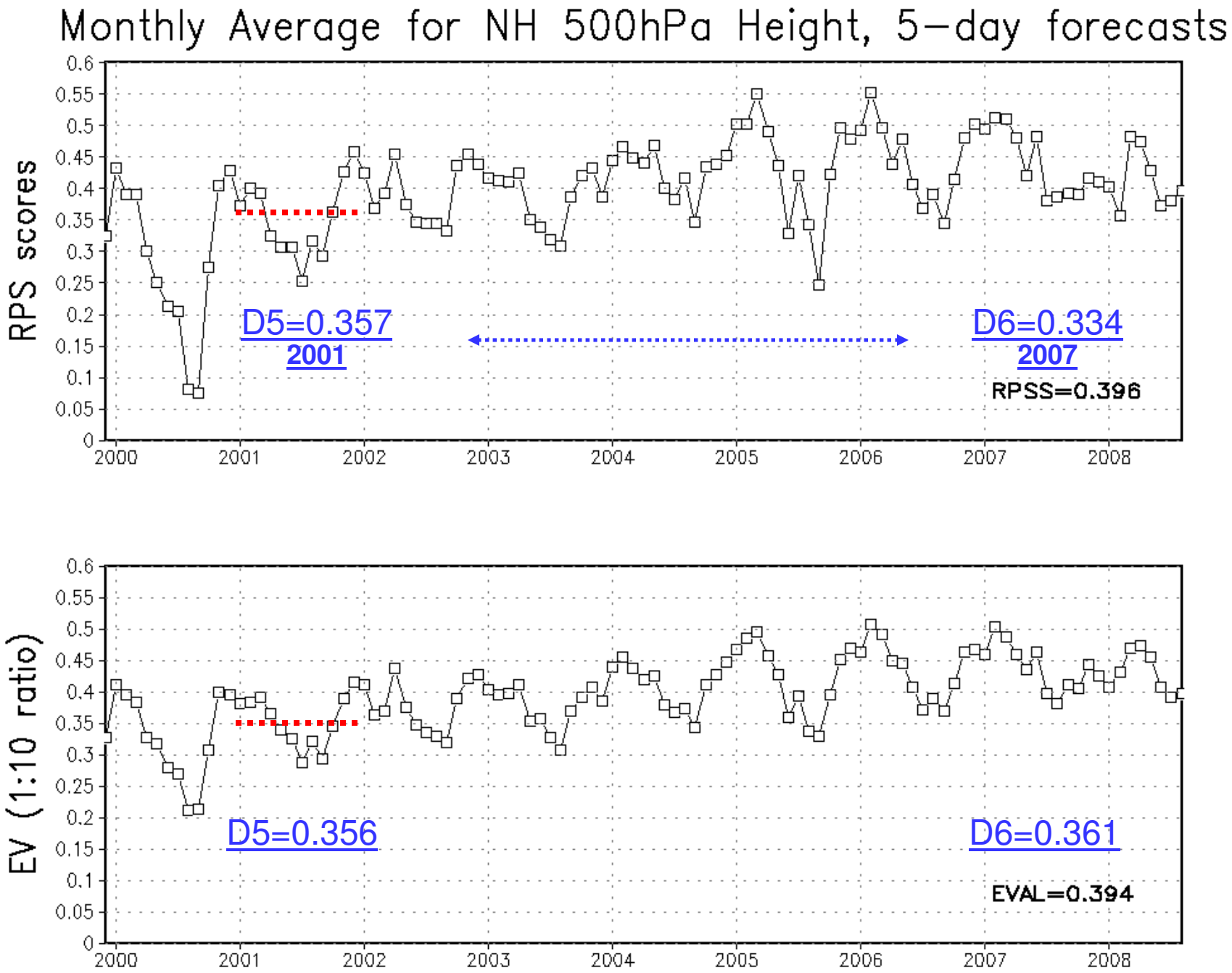
  - Short and long lead-time
- Probabilistic forecast products
  - 10% and 90% probability forecast


Monthly Ave. Scores (NH 500hPa Height, 5-day forecasts)
AC scores
0.9
0.87
0.84
0.81
0.78
0.75
0.72
0.69
0.66
0.63
GFS=0.811
CTL=0.806
ENS=0.821
2001
2002
2003
2004
2005
2006
2007
2008
RMS errors
80
75
70
65
60
55
50
45
40
35
30
25
20
GFS=53.084
CTL=53.387
ENS=50.082
2001
2002
2003
2004
2005
2006
2007
2008


Monthly Ave. Scores (SH 500hPa Height, 5-day forecasts)
AC scores
0.9
0.87
0.84
0.81
0.78
0.75
0.72
0.69
0.66
0.63
GFS=0.788
CTL=0.782
ENS=0.784
2001
2002
2003
2004
2005
2006
2007
2008
RMS errors
90
85
80
75
70
65
60
55
50
45
40
35
30
GFS=65.870
CTL=66.324
ENS=63.634
2001
2002
2003
2004
2005
2006
2007
2008


NH 500 mb Height of GFS (wave 1–20)
Average For Jun 01 – May 31
Anomaly correlation scores
Forecast days
02–03
03–04
04–05
05–06
06–07
07–08
7.2d to 7.6d


NH 500 mb Height of Ensemb
Average For Jun 0
Anomaly correlation scores
Forecast days
02–03
03–04
04–05
05–06
06–07
07–08
7.5 to 8.3d


Monthly Average for NH 500hPa Height, 5-day forecasts
RPS scores
0.6
0.55
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
2000
2001
2002
2003
2004
2005
2006
2007
2008
D5=0.357
2001
D6=0.334
2007
RPSS=0.396
EV (1:10 ratio)
D5=0.356
D6=0.361
EVAL=0.394

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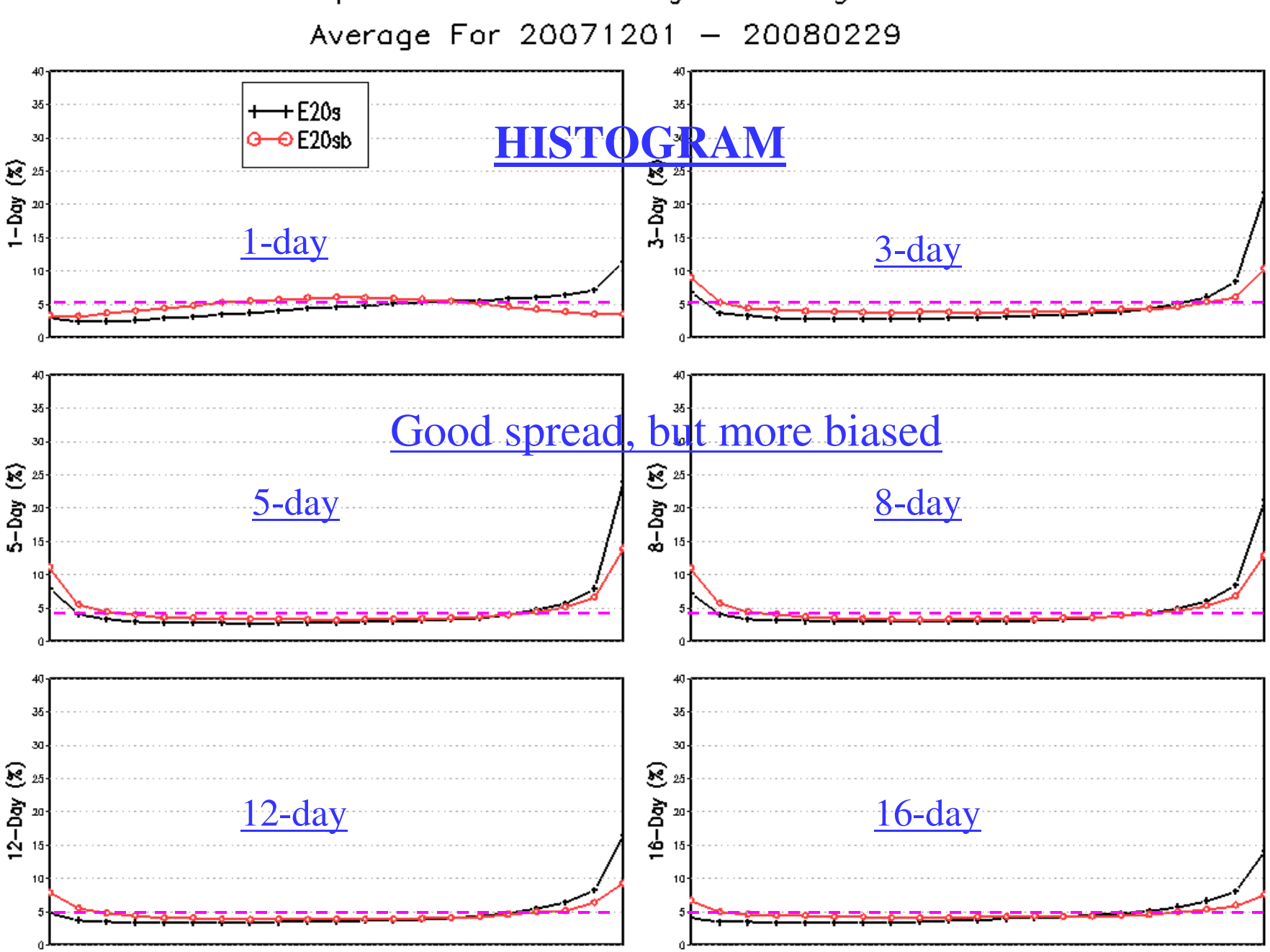
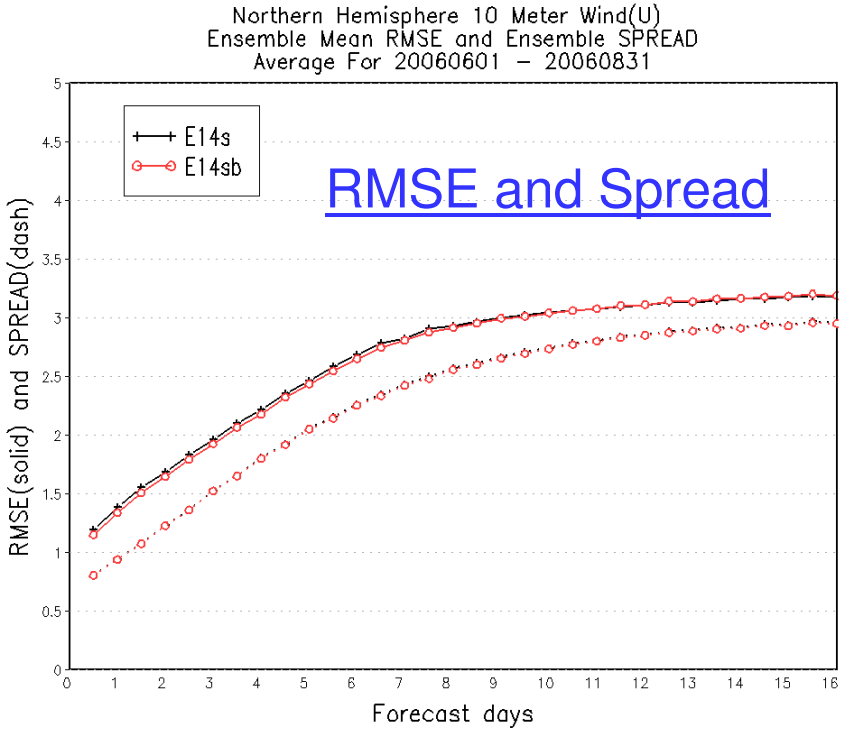
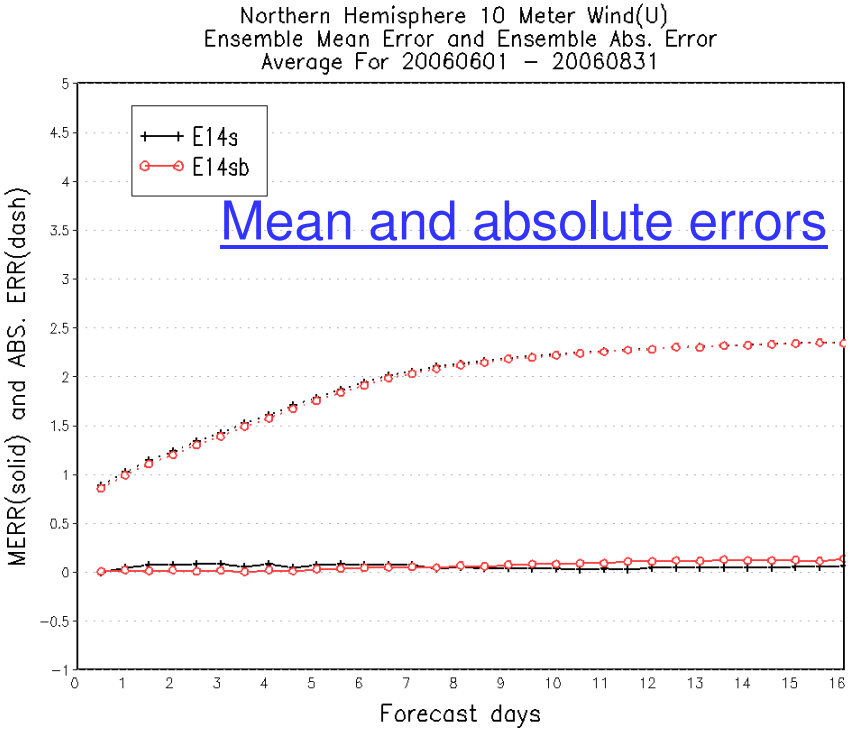
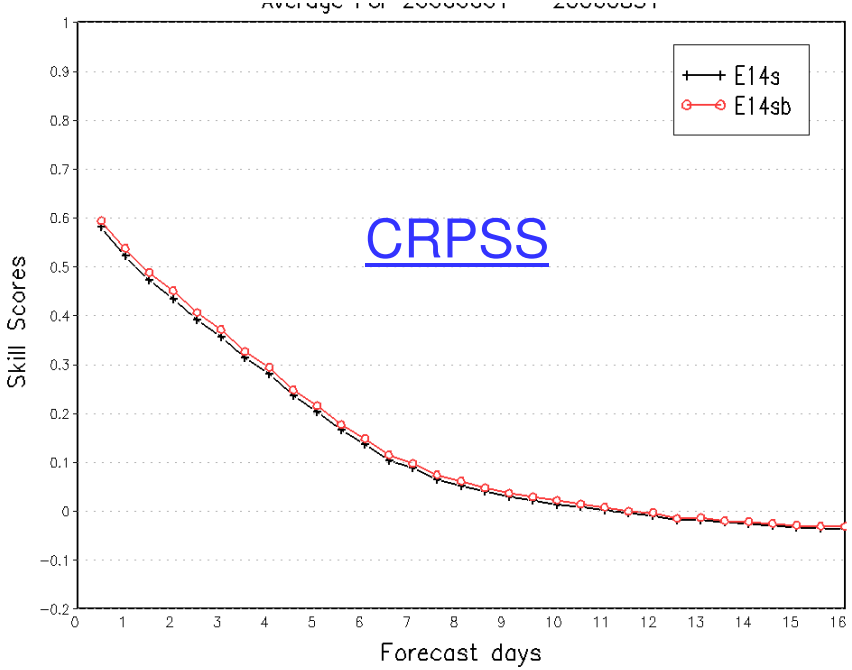

Northern Hemisphere 500hPa Height Histogram Distribution
Average For 20071201 – 20080229
E20s
E20sb
HISTOGRAM
1-Day (%)
1-day
3-Day (%)
3-day
Good spread, but more biased
5-Day (%)
5-day
8-Day (%)
8-day
12-Day (%)
12-day
16-Day (%)
16-day



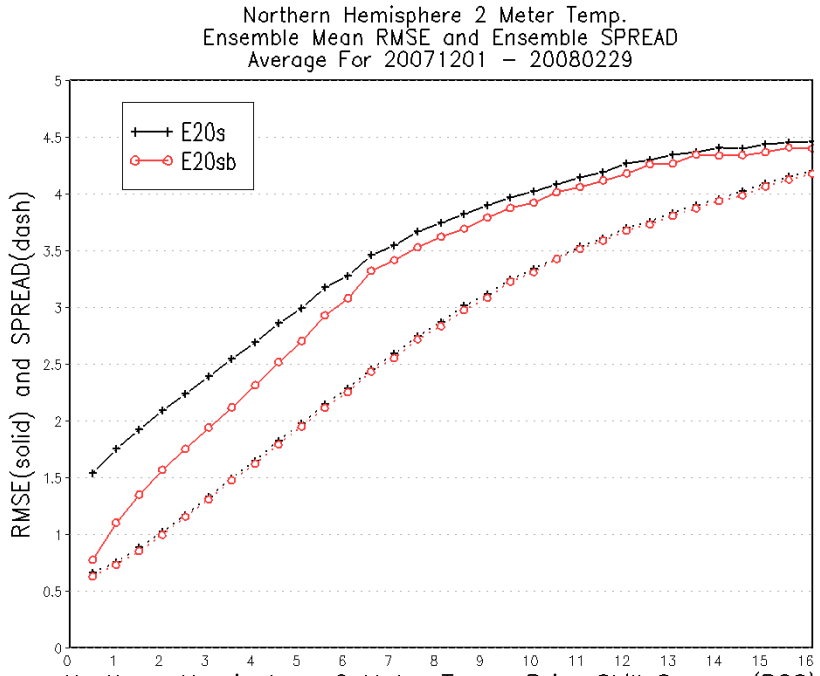


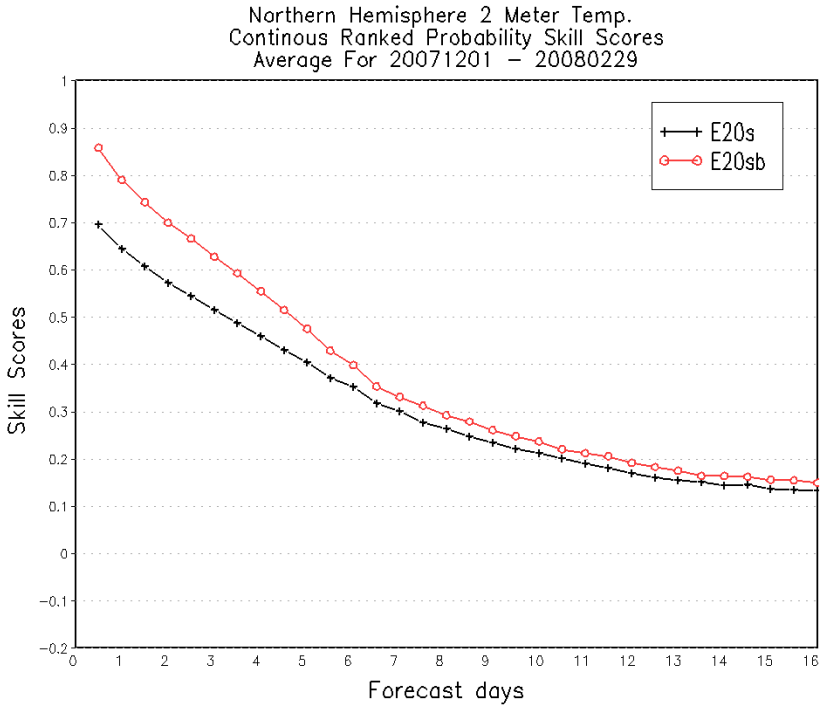


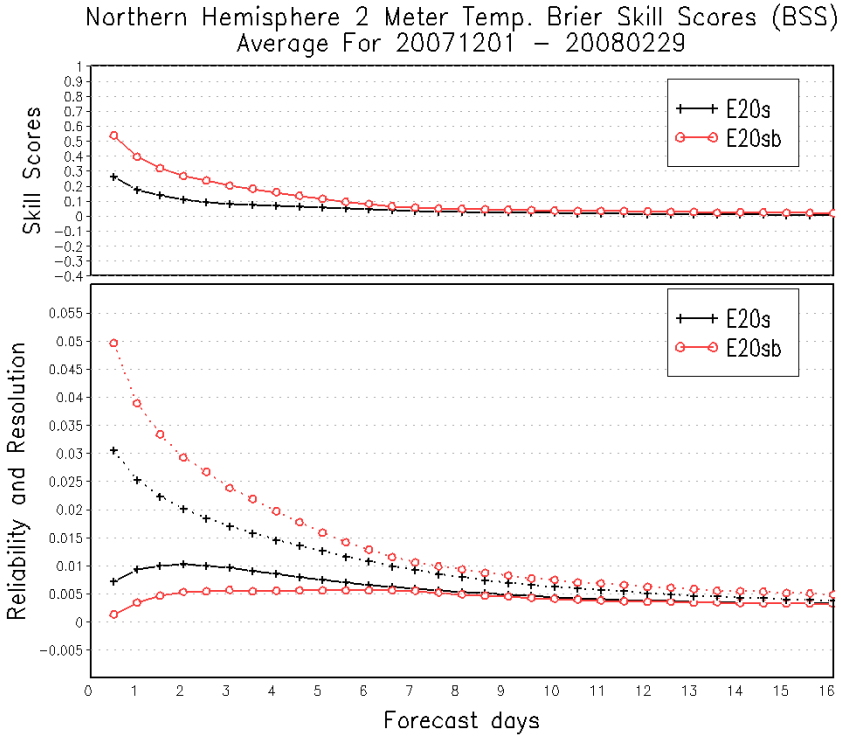
10 meter wind (u-component)

Less biased,

There is less room to improve the skill by bias-correction only







2-meter temperature
There is more room to improvement
Benefit from hybrid GFS forecast
for the first 7.5 days

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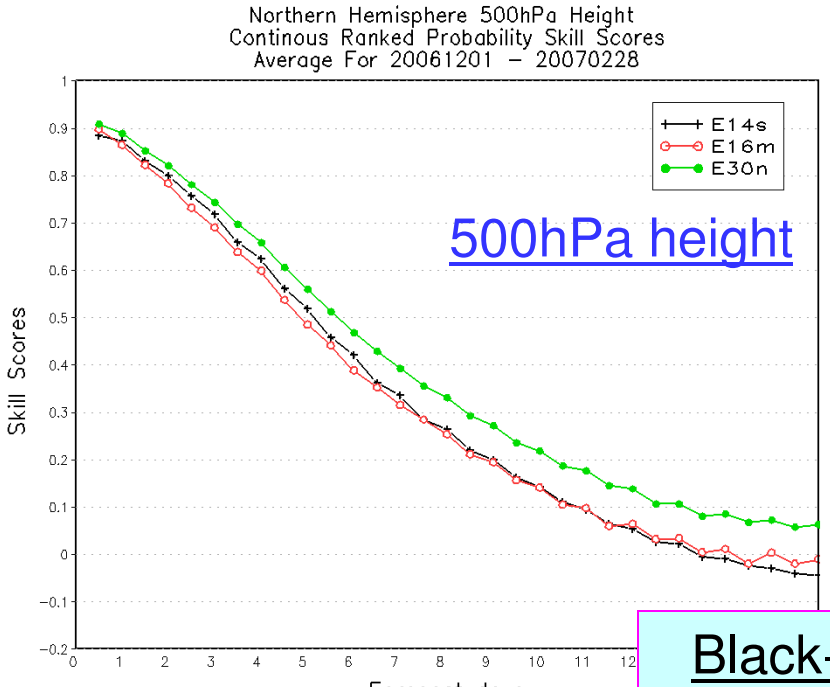
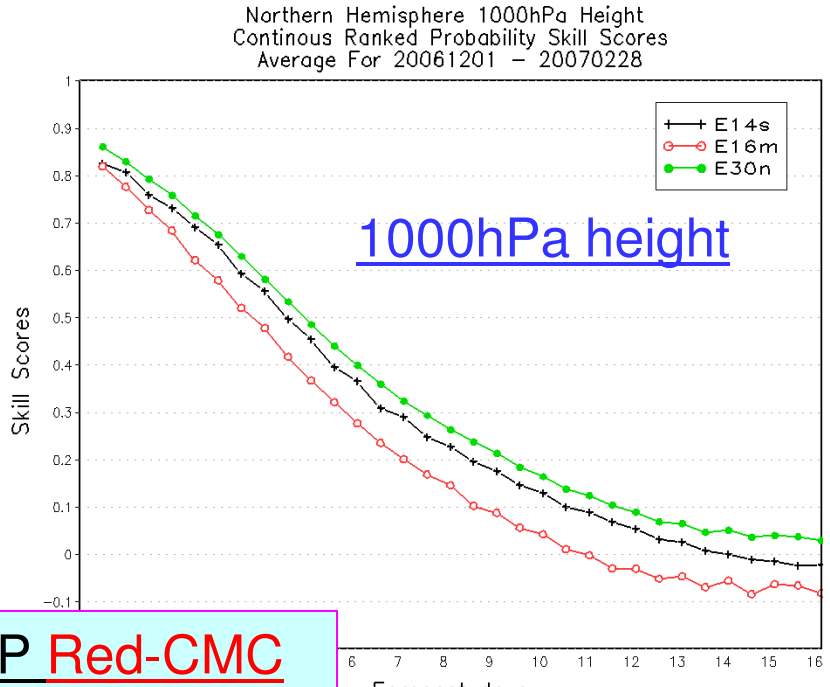
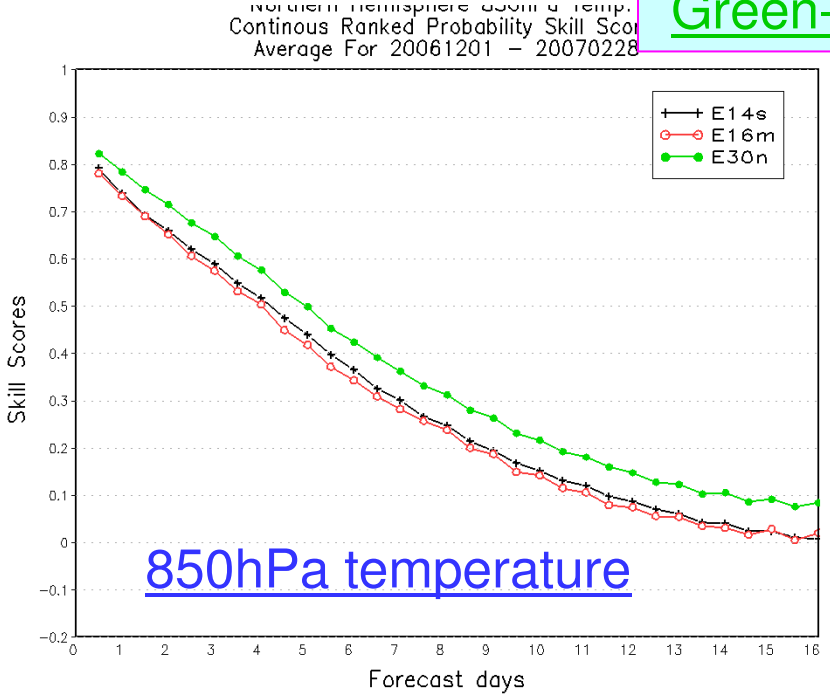


500hPa height

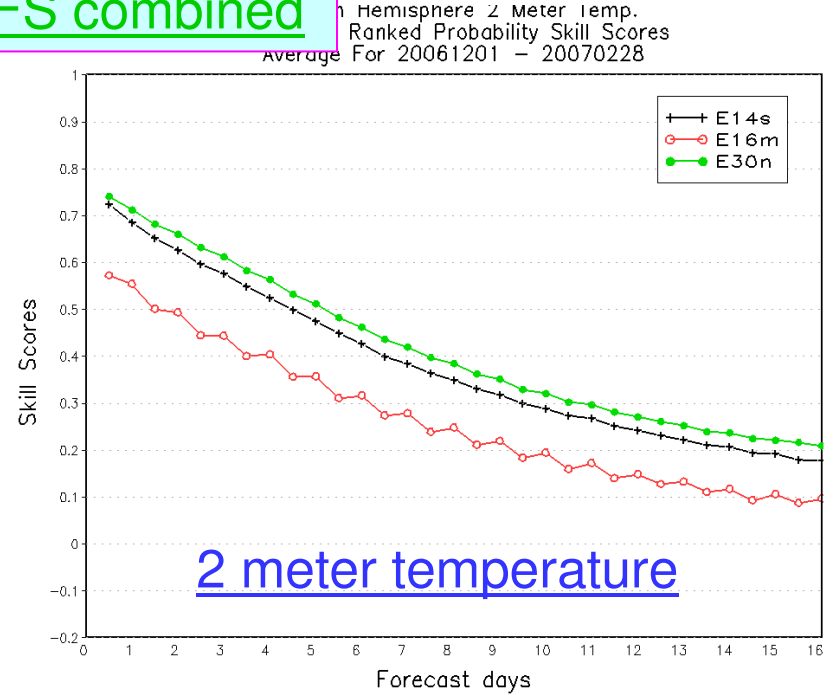


1000hPa height

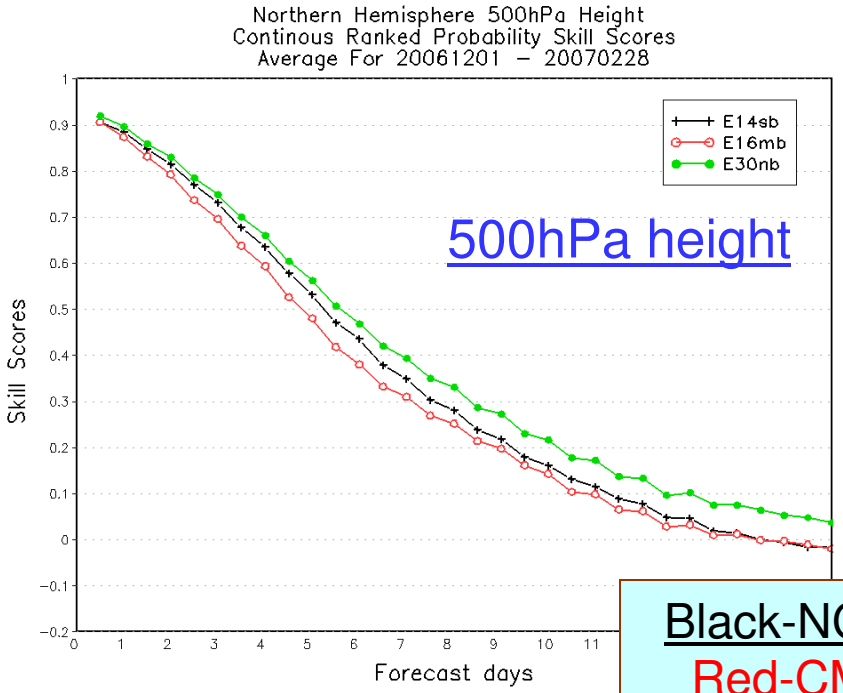
Black-NCEP Red-CMC
Green-NAEFS combined

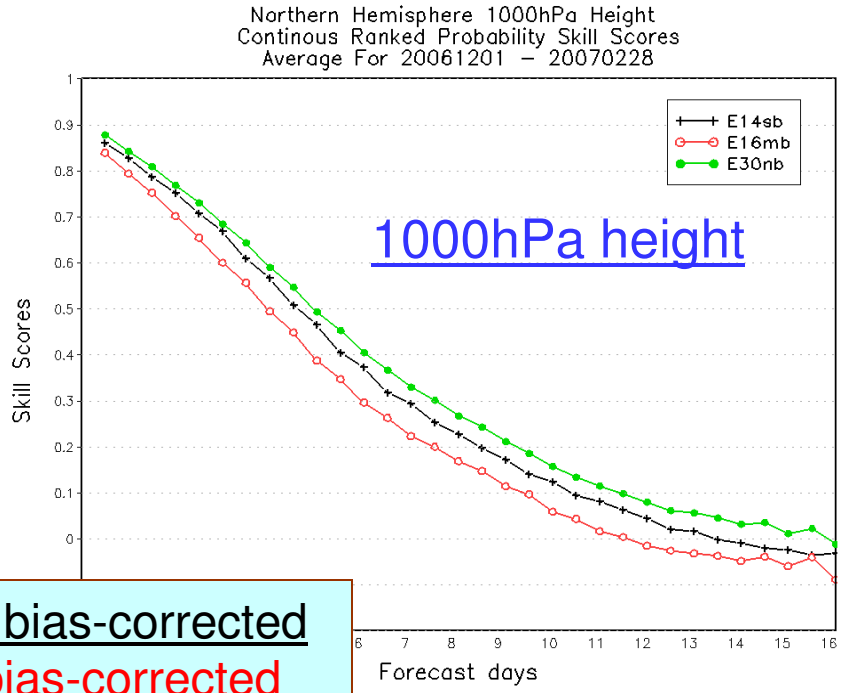


850hPa temperature

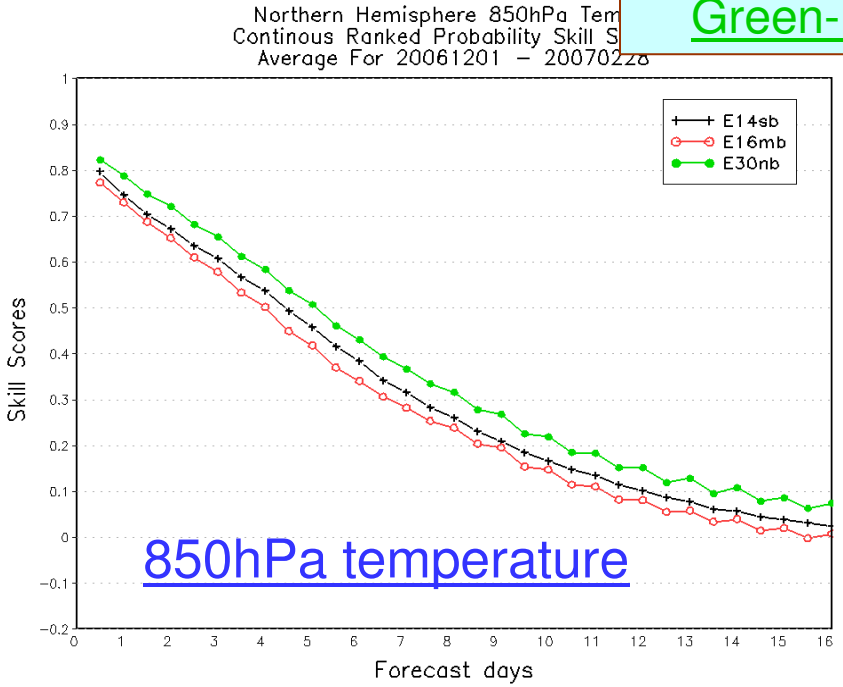


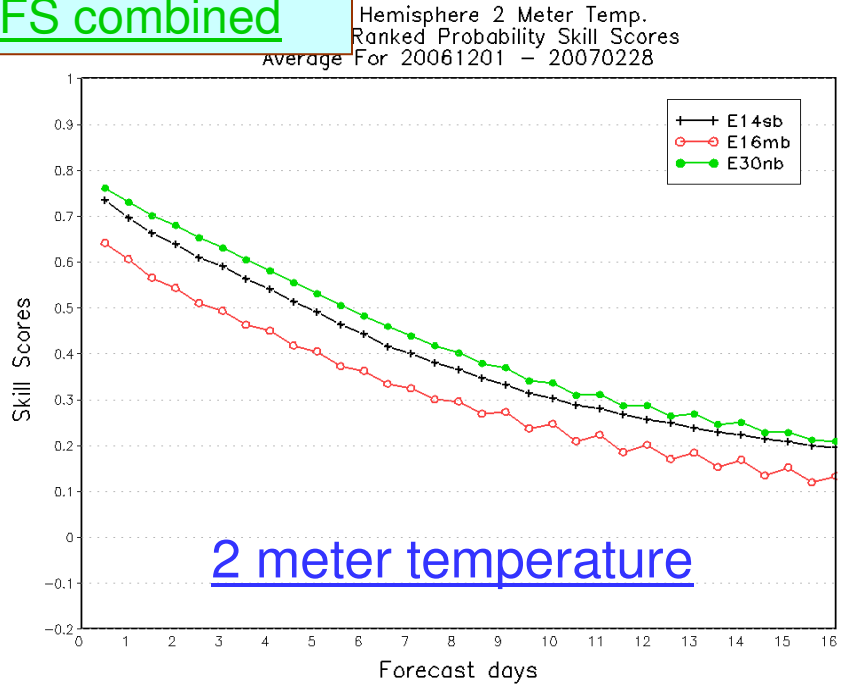
2 meter temperature





Black-NCEP bias-corrected
Red-CMC bias-corrected
Green-NAEFS combined





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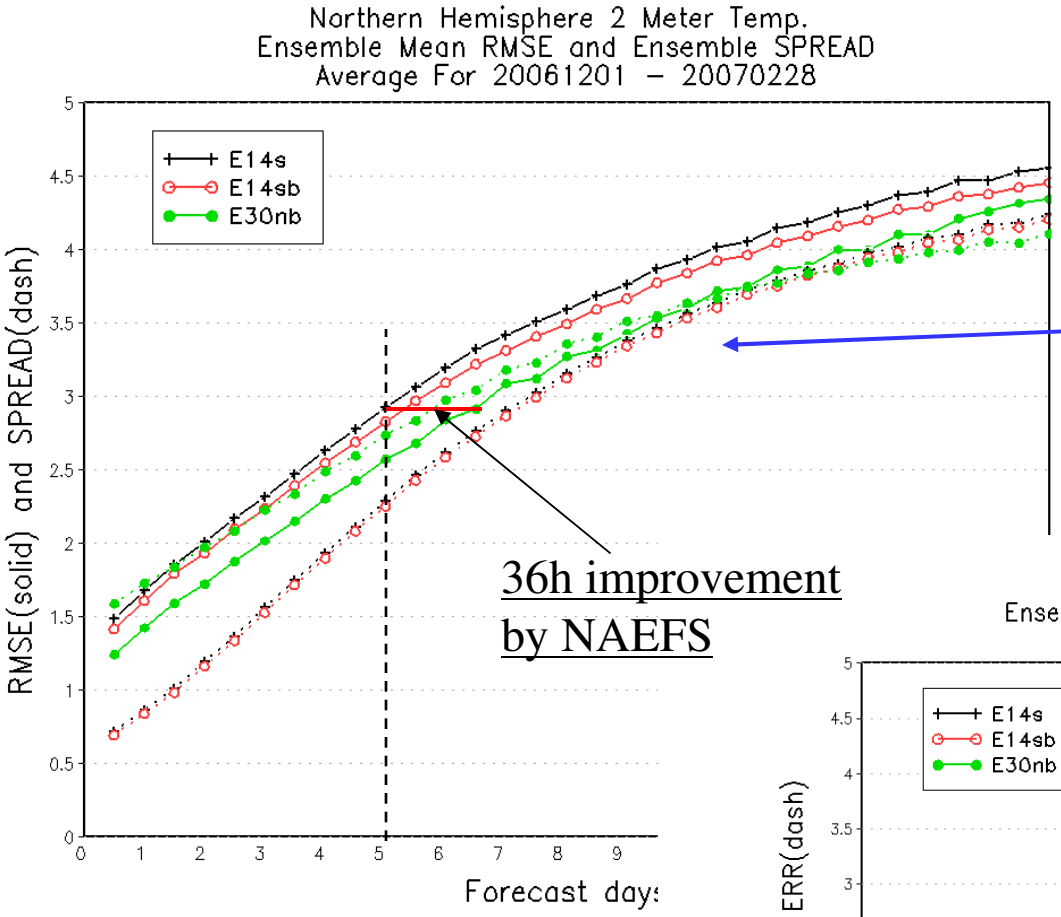
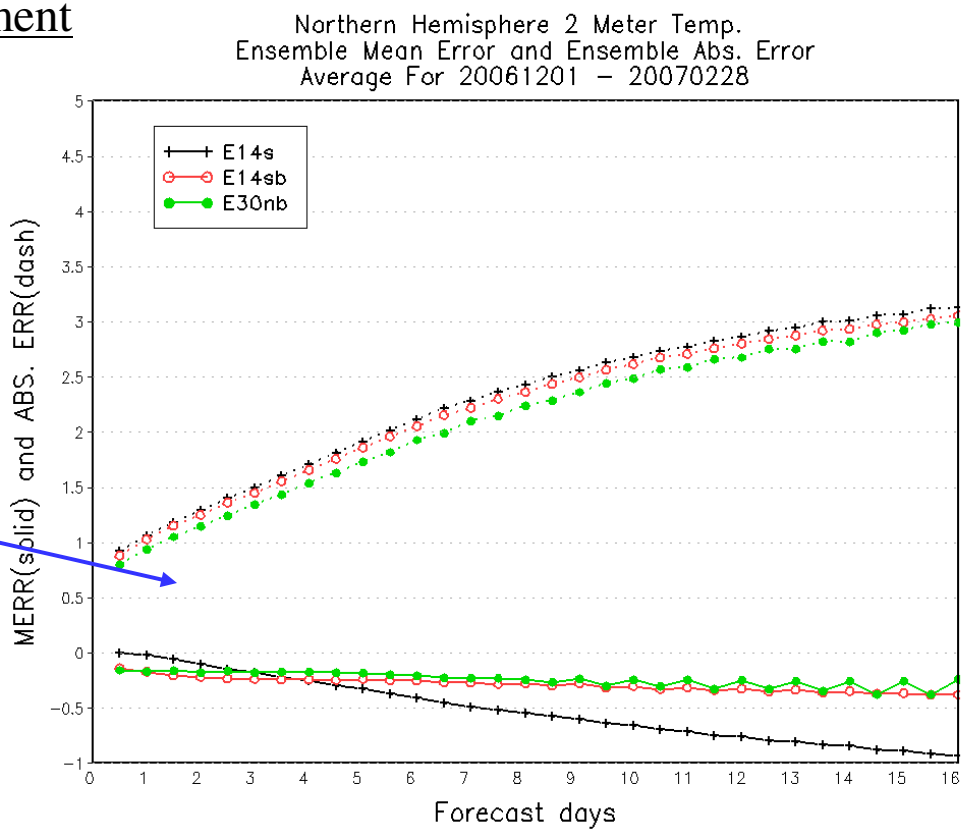


Solid: RMS error

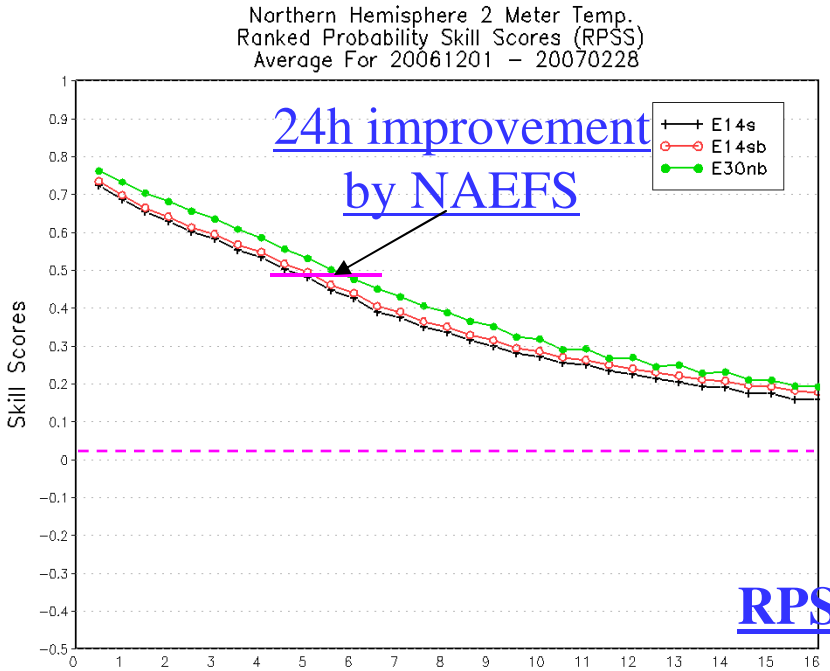
Dash: Spread

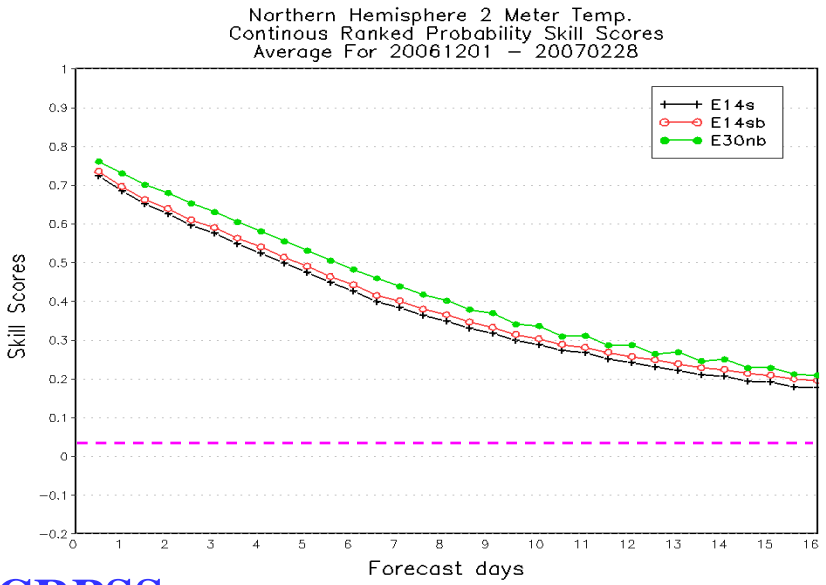
36h improvement by NAEFS



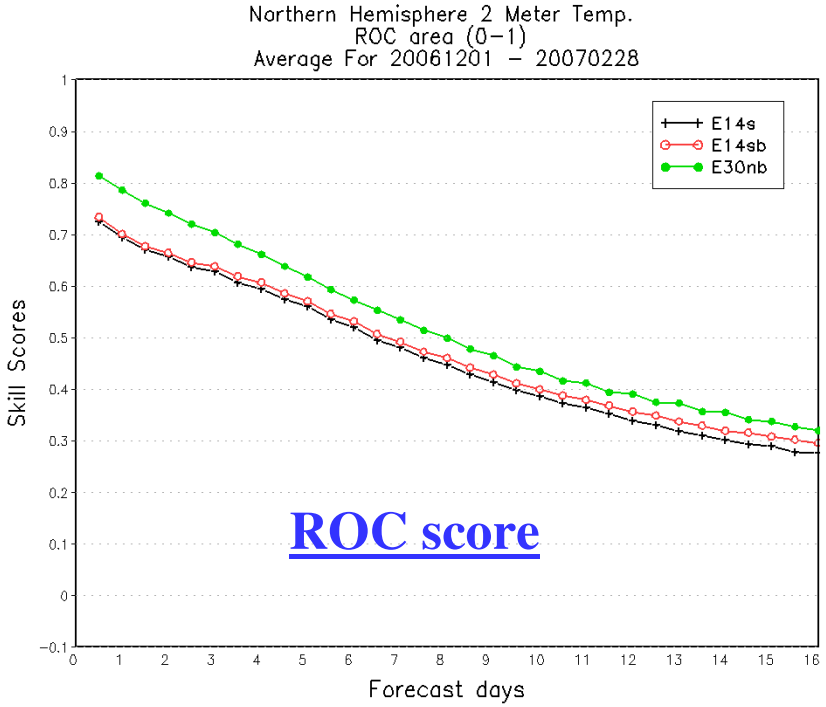
Solid: Mean error (bias)

Dash: Mean absolute error





RPSS .vs CRPSS



Winter 2006-2007

NH 2m temperature

For

NCEP raw forecast (black)

NCEP bias corrected forecast (red)

NAEFS forecast (pink)

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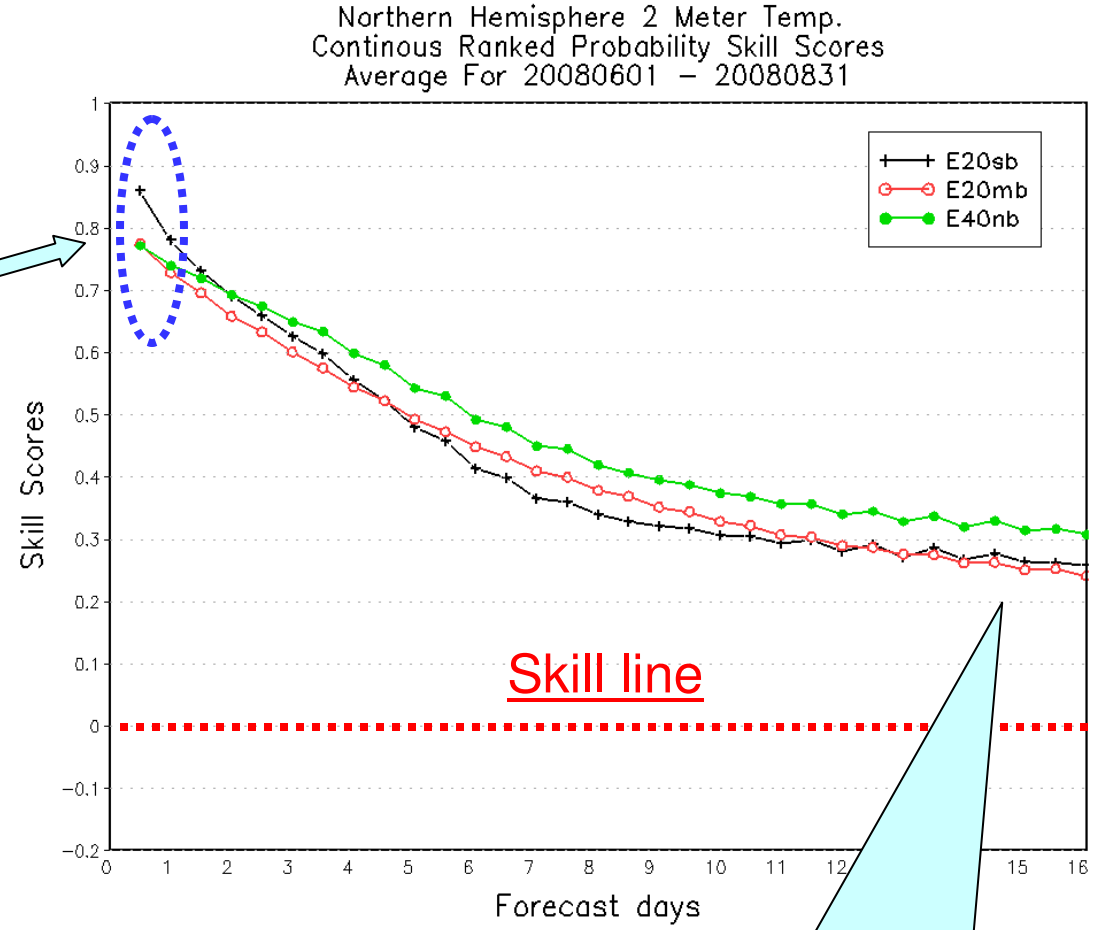
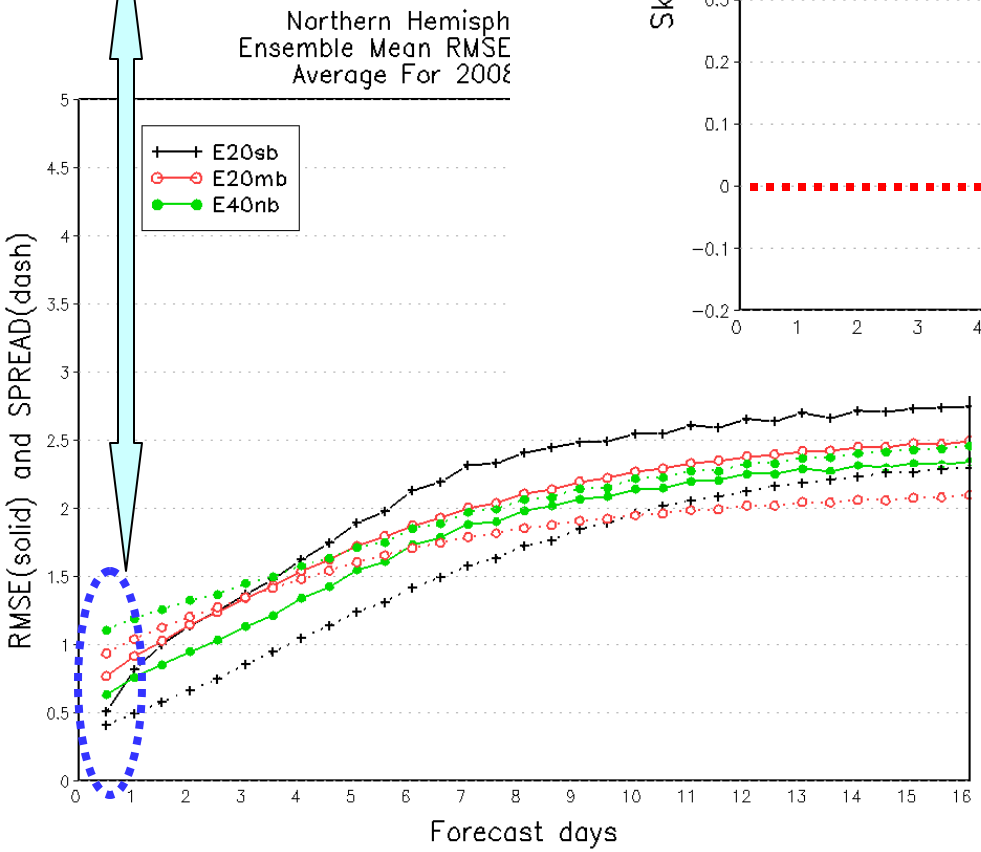
The problems come from our verification?

We need to consider the difference of CMC's and NCEP's analysis

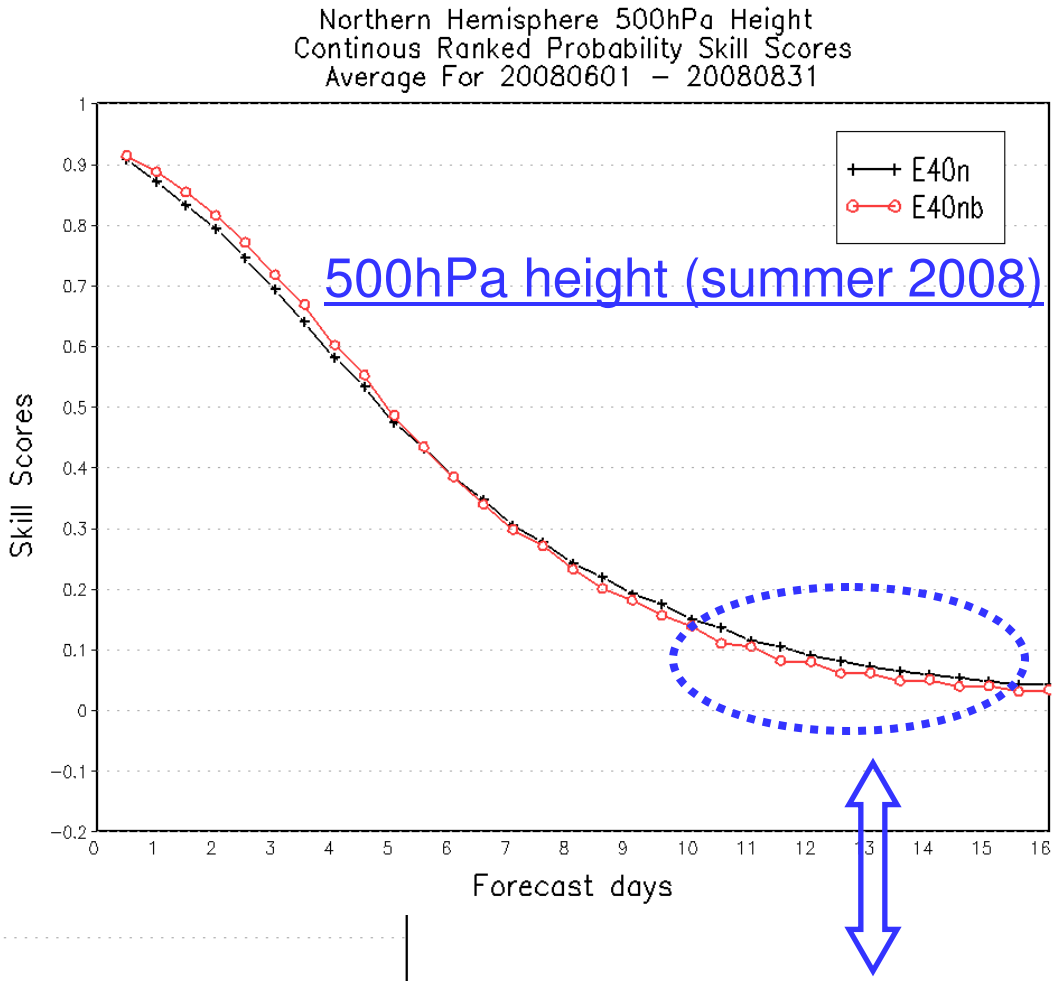




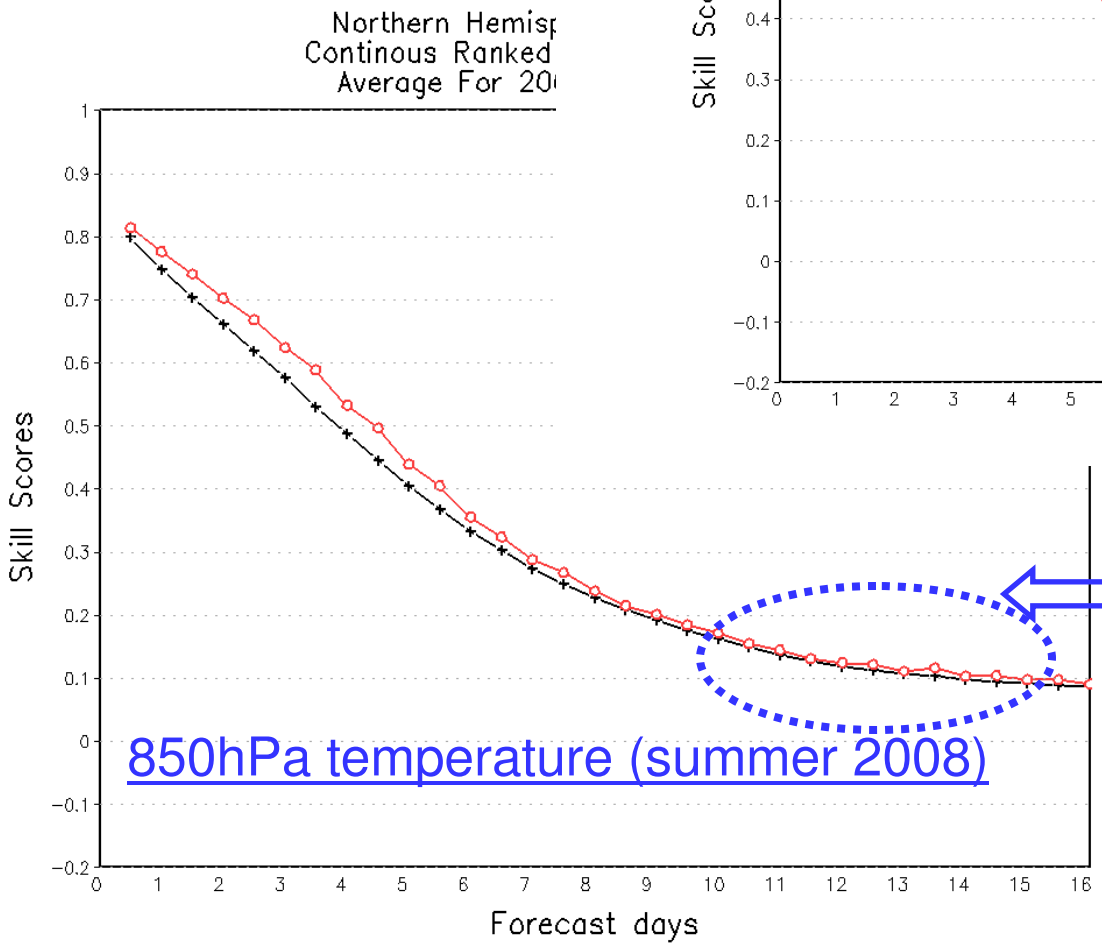
Do we really have that much skills for week-2 forecast?

We need to consider the difference between CDAS and GDAS.

Do we need hindcast for the calibration of week-2 forecast?



500hPa height (summer 2008)



850hPa temperature (summer 2008)

There is no skill improvement of week-2 forecast for some season and variables.

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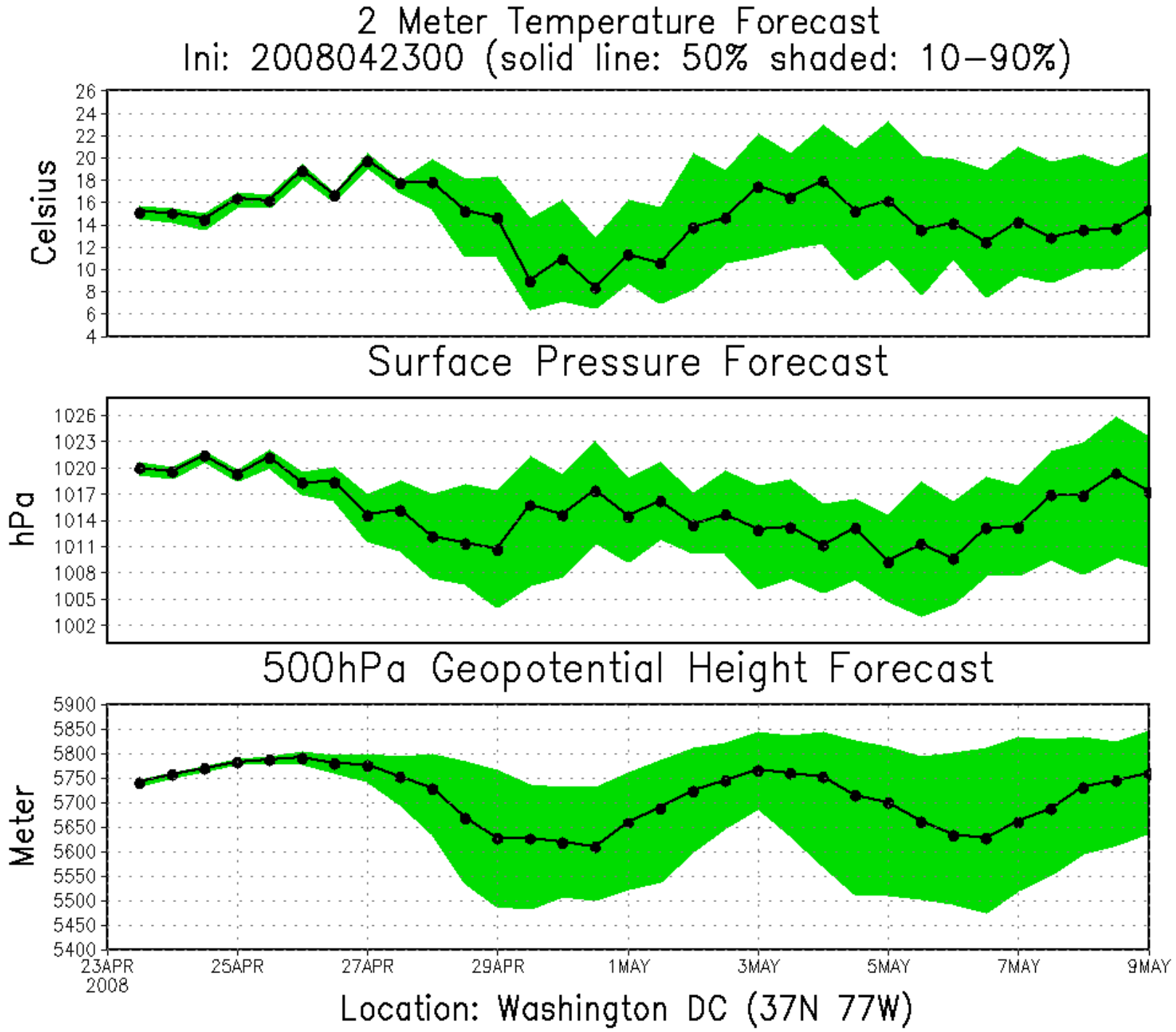
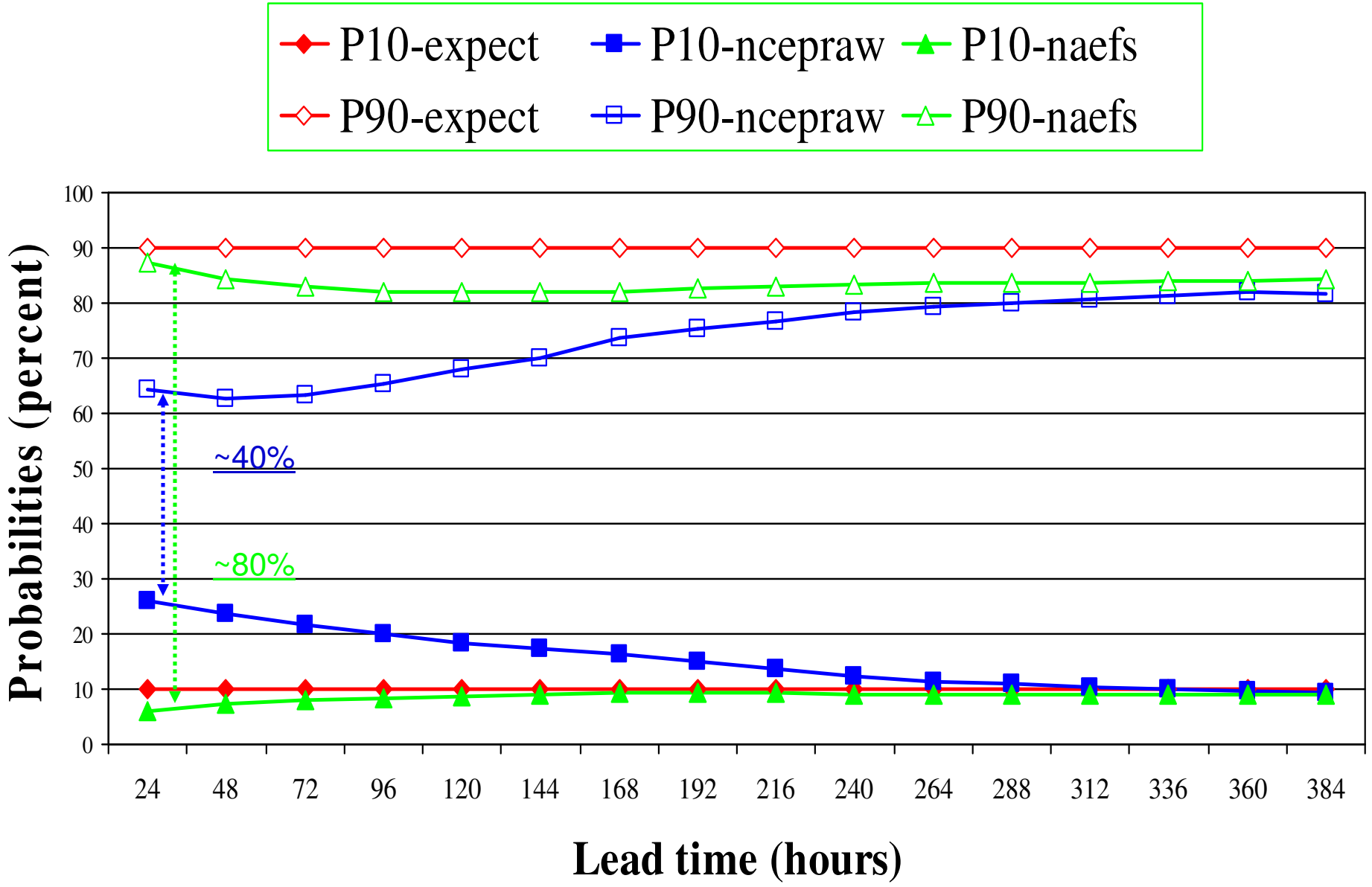
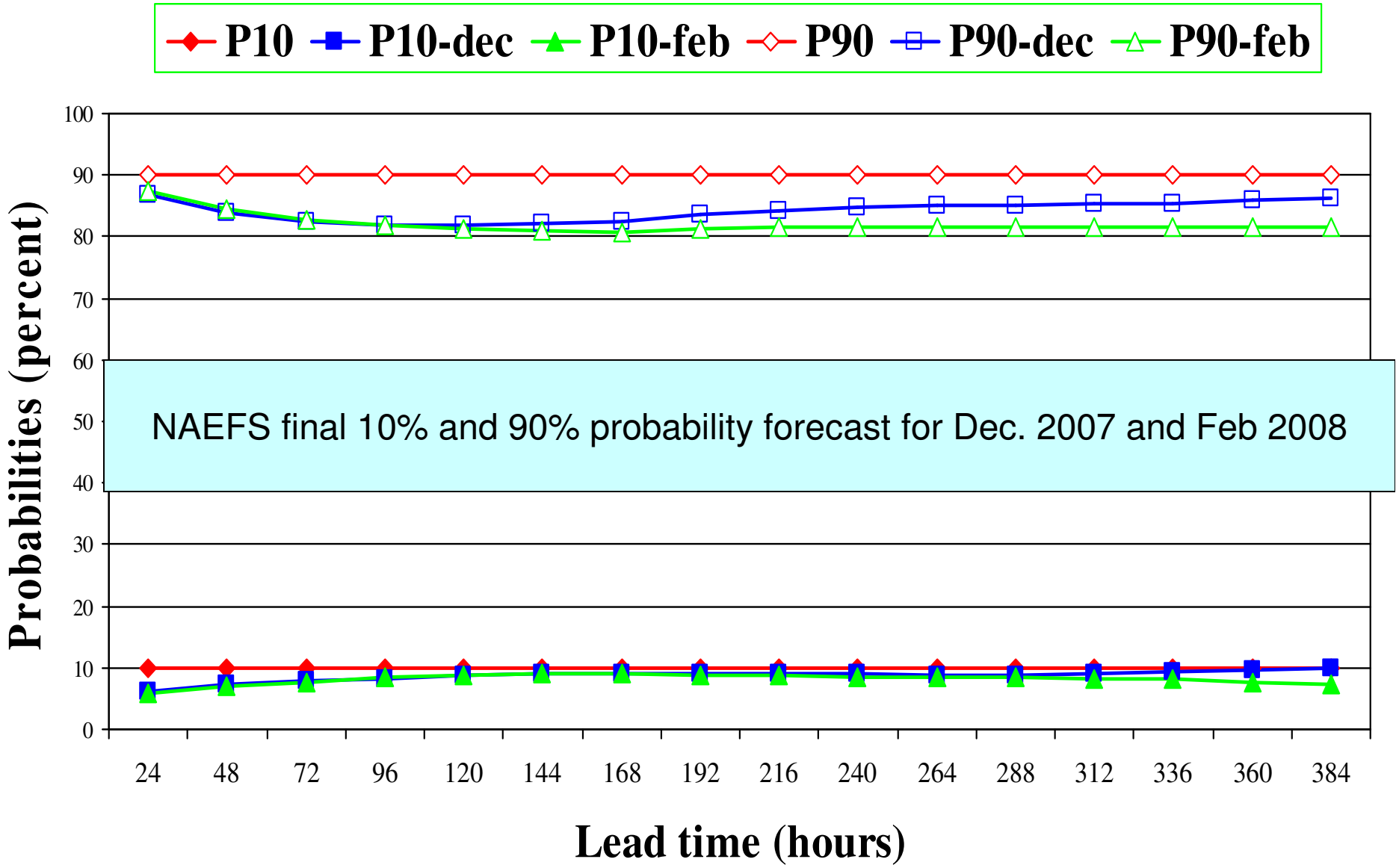

2 Meter Temperature Forecast
Ini: 2008042300 (solid line: 50% shaded: 10-90%)
Celsius
26
24
22
20
18
16
14
12
10
8
6
4
Surface Pressure Forecast
hPa
1026
1023
1020
1017
1014
1011
1008
1005
1002
500hPa Geopotential Height Forecast
Meter
5900
5850
5800
5750
5700
5650
5600
5550
5500
5450
5400
23APR
2008
25APR
27APR
29APR
1MAY
3MAY
5MAY
7MAY
9MAY
Location: Washington DC (37N 77W)

2-meter temperature 10/90 probability forecast verification
Northern Hemisphere, period of Dec. 2007 – Feb. 2008



2-meter temperature 10/90 probability forecast verification Northern Hemisphere, seasonal variation for NAEFS



Monitoring/Verification System for NAEFS and Down-scaling Forecast

Developed by
Bo Cui
EMC/NCEP/NWS/NOAA

http://wwwt.emc.ncep.noaa.gov/gmb/ens/NAEFS/NAEFS-eval.html

http://www.emc.ncep.noaa.gov/gmb/wx20cb/rtma/

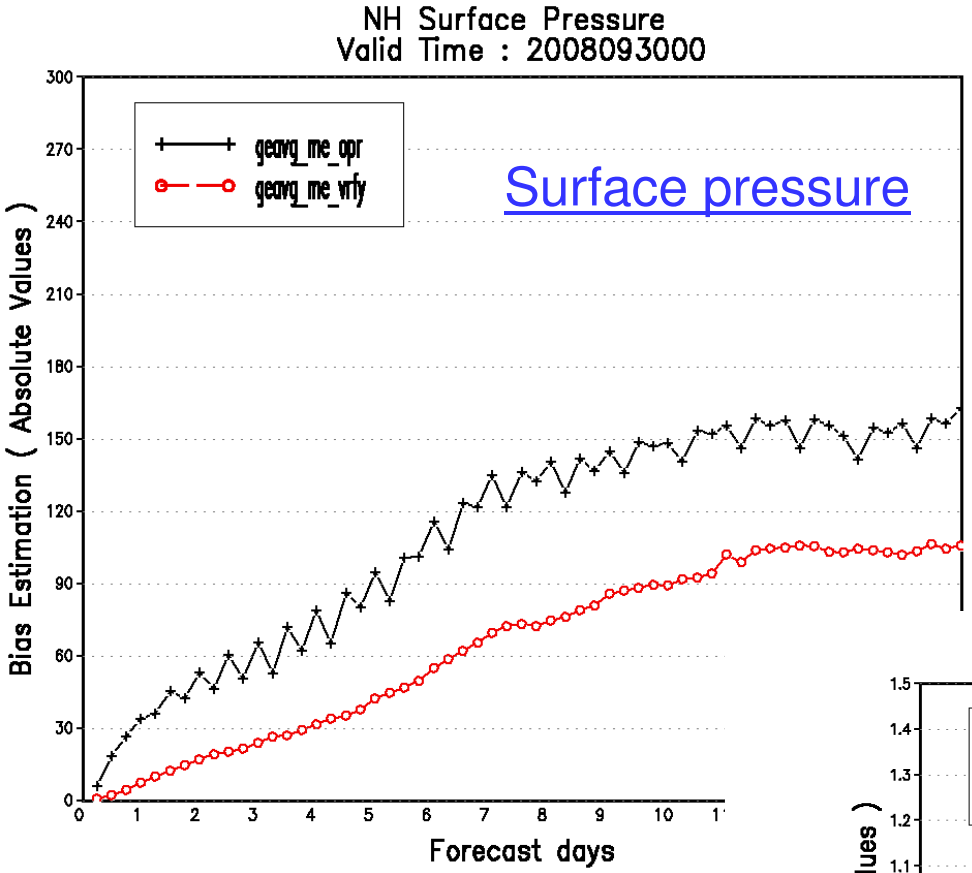
North American Ensemble Forecast System

Experimental Products

- **Bias Comparison Statistics which includes:**
  - **NCEP Raw and Bias-Corrected Ensemble Domain Averaged Bias**
  - **NCEP Raw and Bias-Corrected Ensemble Domain Averaged Bias Reduction (Percents)**
  - **CMC Raw and Bias-Corrected Control Forecast Domain Averaged Bias**
  - **CMC Raw and Bias-Corrected Control Forecast Domain Averaged Bias Reduction (Percents)**

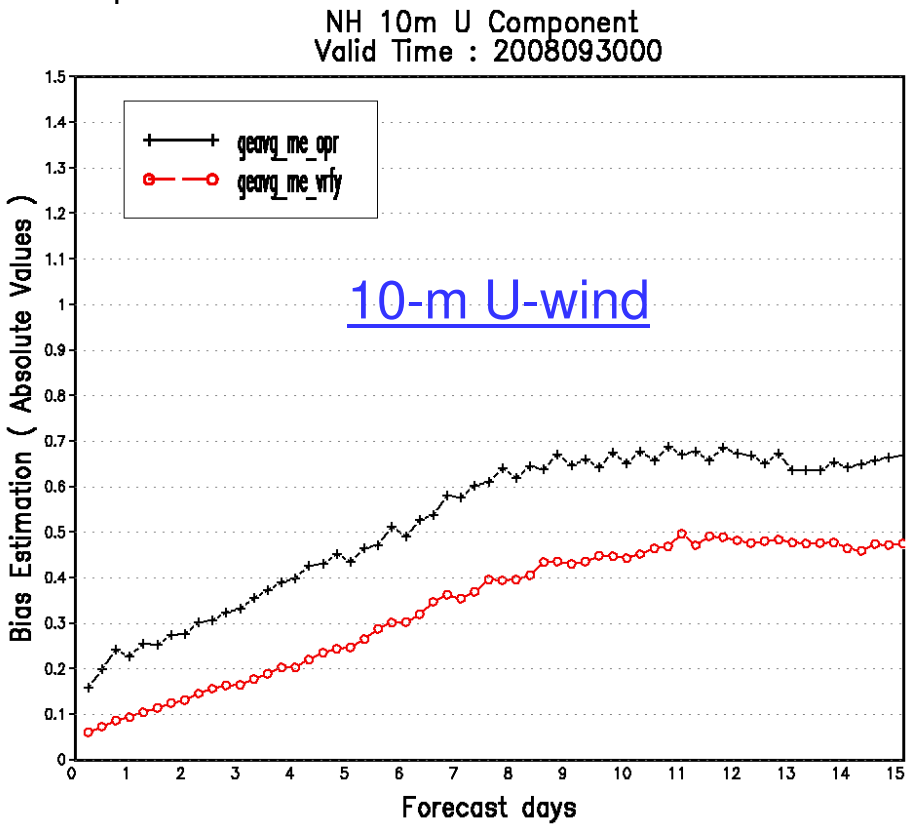
- **Probabilistic Evaluation (under developing) which includes:**
  - **PAC, RMS, ROC and RPSS**

NAEFS | **EMC Ensemble Products**



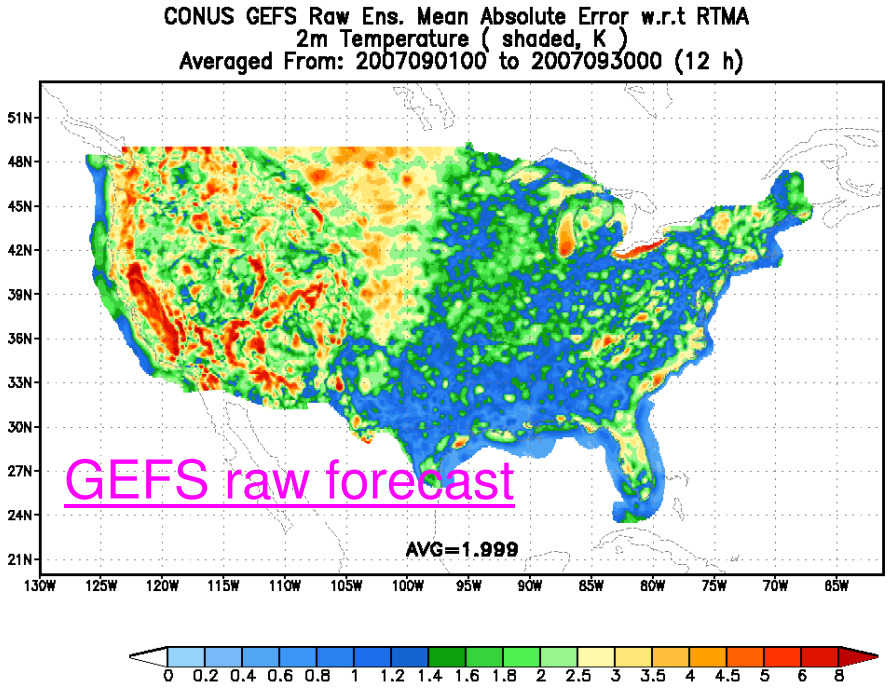
Surface pressure

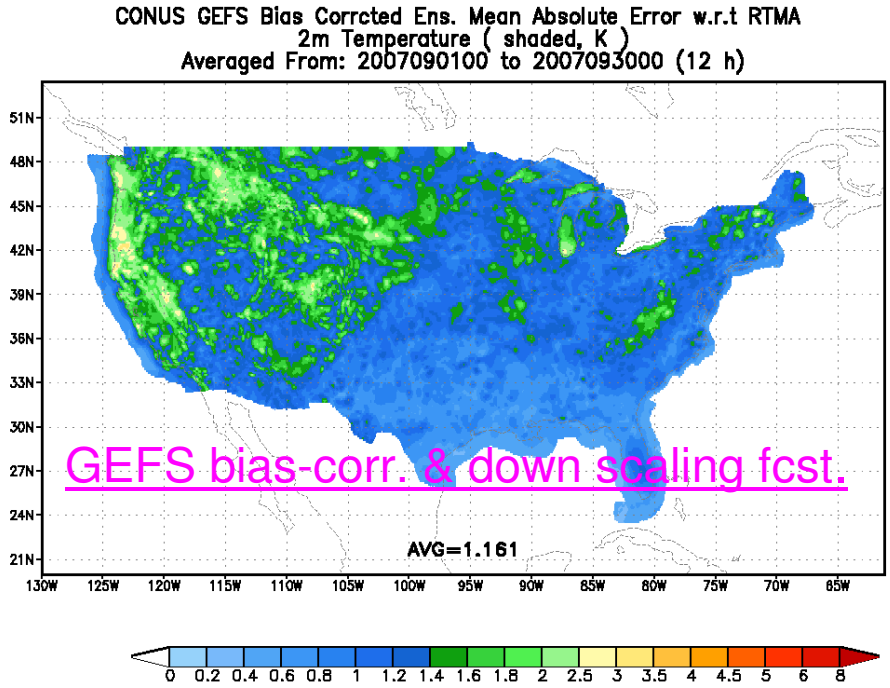
0.02 weight decaying average, updating every day

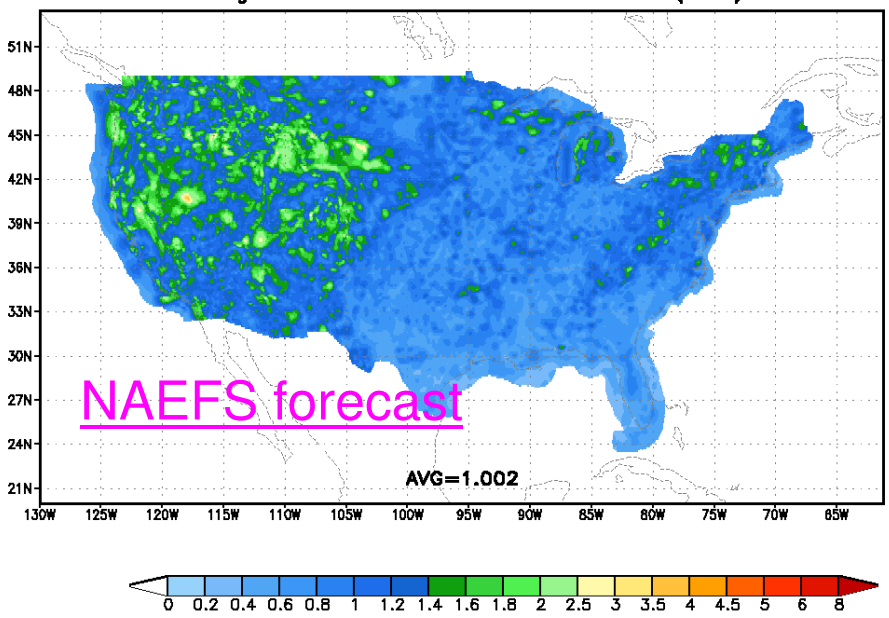


10-m U-wind

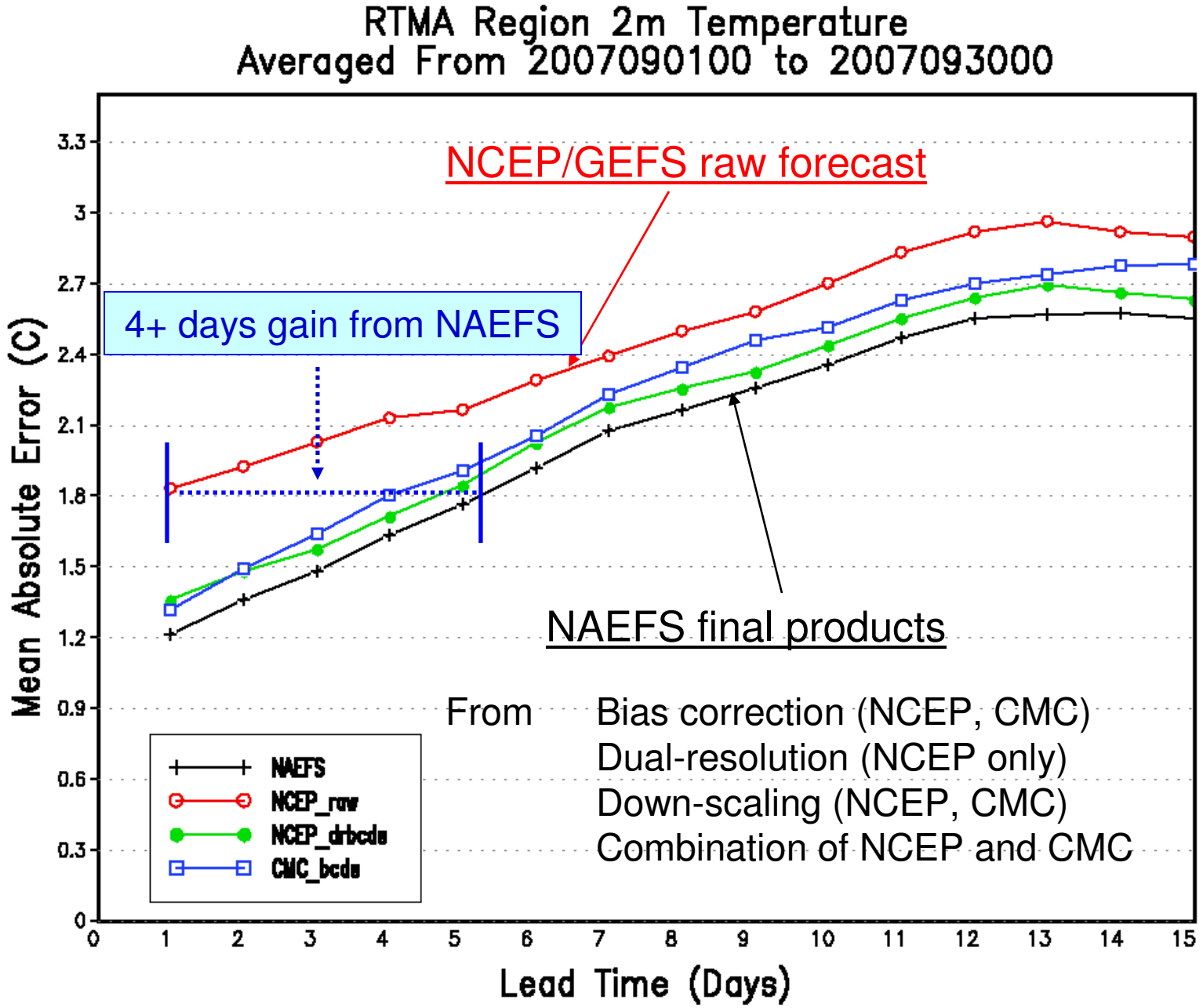
For all 35 bias corrected variables



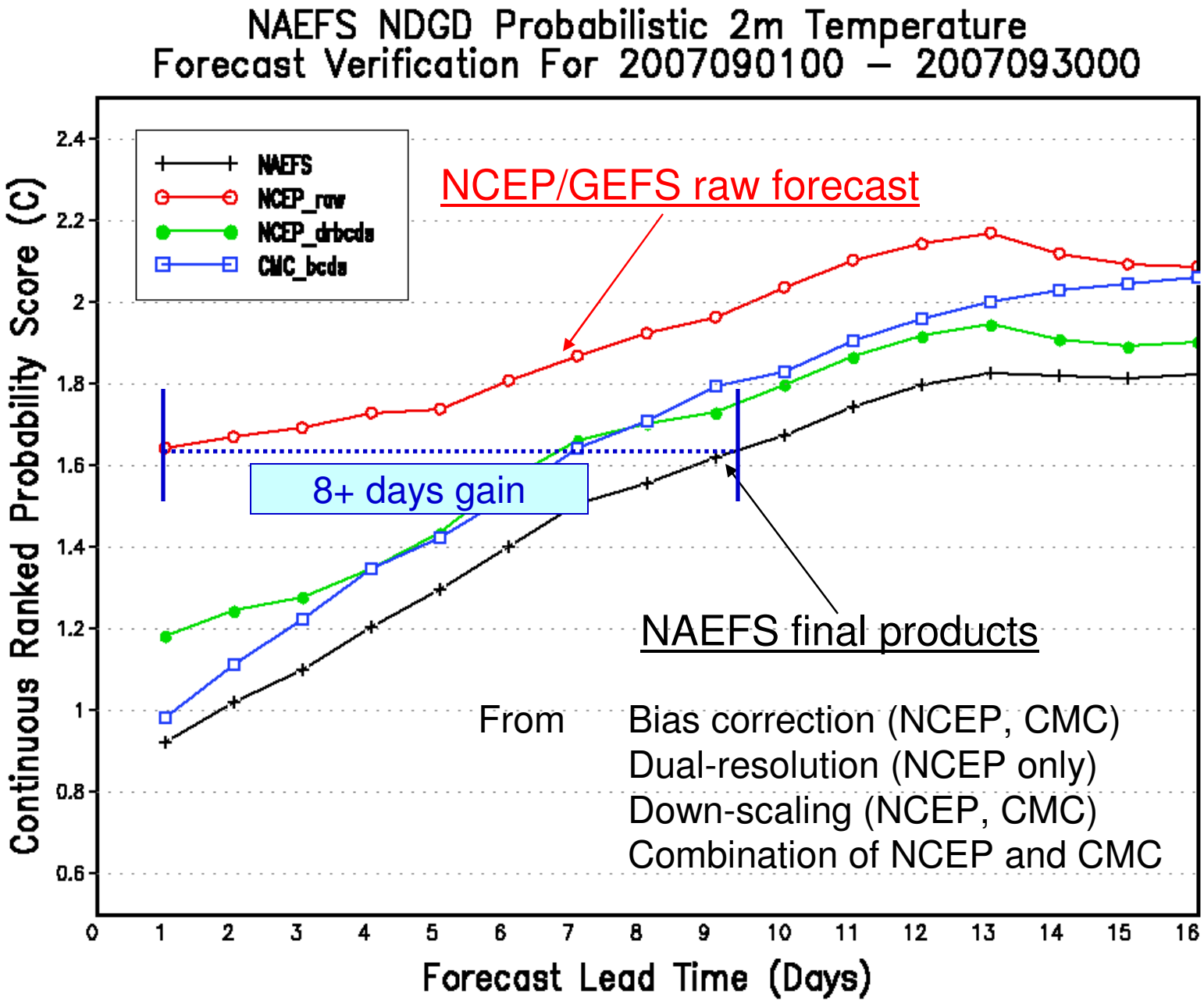


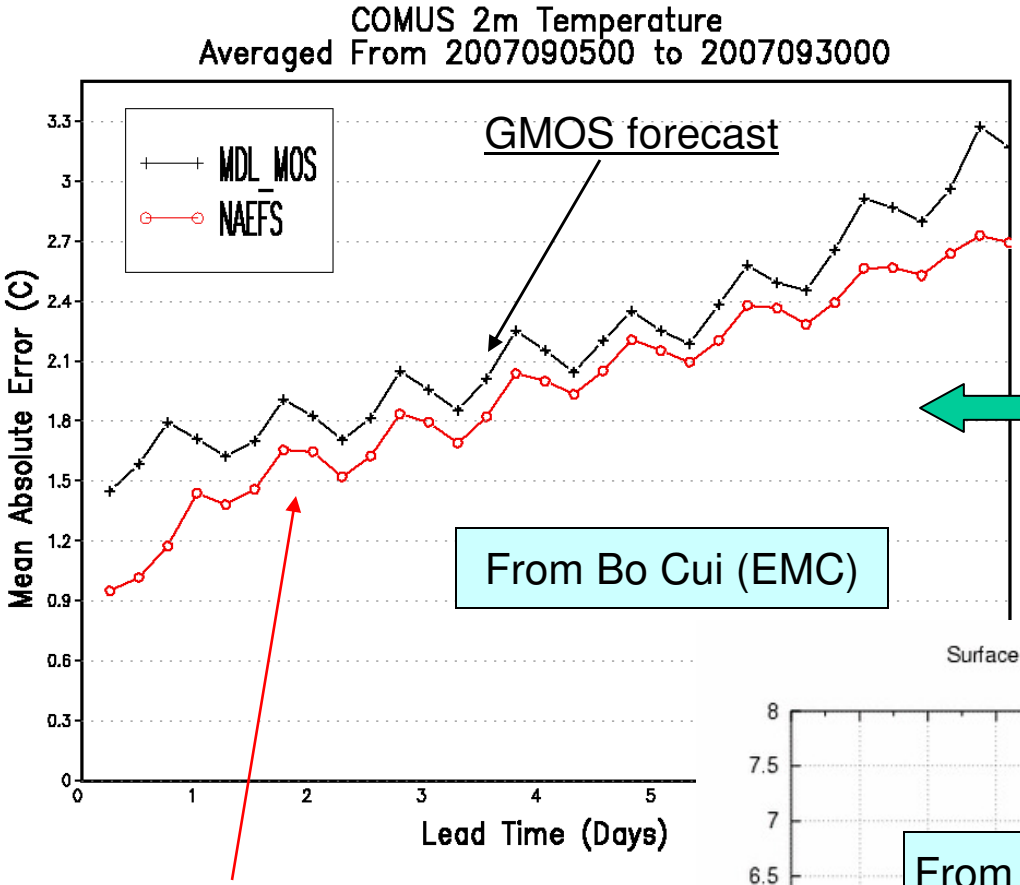


12hr 2m T forecast
Mean Absolute Error
w.r.t RTMA for CONUS
Average for September



BO CUI, GCWMB/EMC/NCEP/NOAA


NAEFS NDGD Probabilistic 2m Temperature
Forecast Verification For 2007090100 – 2007093000
NAEFS
NCEP_raw
NCEP_drbcds
CMC_bcds
NCEP/GEFS raw forecast
8+ days gain
NAEFS final products
From
Bias correction (NCEP, CMC)
Dual-resolution (NCEP only)
Down-scaling (NCEP, CMC)
Combination of NCEP and CMC
Continuous Ranked Probability Score (C)
Forecast Lead Time (Days)
2.4
2.2
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16



GMOS forecast

From Bo Cui (EMC)

NAEFS final products

From :
Bias correction (NCEP, CMC)
Dual-resolution (NCEP only)
Down-scaling (NCEP, CMC)
Combination of NCEP and CMC

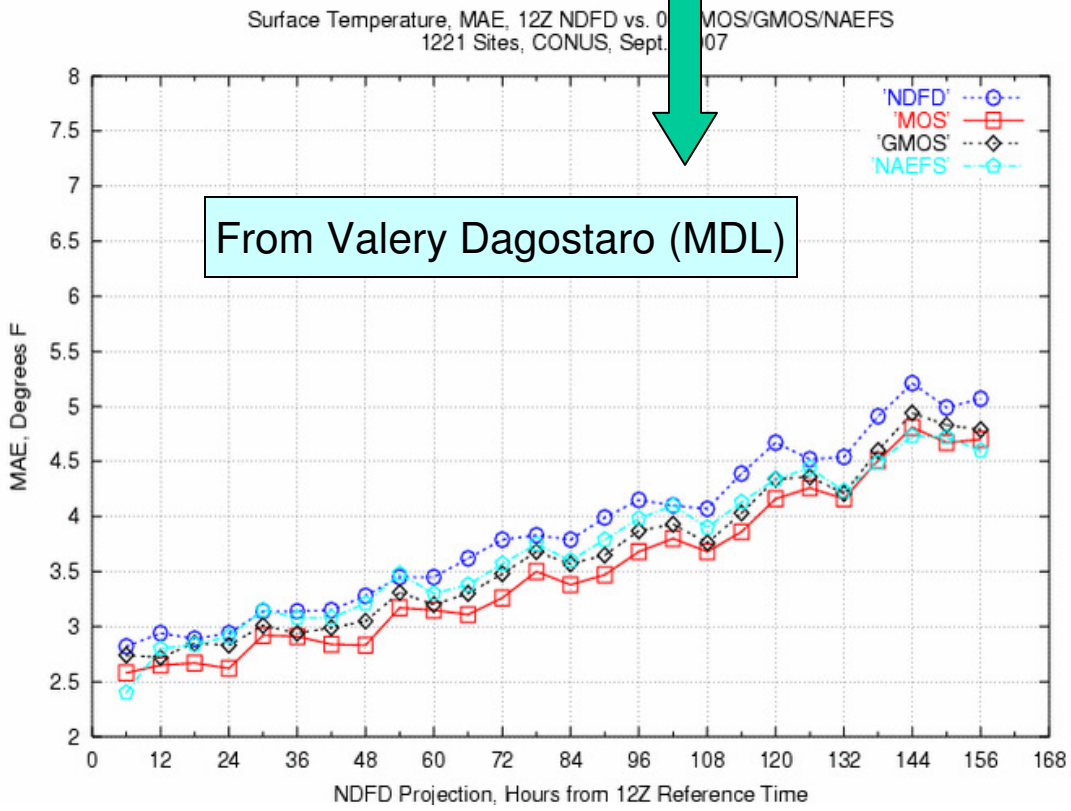
CONUS 2m Temperature

For September 2007

Verify against RTMA

Verify against observation

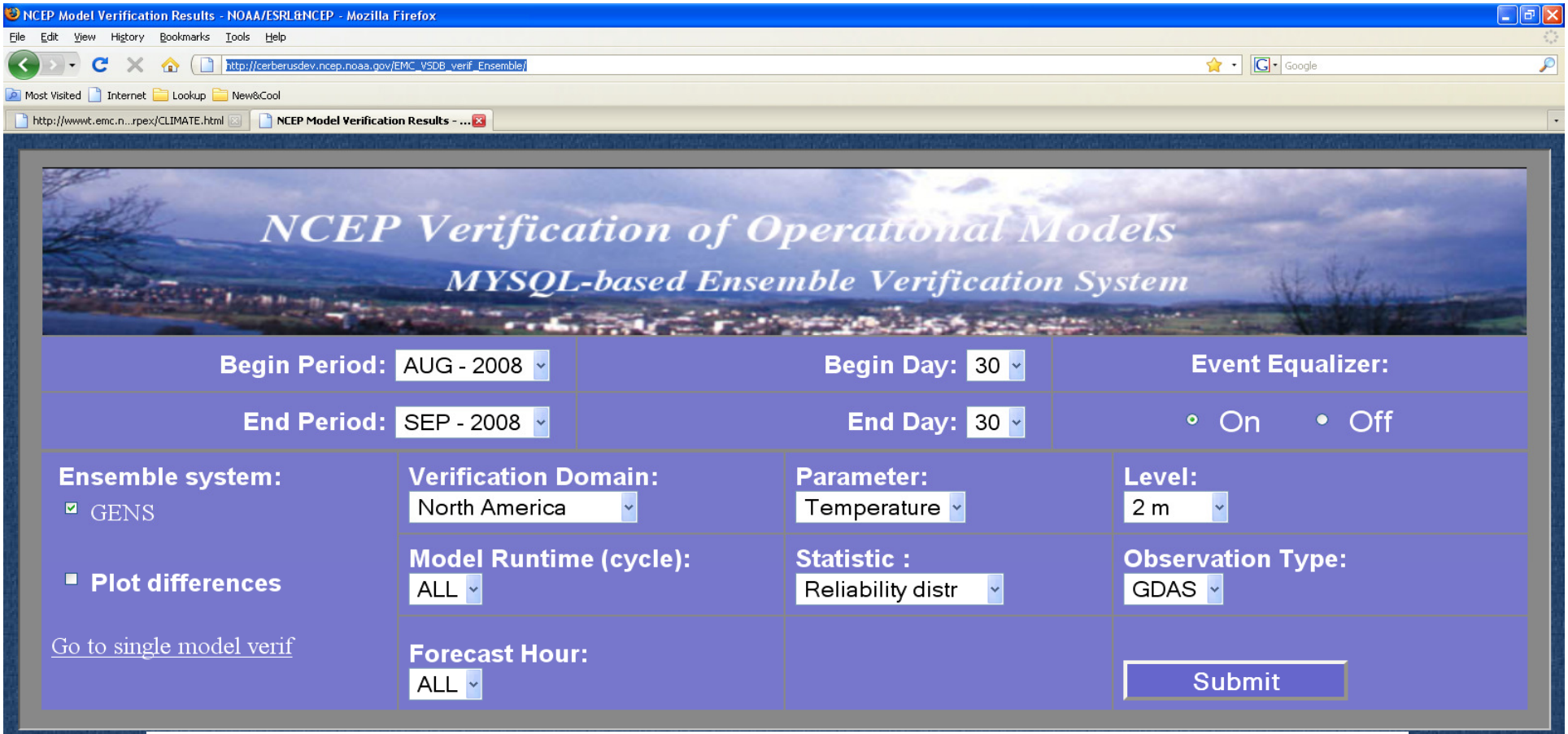
From Valery Dagostaro (MDL)



MYQSL-based Ensemble Verification System

Developed by
Binbin Zhou
EMC/NCEP/NWS/NOAA

http://cerberusdev.ncep.noaa.gov/EMC_VSDB_verif_Ensemble/



Features:

(1) MYSQL: relational database like **Oracle**, but open source and 100% free from Sun Inc.

(2) Efficient in data storage and management

(3) All of plots are generated on fly, saving space

(4) Fast access online

(5) Still NCEP in-house, not accessible from outside

NCEP Model Verification Results - NOAA/ESRL&NCEP - Mozilla Firefox

File Edit View History Bookmarks Tools Help

http://cerberusdev.ncep.noaa.gov/EMC_VSDB_verif_Ensemble/index.cgi?jsdate_from_perl=1222777781000&begin_time_period=DEC+-+2007&begin_day=01&end_time_period=FEB+-+2008&end_day=288

Most Visited Internet Lookup New&Cool

http://wwwt.emc.n...rpex/CLIMATE.html | NCEP Model Verification Results - ...

NCEP Verification of Operational Models

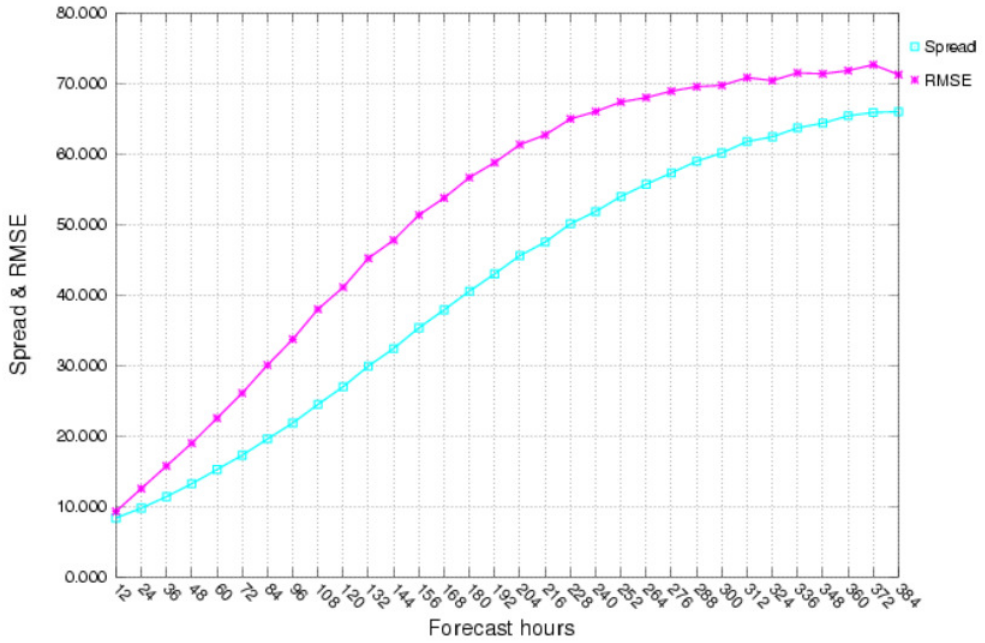
MYSQL-based Ensemble Verification System

Begin Period: DEC - 2007	Begin Day: 01	Event Equalizer:
End Period: FEB - 2008	End Day: 28	On / Off

Ensemble system: GENS	Verification Domain: North Hemisphere	Parameter: Height	Level: 1000mb
Plot differences	Model Runtime (cycle): ALL	Statistic : Spread & RMSE	Observation Type: GDAS
Go to single model verif	Forecast Hour: ALL		Submit

Data Tables: GENS PDF graphic Email comments

1000 hPa Height, Runtime: 01 DEC 2007 - 28 FEB 2008, Cycle: ALL, North Hemisphere



Done

start | dew - PuTTY | dew - PuTTY | dew - PuTTY | dew - PuTTY | X Server display:0 | naefs-worksho... | IMP_MEETING | Inbox for Yueji... | NCEP Model Ver... | Microsoft Powe... | EN | 12:35 PM

RTVSimage_24748.pdf (application/pdf Object) - Mozilla Firefox

File Edit View History Bookmarks Tools Help

http://cerberusdev.ncep.noaa.gov/EMC_VSDB_verif_Ensemble/temp_dir/RTVSimage_24748.pdf

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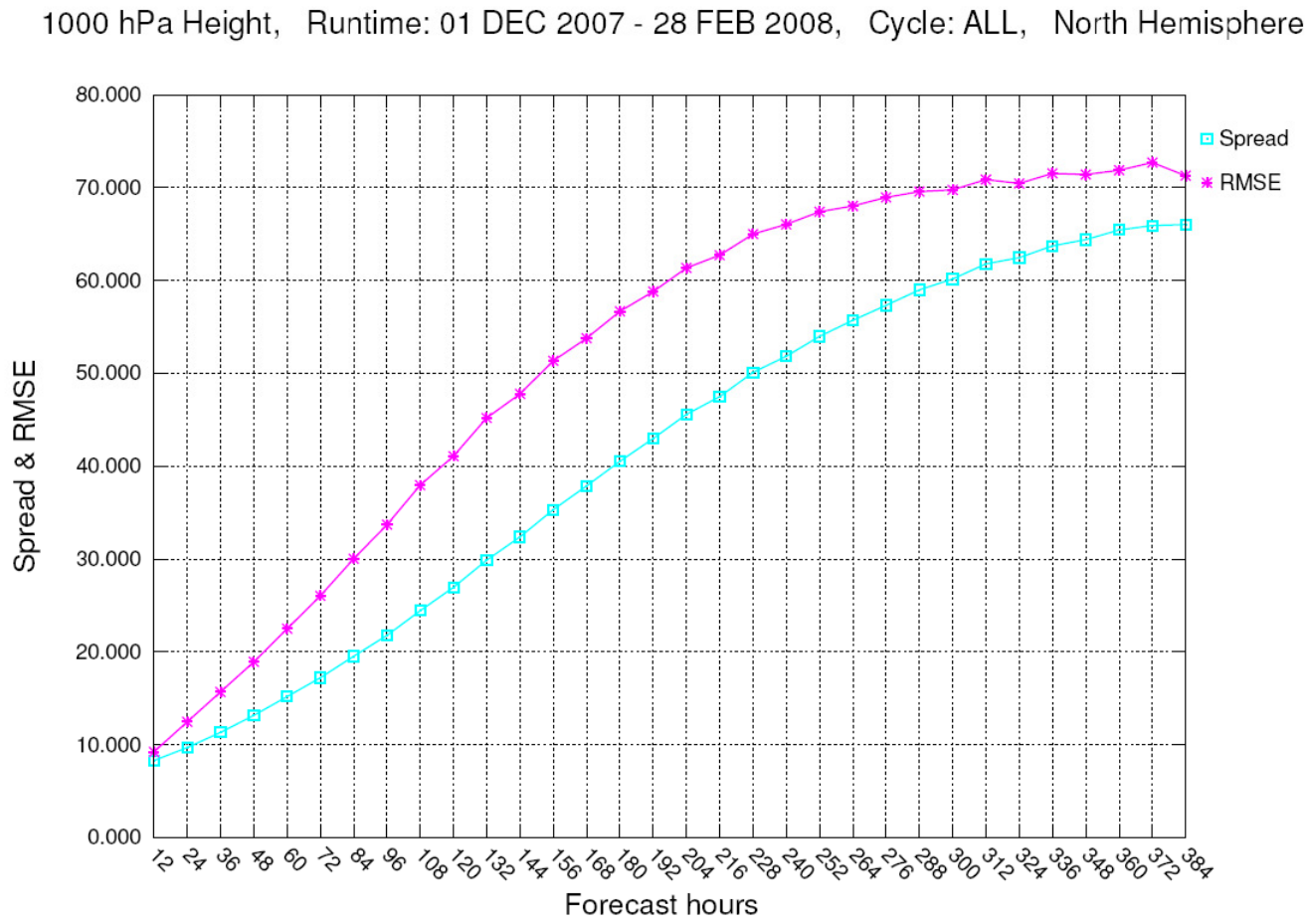
http://wwwt.emc.n...rpex/CLIMATE.html RTVSimage_24748.pdf (applicati...

1 / 1 106% Find

Click to show one page at a time



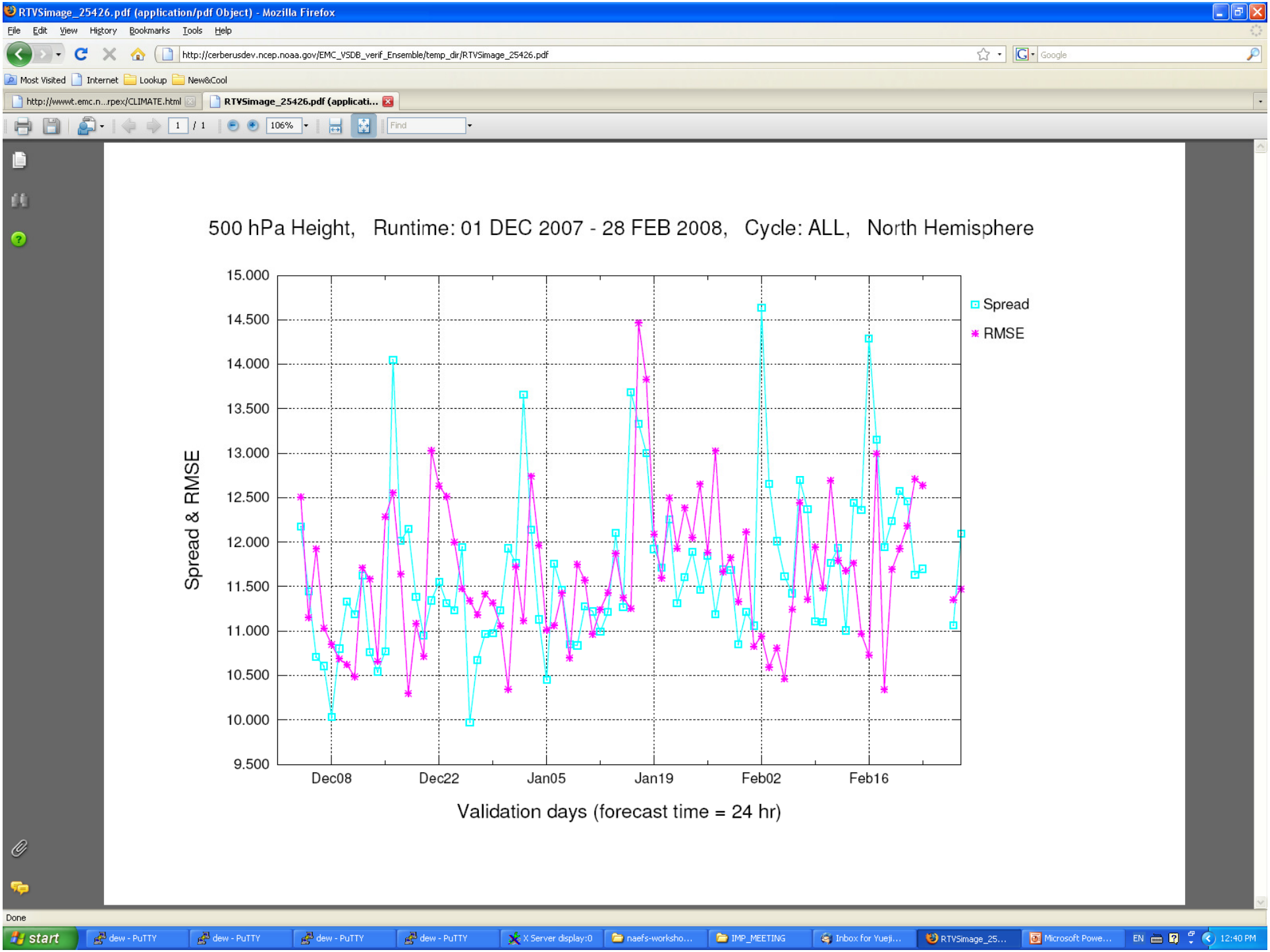
Done

start dew - PuTTY dew - PuTTY dew - PuTTY dew - PuTTY X Server display:0 naefs-worksho... IMP_MEETING Inbox for Yueji... RTVSimage_24... Microsoft Powe... EN 12:36 PM

GENS Verification Results

1000 hPa Height, Runtime: ALL, Forecast Hour: ALL, 01 JAN 2008 - 28 FEB 2008
North Hemisphere, Observation Type: GDAS

Middle Date	Forecast Hour	Spread	RMS
2008-01-30	12	8.428	9.461
2008-01-30	24	9.761	12.739
2008-01-30	36	11.369	15.999
2008-01-30	48	13.180	19.155
2008-01-30	60	15.286	22.858
2008-01-30	72	17.230	26.108
2008-01-30	84	19.602	30.256
2008-01-30	96	21.790	33.387
2008-01-30	108	24.421	37.724
2008-01-30	120	26.868	40.469
2008-01-30	132	29.754	44.596
2008-01-30	144	32.152	47.046
2008-01-30	156	35.139	50.706
2008-01-30	168	37.363	52.675
2008-01-30	180	40.293	55.785
2008-01-30	192	42.372	57.804
2008-01-30	204	45.221	60.260
2008-01-30	216	46.765	62.162
2008-01-30	228	49.522	64.120
2008-01-30	240	50.976	66.393
2008-01-30	252	53.416	67.311
2008-01-30	264	54.762	69.254
2008-01-30	276	56.885	69.531
2008-01-30	288	58.064	71.360
2008-01-30	300	59.656	70.696
2008-01-30	312	60.760	72.153
2008-01-30	324	61.818	71.055
2008-01-30	336	62.978	72.073
2008-01-30	348	63.748	71.814
2008-01-30	360	64.897	72.246
2008-01-30	372	65.322	73.131
2008-01-30	384	65.146	71.497

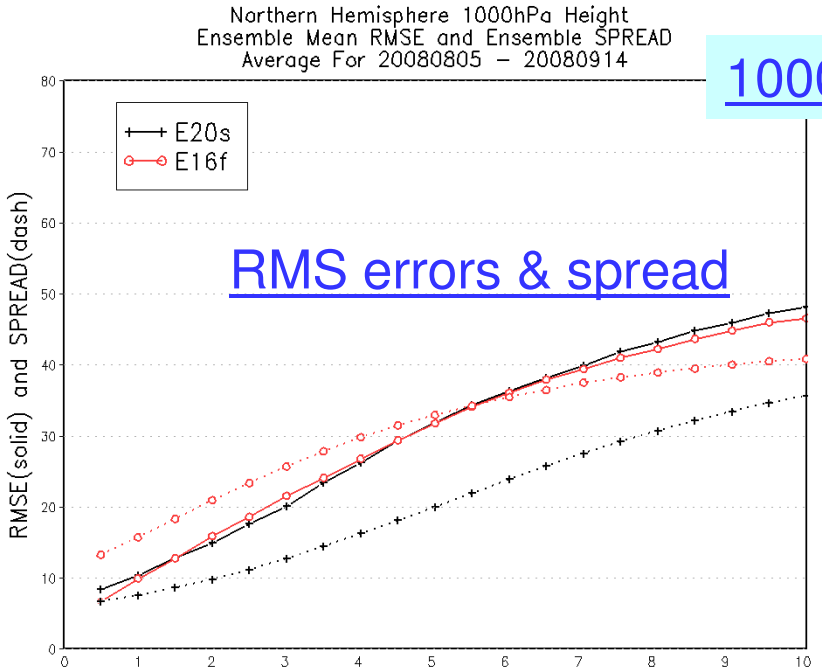

RTVSimage_25426.pdf (application/pdf Object) - Mozilla Firefox
File Edit View History Bookmarks Tools Help
http://cerberusdev.ncep.noaa.gov/EMC_VSDB_verif_Ensemble/temp_dir/RTVSimage_25426.pdf
Most Visited Internet Lookup New&Cool
http://wwwt.emc.n...rpex/CLIMATE.html
RTVSimage_25426.pdf (applicati...
500 hPa Height, Runtime: 01 DEC 2007 - 28 FEB 2008, Cycle: ALL, North Hemisphere
Spread
RMSE
Spread & RMSE
15.000
14.500
14.000
13.500
13.000
12.500
12.000
11.500
11.000
10.500
10.000
9.500
Dec08
Dec22
Jan05
Jan19
Feb02
Feb16
Validation days (forecast time = 24 hr)
Done
start
dew - PuTTY
dew - PuTTY
dew - PuTTY
dew - PuTTY
X Server display:0
naefs-worksho...
IMP_MEETING
Inbox for Yueji...
RTVSimage_25...
Microsoft Powe...
EN
12:40 PM

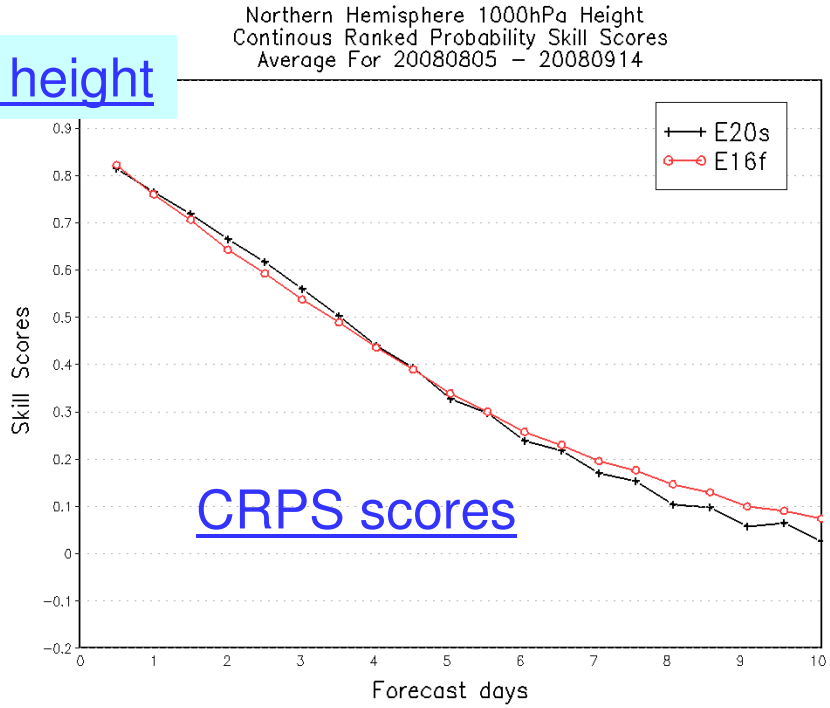
FNOMC global ensemble evaluation plan (draft)

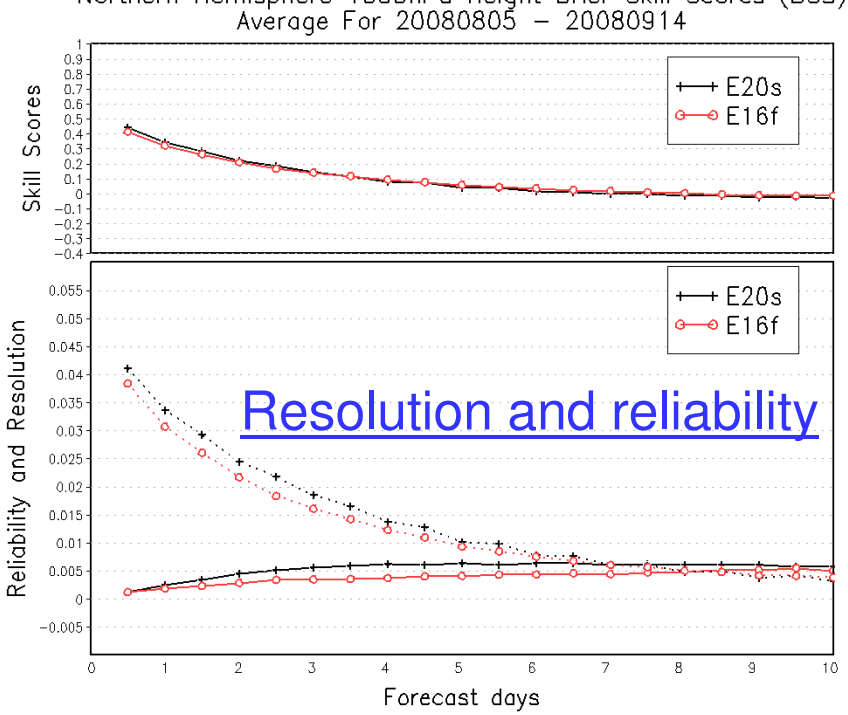
- Yuejian Zhu

- Basic evaluation
  - FNMOC raw forecast only
  - Against FNMOC analysis
  - 2.5 degree resolution
  - Globally, NH, SH and tropical
  - 500hPa height and 850hPa temperature
  - 1000hPa height and other surface variables, such as T2m, U10m and V10m
  - Anomaly correlation and RMS error for ensemble mean
  - Probabilistic evaluations, such as CRPS, reliability and resolution…
- Further evaluation
  - Raw ensemble forecast only
  - Against consensus analysis
  - Joined FNMOC with NCEP's
  - Joined FNMOC with CMC's
  - Joined FNMOC with current NAEFS's
  - Evaluation methods and variables are the same as basic evaluation
- Evaluation after post process
  - All bias corrected forecasts
  - Against NCEP analysis
  - Joined FNMOC with NCEP's
  - Joined FNMOC with CMC's
  - Joined FNMOC with NAEFS's
  - Evaluation methods and variables are the same as basic evaluation
- Evaluation after statistical downscaling process
  - Pending on the resources
  - CONUS only for 5km resolution
  - Against RTMA analysis
  - Variables: T2m, surface pressure
  - RMS error, mean error, absolute error for ensemble mean
  - CRPS for ensemble distribution
- Possible problem: (solve it)
  - We don't have T2m analysis
  - Possible to ask FNMOC to send T2m analysis with deterministic analysis/forecast package (check with Michael Sestak)

1000hPa height







NCEP/GEFS .vs FNMOC/GEFS

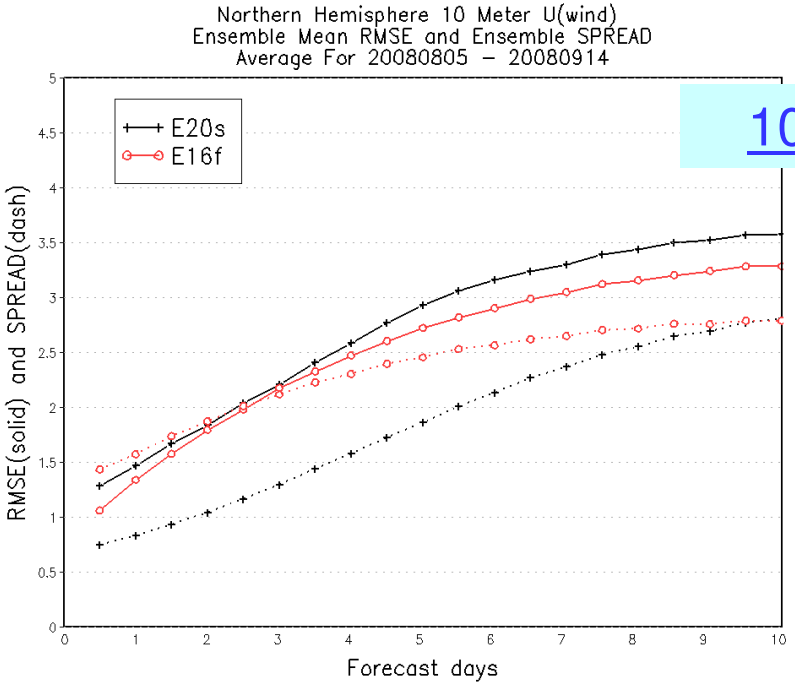
Raw ensemble forecasts

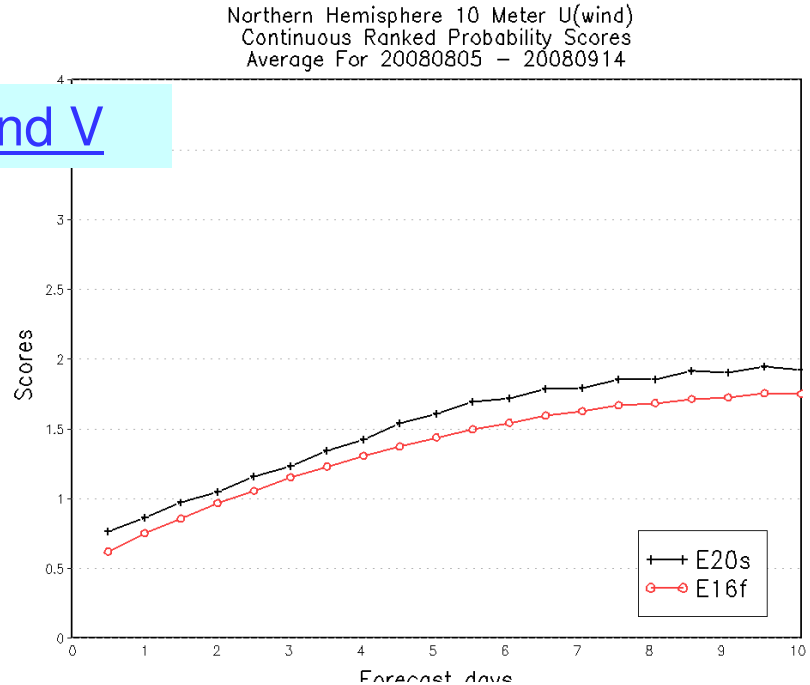
Very closed skills

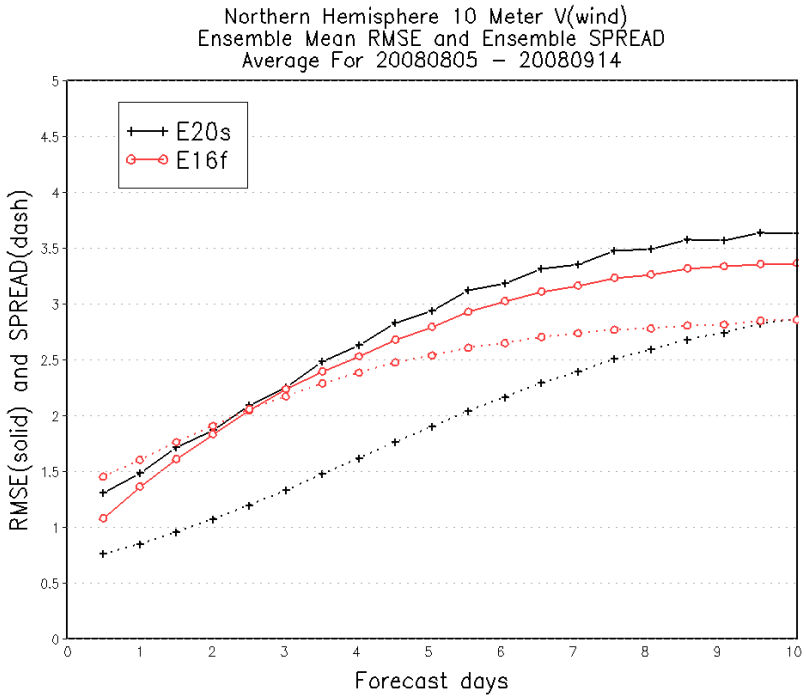
NCEP has more resolution

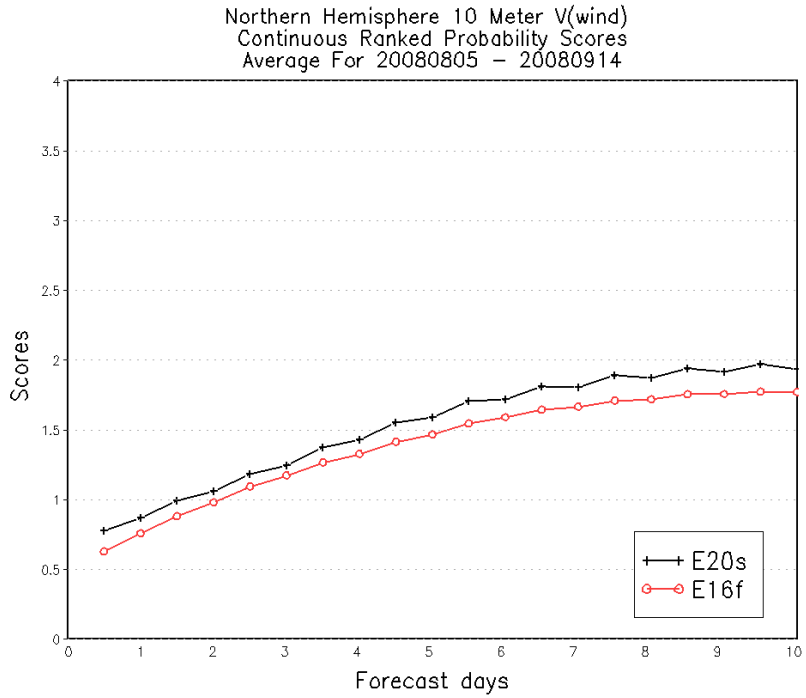
FNMOC is more reliable

10-m U and V

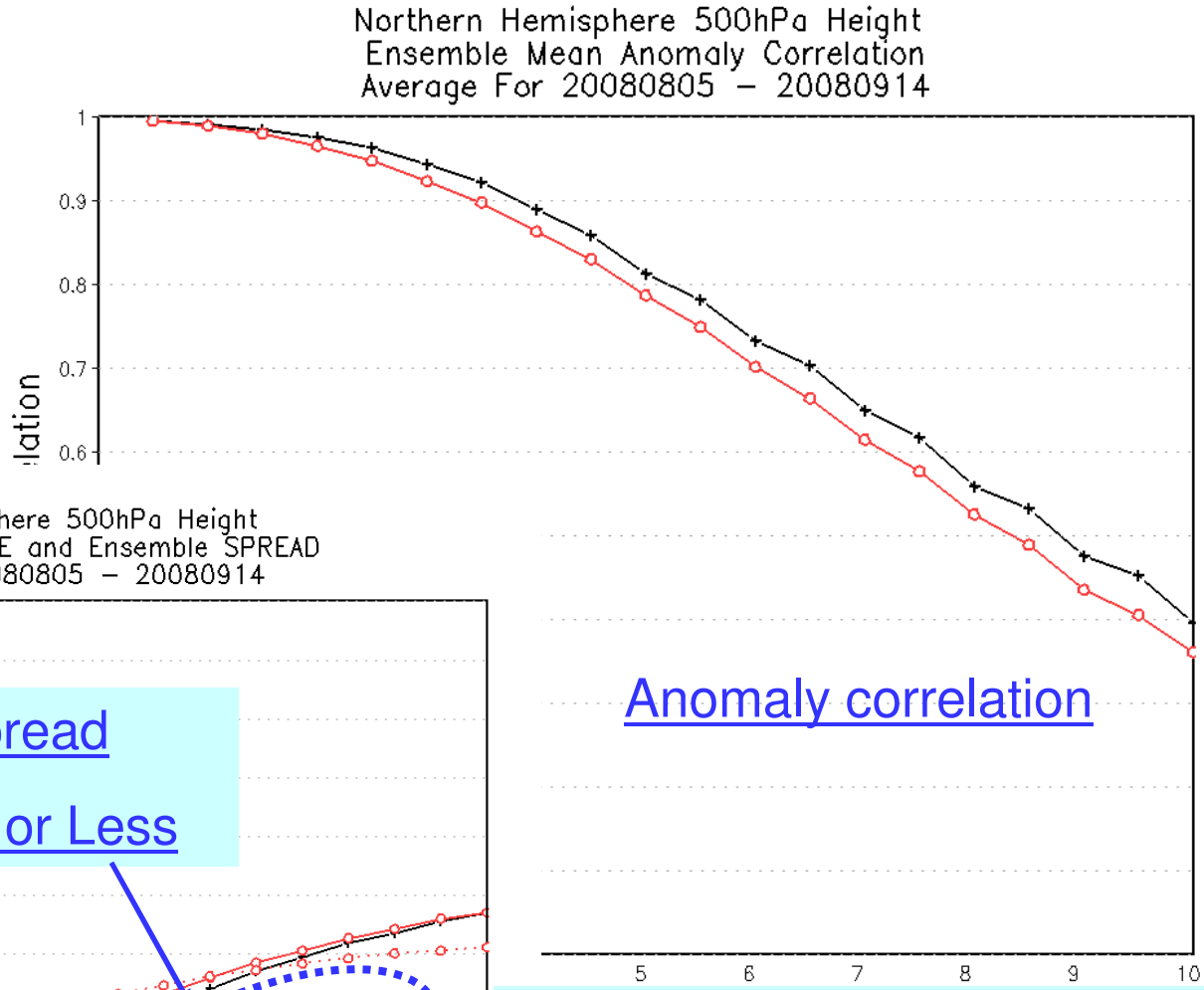

Northern Hemisphere 10 Meter U(wind)
Ensemble Mean RMSE and Ensemble SPREAD
Average For 20080805 - 20080914
E20s
E16f
RMSE(solid) and SPREAD(dash)
Forecast days


Northern Hemisphere 10 Meter U(wind)
Continuous Ranked Probability Scores
Average For 20080805 - 20080914
Scores
E20s
E16f

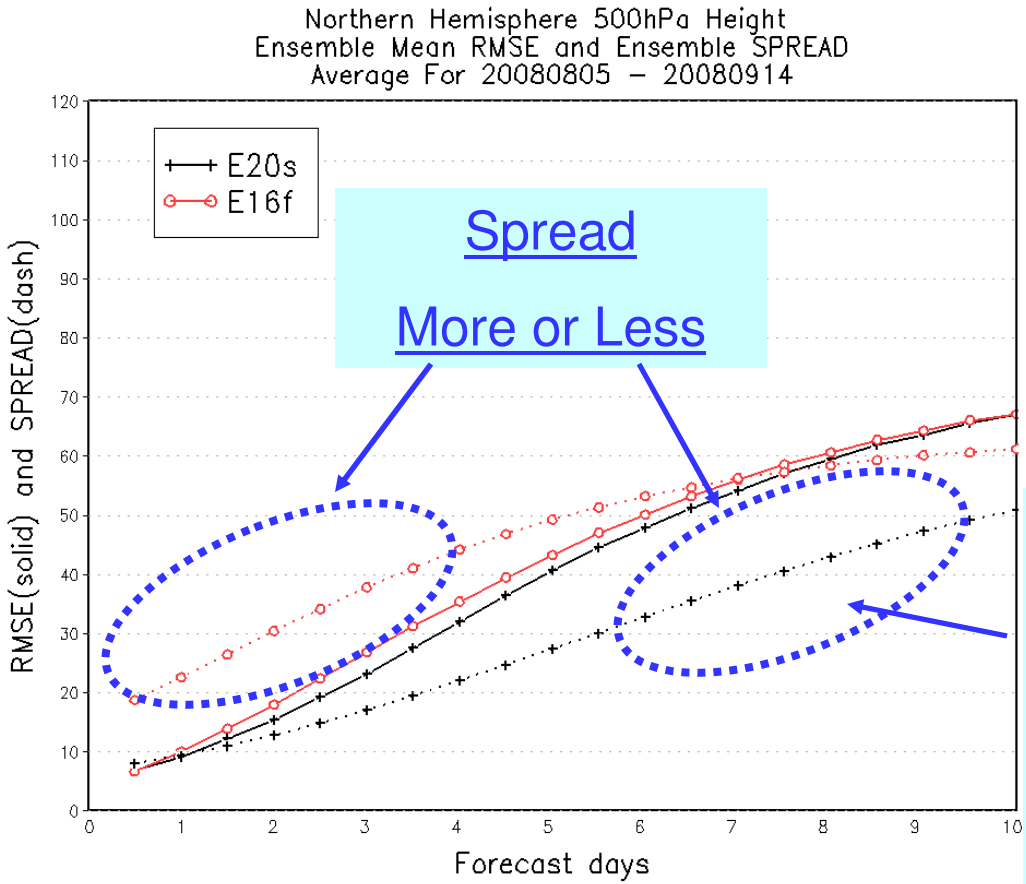

Northern Hemisphere 10 Meter V(wind)
Ensemble Mean RMSE and Ensemble SPREAD
Average For 20080805 - 20080914
E20s
E16f
RMSE(solid) and SPREAD(dash)
Forecast days


Northern Hemisphere 10 Meter V(wind)
Continuous Ranked Probability Scores
Average For 20080805 - 20080914
Scores
E20s
E16f
Forecast days

500hPa height



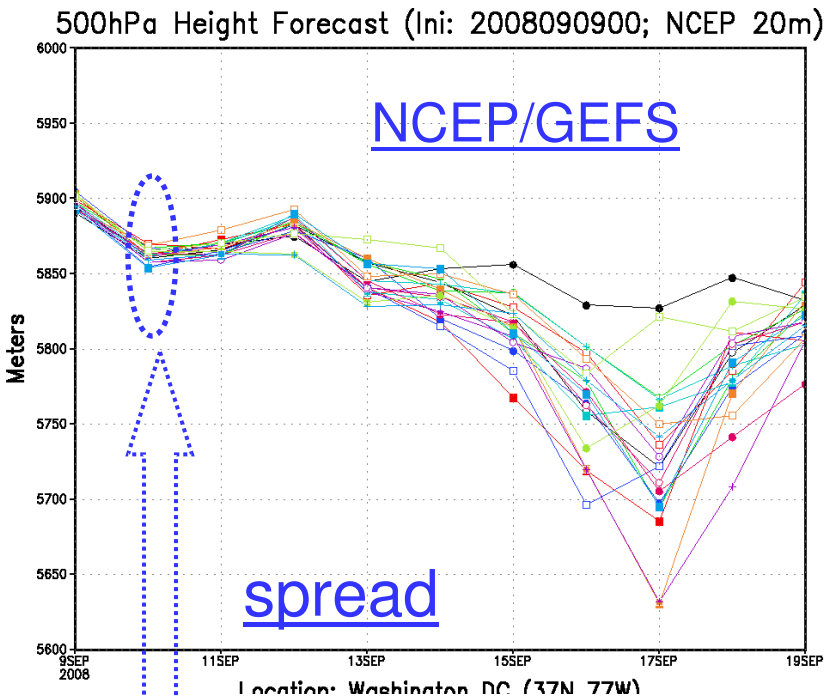
Anomaly correlation

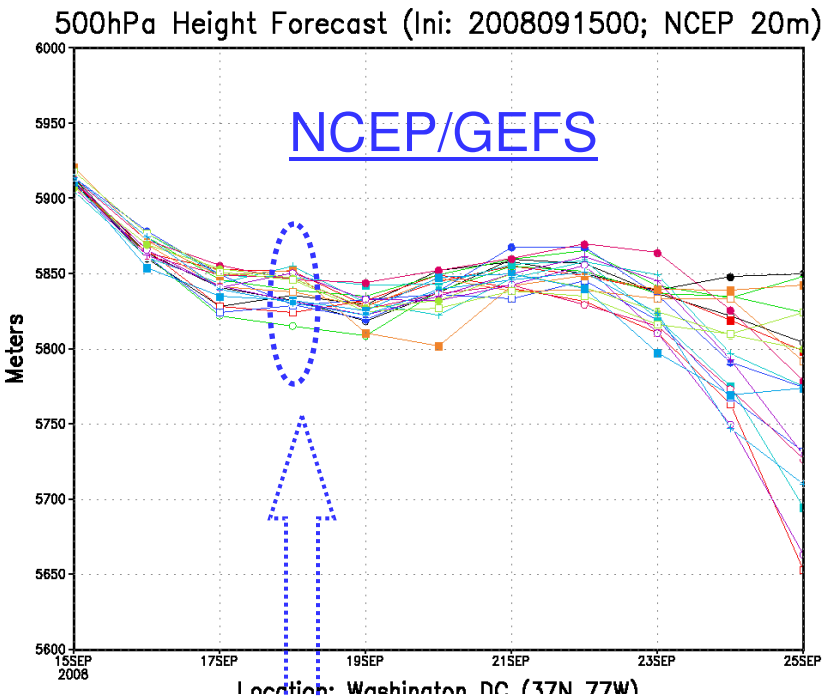


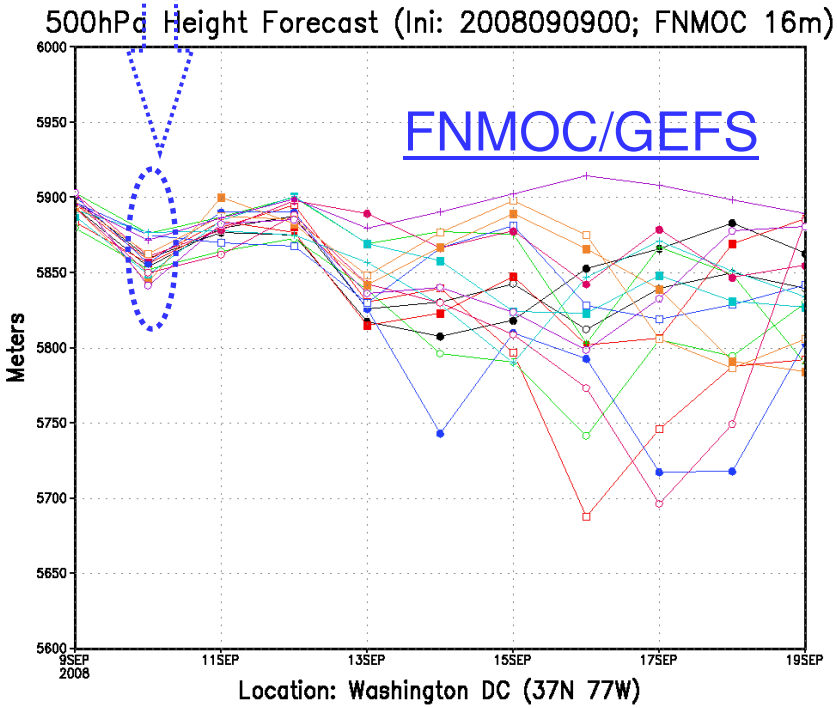
Spread

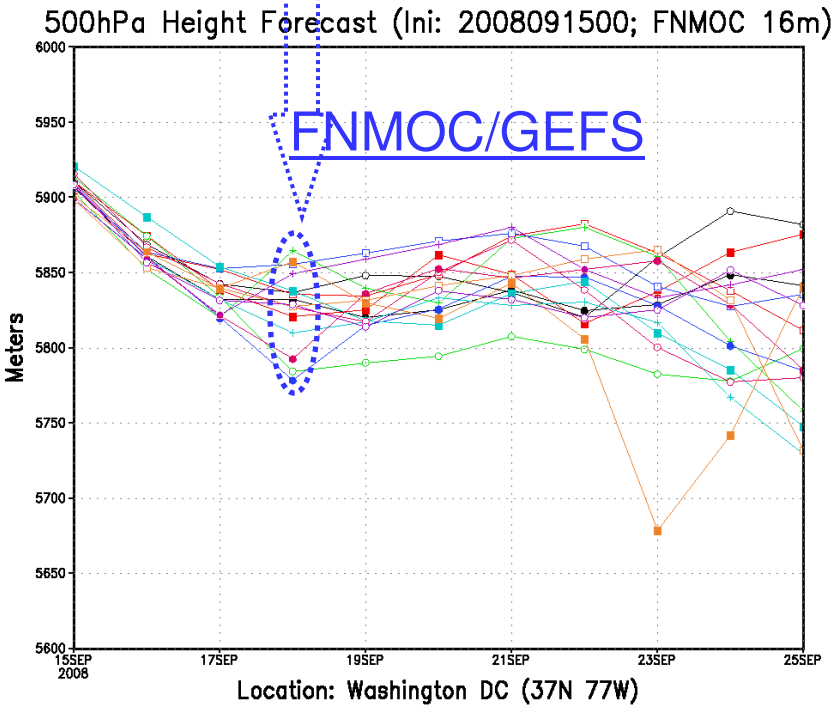
More or Less

It will be much improved from coming NCEP/GEFS implementation, through introduce stochastic scheme and high orders horizontal diffusion


500hPa Height Forecast (Ini: 2008090900; NCEP 20m)
NCEP/GEFS
spread
Meters
6000
5950
5900
5850
5800
5750
5700
5650
5600
9SEP 2008
11SEP
13SEP
15SEP
17SEP
19SEP


500hPa Height Forecast (Ini: 2008091500; NCEP 20m)
NCEP/GEFS
Meters
6000
5950
5900
5850
5800
5750
5700
5650
5600
15SEP 2008
17SEP
19SEP
21SEP
23SEP
25SEP


500hPa Height Forecast (Ini: 2008090900; FNMOC 16m)
FNMOC/GEFS
Meters
6000
5950
5900
5850
5800
5750
5700
5650
5600
9SEP 2008
11SEP
13SEP
15SEP
17SEP
19SEP
Location: Washington DC (37N 77W)


500hPa Height Forecast (Ini: 2008091500; FNMOC 16m)
FNMOC/GEFS
Meters
6000
5950
5900
5850
5800
5750
5700
5650
5600
15SEP 2008
17SEP
19SEP
21SEP
23SEP
25SEP
Location: Washington DC (37N 77W)

Preliminary results

- It is very preliminary
  - There are only about 40 cases
  - Raw ensemble forecast only
- Statistics show
  - FNMOC GEFS has comparable global ensemble system
  - Very good forecast for near surface variables
    - Need to have more samples to verify 2-m temperature
- Need to evaluate different seasons
  - To wait for winter season coming
- Overall consideration
  - To have more variables to be evaluated
  - Have bias correction
  - Combine to exist NAEFS forecast
    - Is there any value added to current system?

Background !!!!!

CRPS Decomposition

$$\overline{CRPS} = \sum_{i=0}^{N} [\overline{\alpha}_i p_i^2 + \overline{\beta}_i (1 - p_i)^2]$$

$$\overline{CRPS} = \sum_{i=0}^{N} \overline{g}_i [(1 - \overline{o}_i) p_i^2 + \overline{o}_i (1 - p_i)^2]$$

$$\overline{CRPS} = \overline{RELI} - \overline{RESO} + \overline{U}$$

$$\overline{RELI} = \sum_{i=0}^{N} \overline{g}_i (\overline{o}_i - p_i)^2$$

$$\overline{RESO} = \sum_{i=0}^{N} \overline{g}_i \overline{o}_i^2$$

$$\overline{U} = \sum_{i=0}^{N} \overline{g}_i \overline{o}_i$$

Where:

$$\overline{g}_i = \overline{\alpha}_i + \overline{\beta}_i$$

$$\overline{o}_i = \frac{\overline{\beta}_i}{\overline{\alpha}_i + \overline{\beta}_i}$$