NOAA Earth System
Research Laboratory

A method to estimate analysis and forecast errors

Malaquías Peña, Zoltan Toth,
Mozheng Wei and Isidora Jankov

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Forecast Evaluation

• NWP forecasts use *numerical analysis as a proxy for the truth.* Studies rely on this because

— Truth is unknown and analysis is the best representation of the atmosphere at any given time

— Observations are not sufficient to assess all (3D) fields

— Analysis errors << medium range forecast errors

- Bad symptoms of using analysis as proxy for the truth

— Skill scores depend on choice of verifying analysis

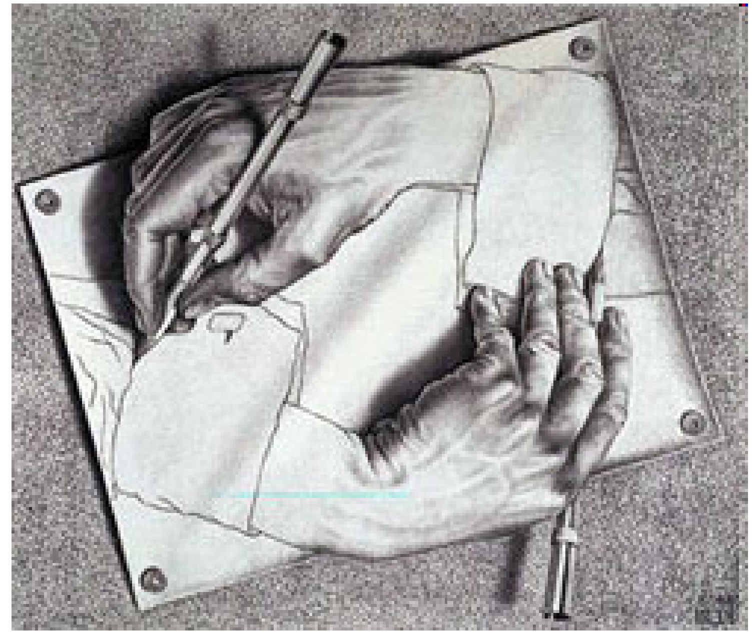
— Errors are underestimated in regions devoid of data or where DA schemes give little weight to observations

Forecast Evaluation (2)

- Candille and Talagrand (2004, 2008)
  — Rewrite skill scores considering uncertainty in the verifying field
  — Skill scores degrade when verifying field uncertainty is taken into account
- Bowler (2006, 2008)
  — Partitions forecast errors into forecasts errors against the truth and the observational uncertainty
  — Skill scores systematically degrade

Estimation of analysis errors

- Analysis errors are scheme dependent
- They correlate with Background errors
- Difficult to attribute errors to either the analysis method or the forecast model



- Simmons and Hollingsworth (2002) use mean square differences, d^2, of ECMWF and MetOffice analyses.

$$d^2 = a_1^2 + a_2^2 - 2a_1 a_2 c_{12}$$

where a_1, a_2 analysis errors and c_{12}, the pattern correlation between a_1 and a_2, are unknown.

It is solved by assuming that the ratio of a_1 and a_2, as well as c_{12}, are the same as the ratio and correlation from short range forecast errors (denoted by x^{24h}):

$$\frac{a_1}{a_2} = \frac{x_1^{24h}}{x_2^{24h}}$$ and $$c_{12} = c_{12}^{24h}$$

Estimation of analysis errors (2)

- Needed to determine amplitude of ensemble perturbations
  - — Breeding and ET at NCEP use climatology of 500hPa of short range forecast errors
  - — Ensemble schemes do not need the full error covariance matrix
- Modern DA schemes can compute analysis errors but are computationally expensive in global variational assimilation schemes
- An analysis estimation method is useful if it is
  - —computationally inexpensive
  - —conservative. That is, the estimated error is larger than the real error to prevent filter divergence
  - —sensitive to observations. That is, it reflects the spatial variance reduction due to observations

Perceived forecast errors

- Define the perceived forecast error variance, d^2, as the squared difference between a forecast, F, and an analysis, A, at the verifying time:

$$d^2 \equiv (F - A)^2$$

- Decomposing it into terms involving the true state, T:

$$d^2 \equiv (F - A)^2 = (F - T)^2 + (A - T)^2 - 2\rho_{fa}(F - T)(A - T)$$

$$= x^2 + x_a{}^2 - 2\rho_{ax}\, x\, x_a$$

where ρ_{fa} is the correlation between true forecast error and true analysis error at the verifying time; x and x_a are the true forecast errors and true analysis errors, respectively.

Perceived forecast errors (2)

- This is the set of equations to solve

$$d_L{}^2 = x_L{}^2 + x_a{}^2 - 2\rho_{ax_L}\; x_L\; x_a$$

where L denotes the forecast lead time.

- Functional analysis problem
- Unknown, flow-dependent functions: x_L, x_0 and ρ and noisy measurements of d

- Stage 1 of this study neglects flow-dependency and uses historical data to obtain robust estimates of the unknowns.
- Will provide sensible and minimal number of assumptions

Assumptions to simplify the problem

- Small initial errors grow *exponentially* and saturate following a logistic function.
  - Theoretical considerations; spread of ensembles; error growth of simplified models.
  - Departures from this evolution of errors will be attributed to model errors, which will be modeled with another continuous function
- Correlation decreases on each analysis cycle at a *power* rate:

$$\rho_m = (\rho_1)^m, m=2,..M$$

where ρ_1 is the correlation at *6h* lead time, $\rho_2 = (\rho_1)^2$ is the correlation at *12h* lead time, $\rho_3 = \rho_1 \rho_2$, is the correlation at *18h*, etc. <u>Only one parameter (ρ_1) needs to be determined.</u>

Estimation Procedure

- In a logistic forecast model, four parameters are estimated: Initial error (x_0), error growth (α), saturation error (s_∞*), and 6h* lead-time correlation (ρ_1)

$$\hat{d}_1^2 = x_1{}^2 + x_0{}^2 - 2\rho_1 \; x_1 \; x_0$$

$$\hat{d}_2^2 = x_2{}^2 + x_0{}^2 - 2\rho_1^2 \; x_2 \; x_0$$

$$\hat{d}_3^2 = x_3{}^2 + x_0{}^2 - 2\rho_1^3 \; x_3 \; x_0$$

and so on

where $$x_L = \frac{s_\infty \cdot c}{e^{-\alpha \cdot L} + c}$$

and $$c = x_0 / (s_\infty - x_0)$$

- Unless the saturation is known, the calculations will be made using an *exponential function* to model the forecast error to be consistent with the assumptions made (i.e., error is local at short leads). In this case, only x_0, α and ρ_1 are determined, and the forecast error at lead L is:

$$x_L = x_0 e^{-\alpha \cdot L}$$

- For data points L larger than 4, the set of equations becomes overdetermined. It is solved by minimizing the following cost function

$$J = \max(\left| d_i{}^2 - \hat{d}_i{}^2 \right| \cdot w_i{}^{-1}), \quad i = 6h, 12h, 18h, \ldots$$

Tests with the Lorenz 63 model

Experimental setup: Perfect model scenario

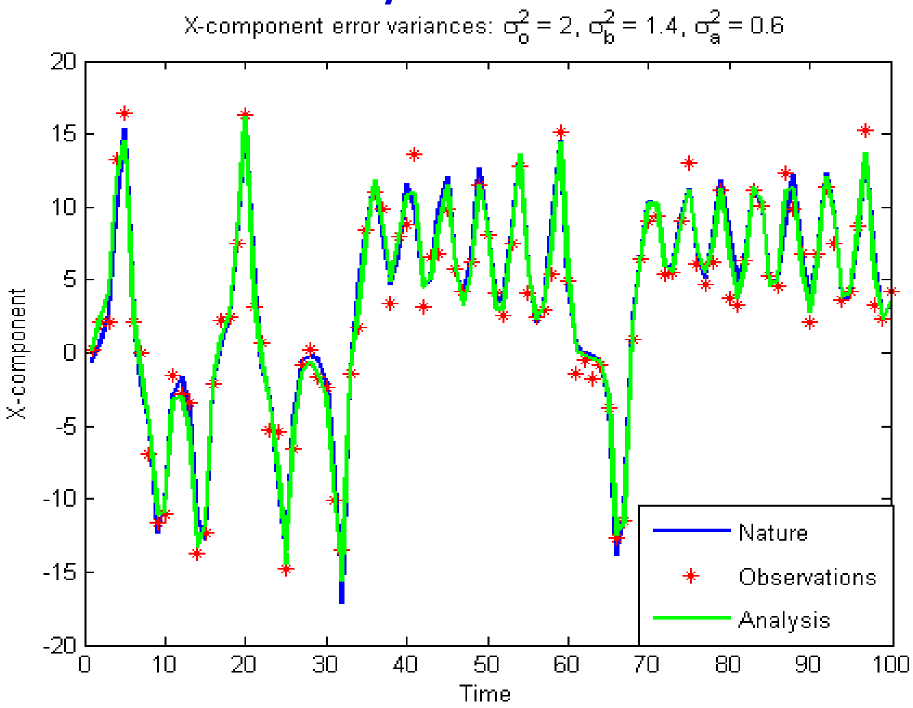
• Produce "nature" time series of size N=2x10^4, with the usual Lorenz 3-variables params

• Produce "observations" from this nature by adding to the true state a value drawn randomly from a normal distribution with mean zero and standard deviation σ_o.

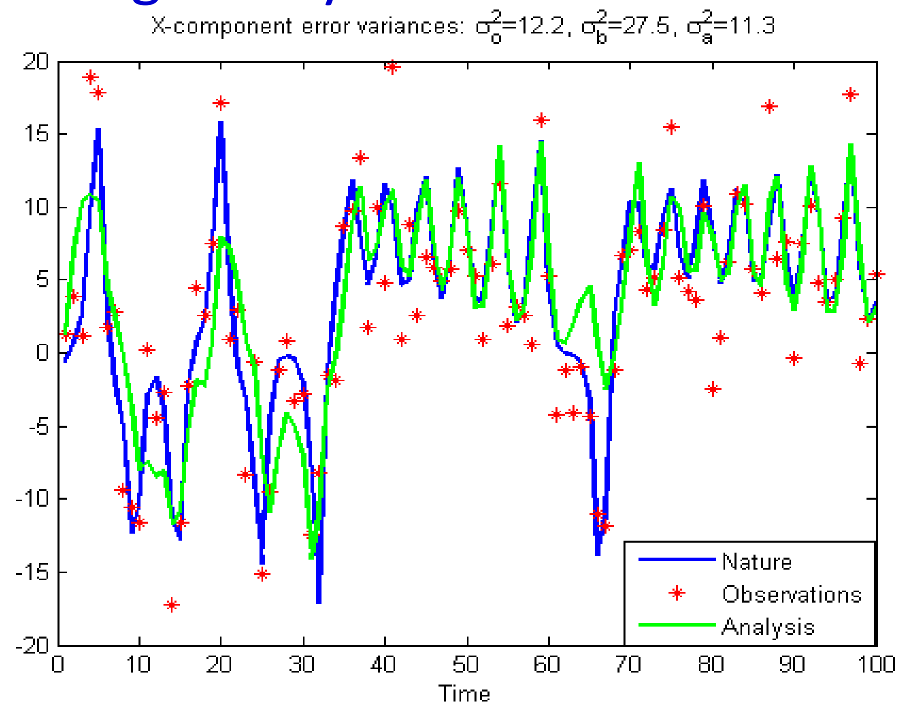
• Assimilate observations every 15 time steps using a 3DVar scheme with a fixed background error covariance generated from a different long time series of (perfect) forecasts using the NMC method. We use a tuning coefficient to modify amplitude of B.

• Use the analysis as initial conditions to produce forecasts out to 40 DA cycle units

Small analysis error variance



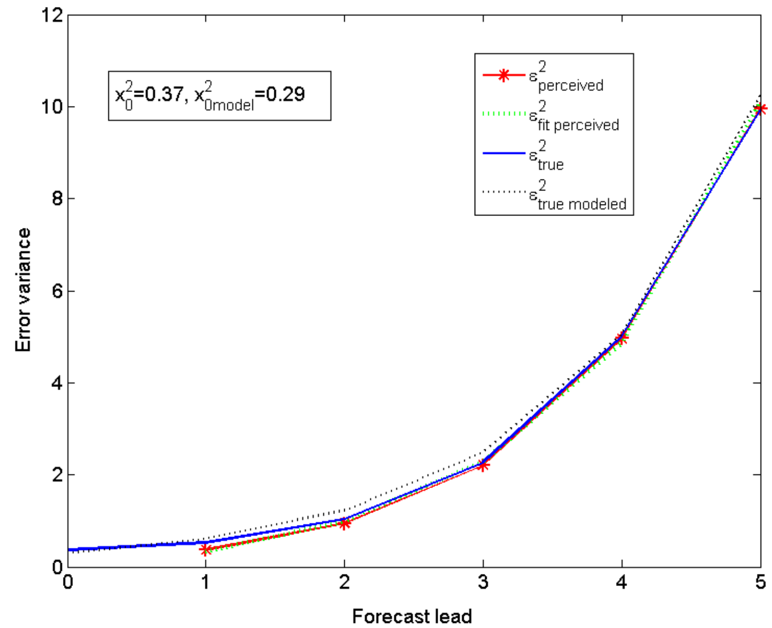
Large analysis error variance



Experimental results

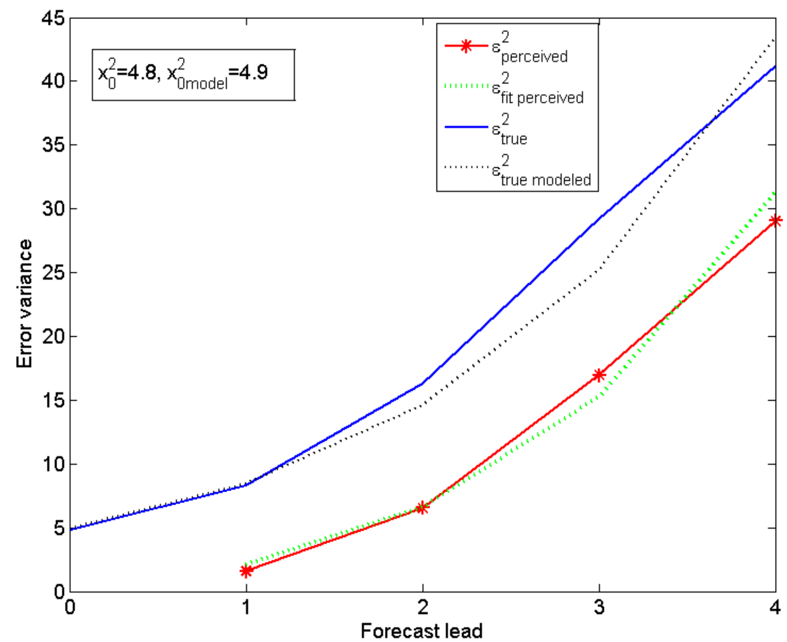
- Perfect model experiment provides:
  - True forecast errors (blue line),
  - Perceived forecast errors (red line)
  - Analysis errors (dotted blue line)
- Measured perceived errors, d (in red) are modeled with ^d (in dotted green) by minimizing the cost function: $J = \max(\left|d_i^{\,2} - \hat{d}_i^{\,2}\right| \cdot w_i^{-1}), \quad i = lead$

- Resulting parameters: [x0 , α, ρ1]

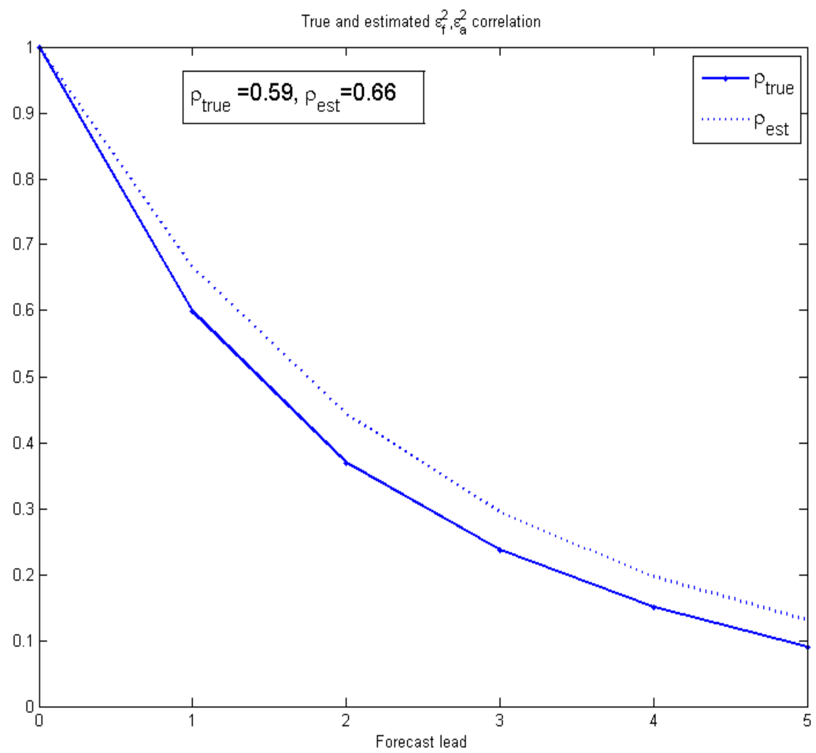


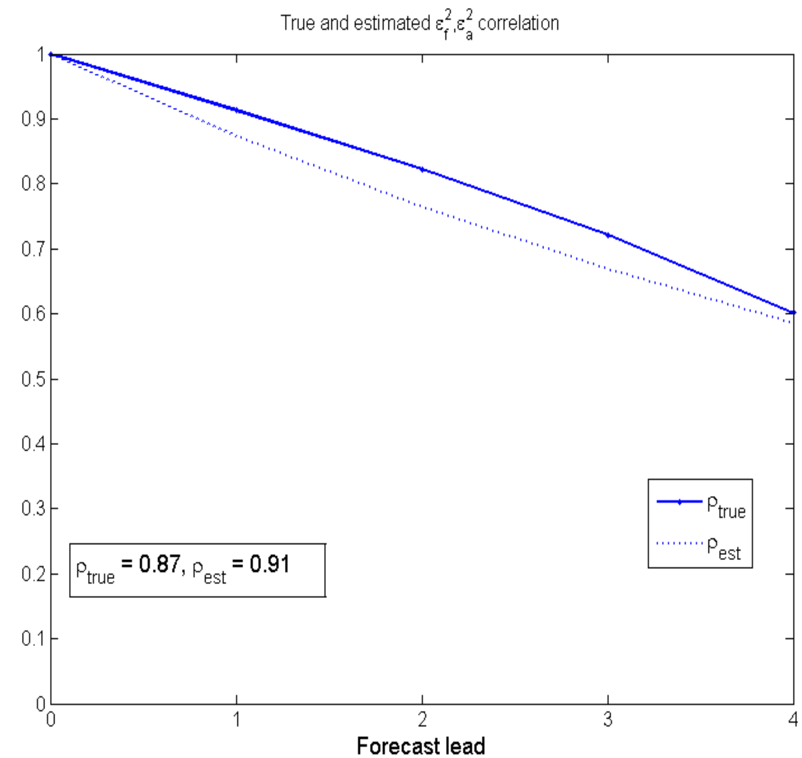
- These parameters are substituted in the exponential equation to create the true modeled analysis and forecast errors (in black)

- Underestimation of true forecast errors by the perceived errors



Error correlation



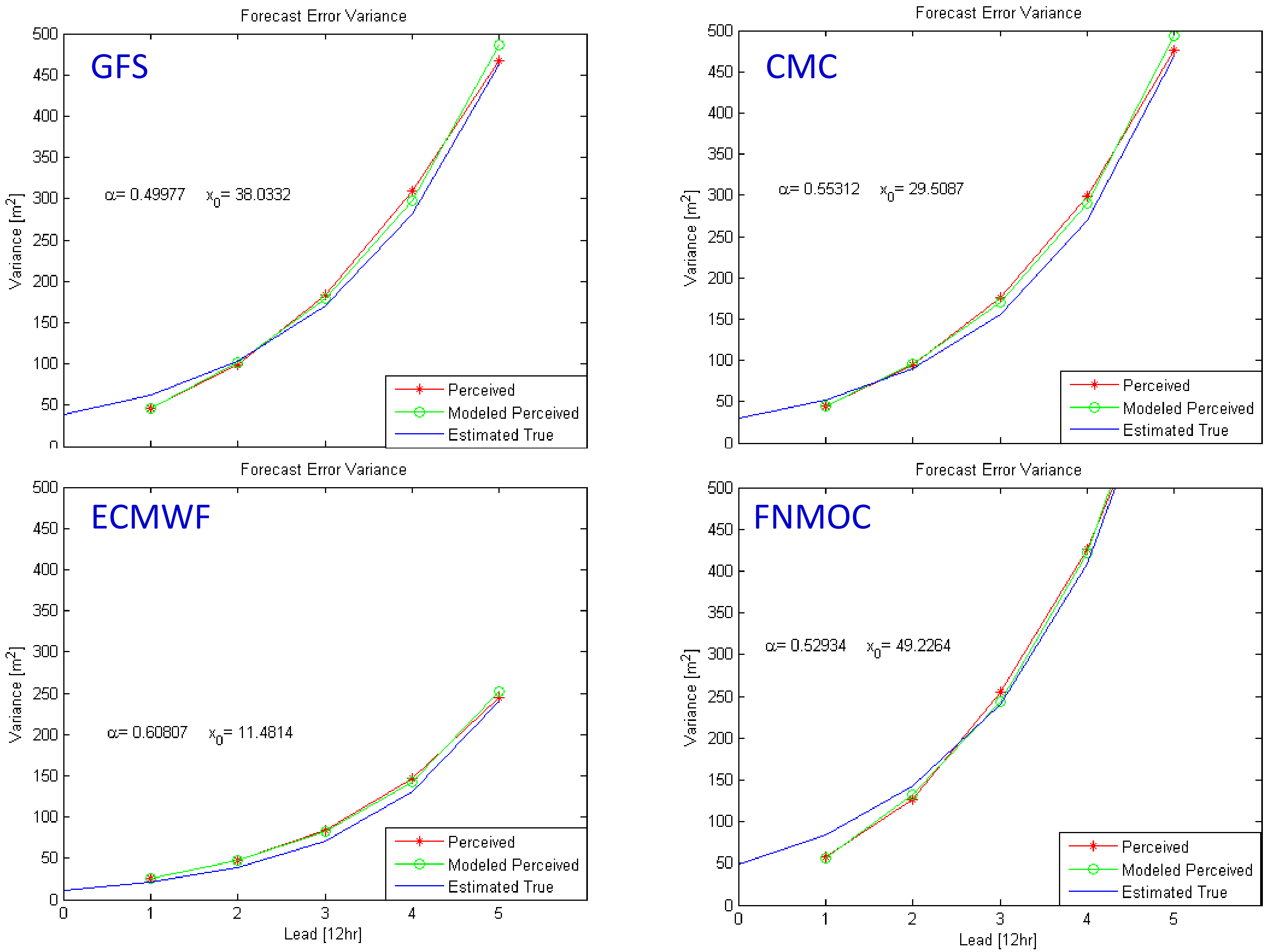


- Estimated correlation dies-off similarly as the true correlation.
- This case is well tuned so the DA scheme is able to extract information independent from the first guess.

- Correlation is too high, probably indicating that the scheme, which is not well calibrated, relies heavily on the FG.
- The shape of the decaying correlation is different from that of the estimated correlation.

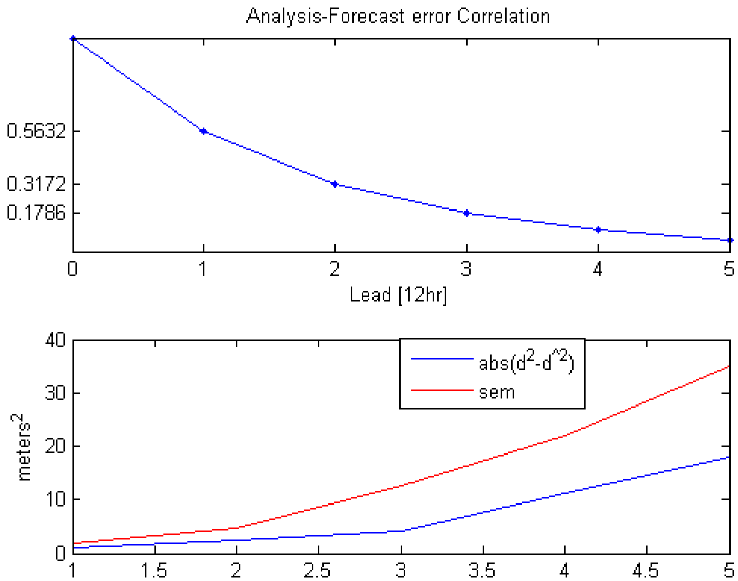
Forecast performance intercomparison

Exponential function fitting. 500hPa height. NH area average. fall 2008



Forecast performance intercomparison

GFS



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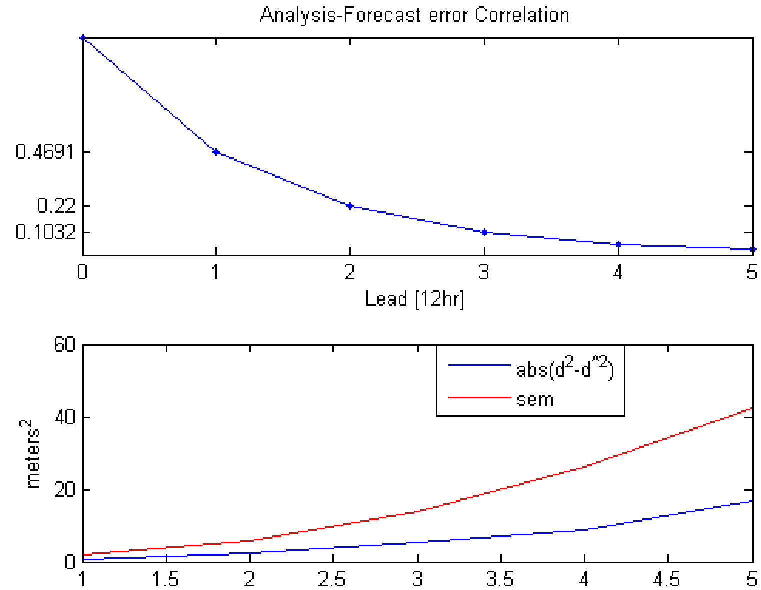
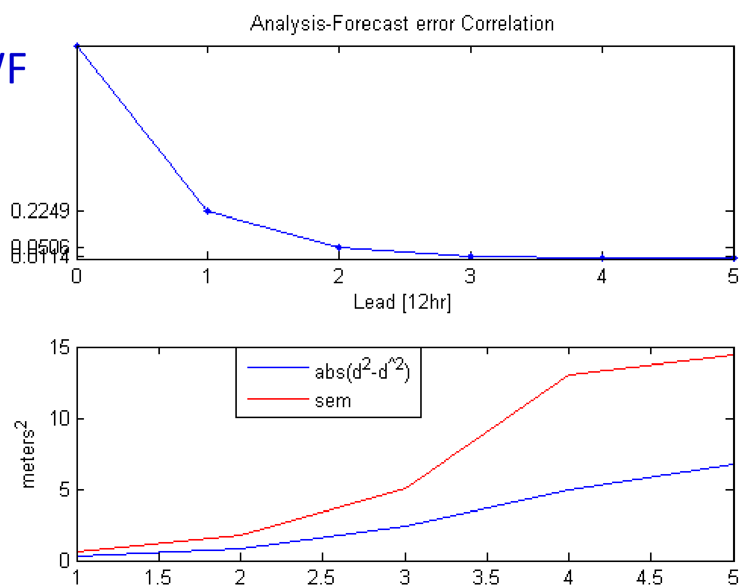
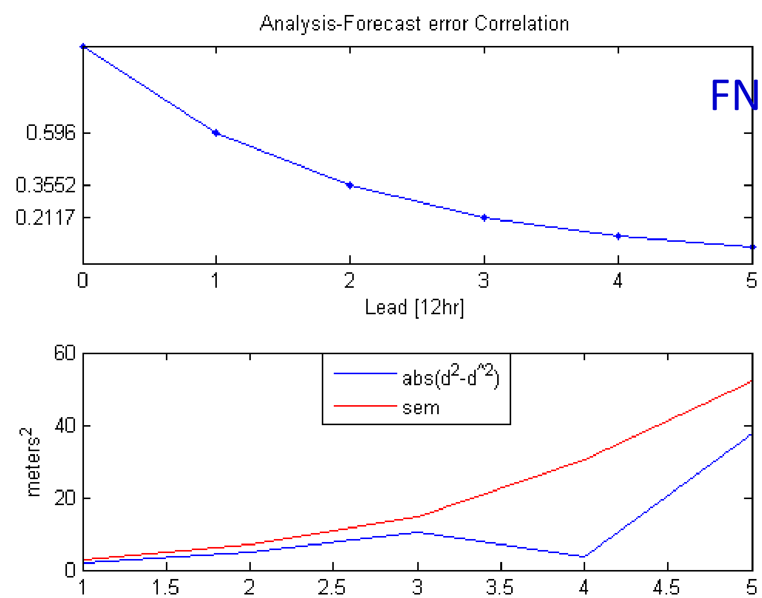


ECMWF

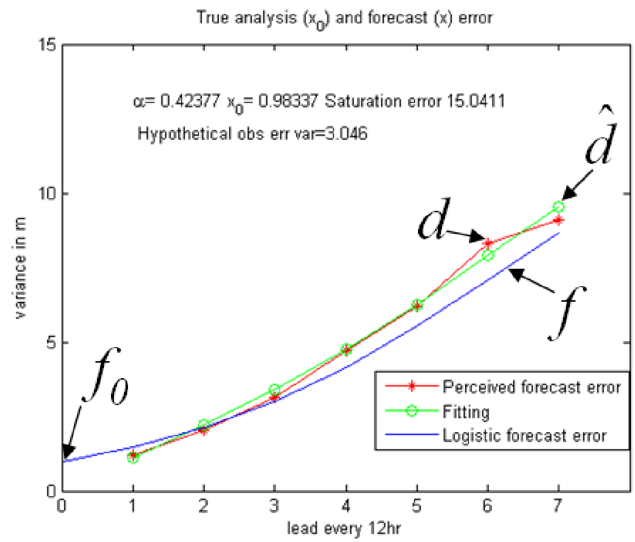


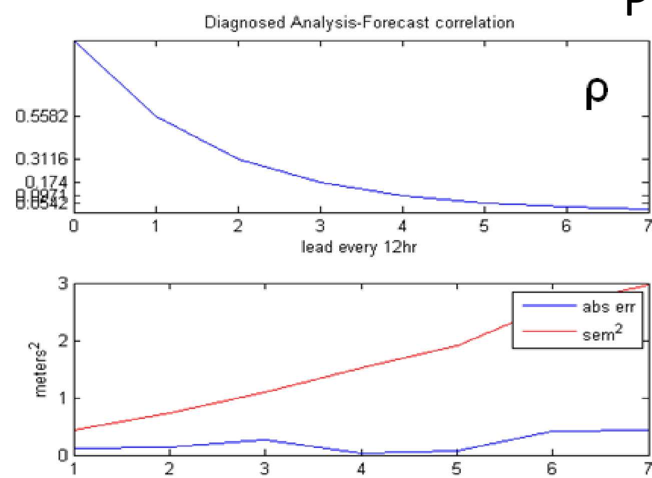
FNMOC



GFS 500 hPa height data. Two gridpoints

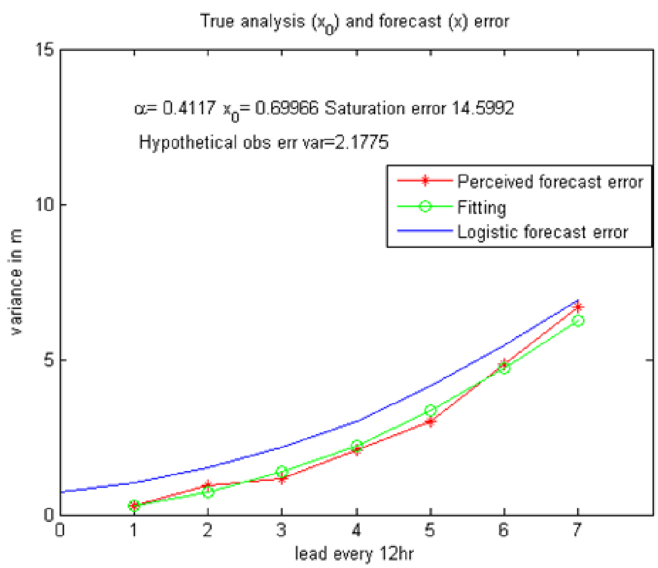
Logistic function fitting

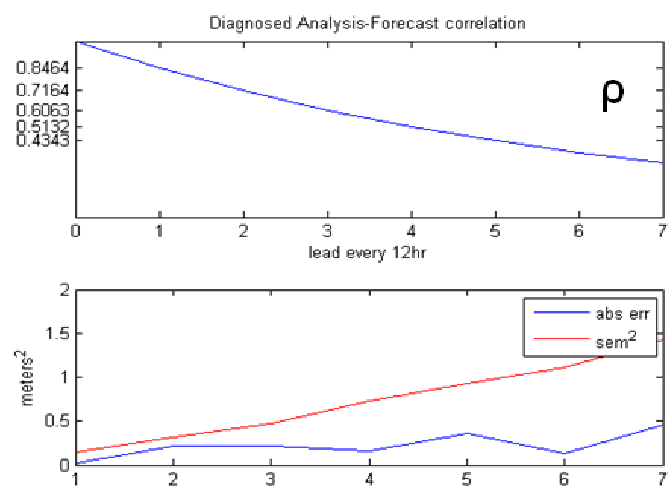




Point in the Extratropics

- Right panel: Estimated A error (f_0) is slightly overestimated
- Left panel: DA assimilates enough new information (from obs) to reduce correlation with forecast errors





Point in the tropics

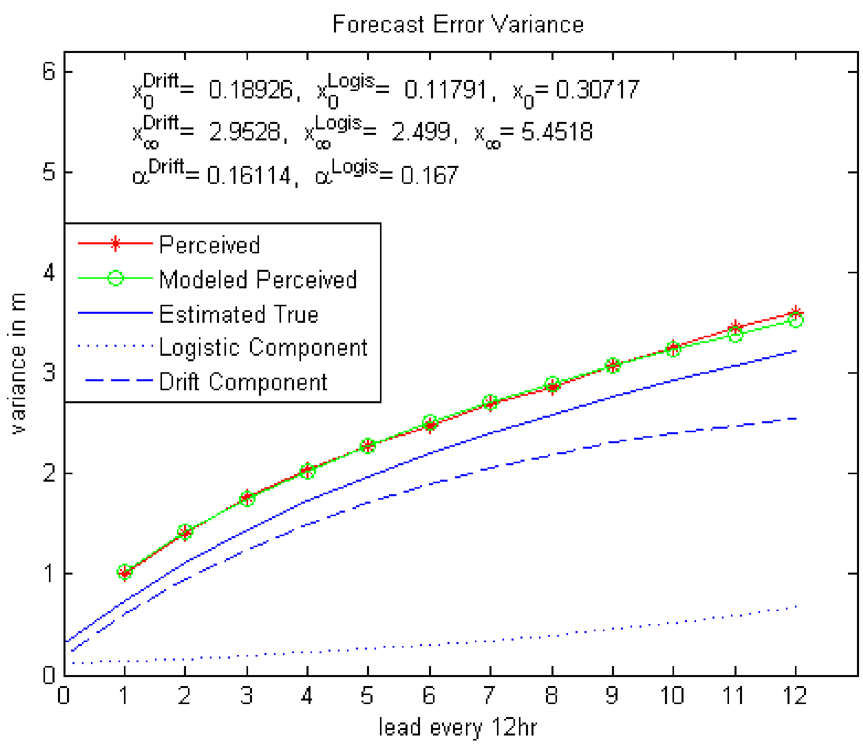
- Right panel: Both A error and F errors are overestimated.
- Left panel: High correlation indicates not enough new observation in the DA scheme

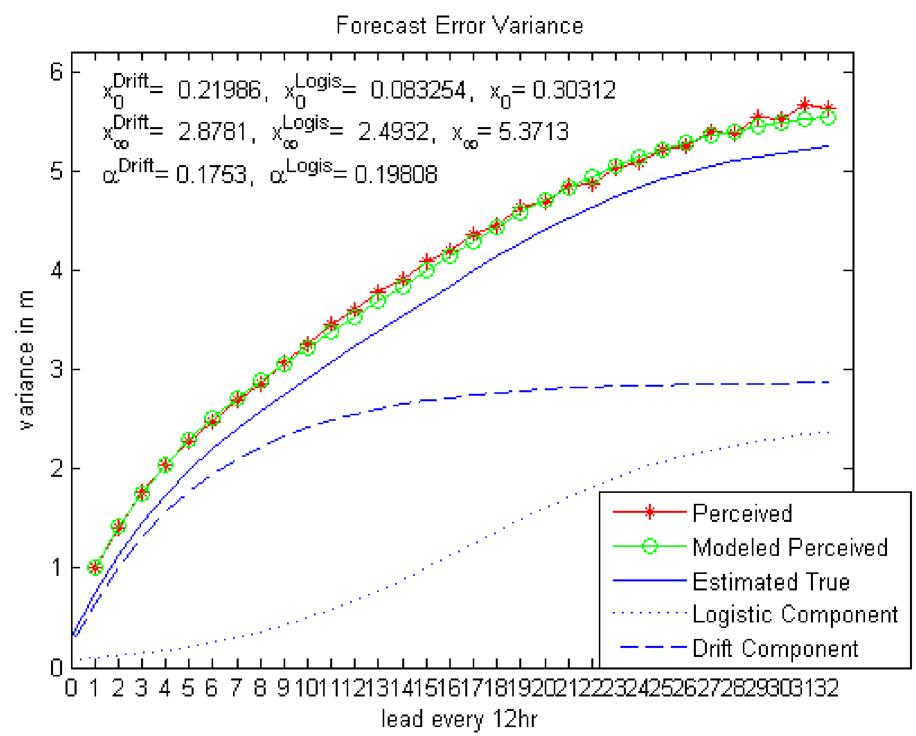
GFS 850 hPa U-component Tropics

- Complex perceived errors can be fit with more sophisticated functions. Here the use of 7 parameters (logistic + saturating exponential functions) is used in a two steps process.
  - In the first step, the short lead data points are fit to a saturating exponential:

$$x = x_a \cdot (s_a - e^{-\alpha t})$$

  - In the second step, the parameters obtained in step 1 are prescribed in the 7 parameter function using many more data points



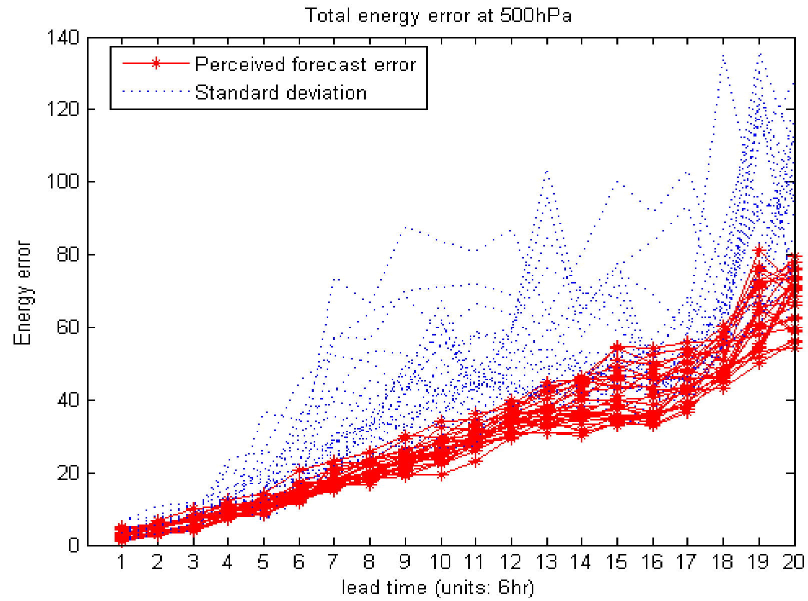


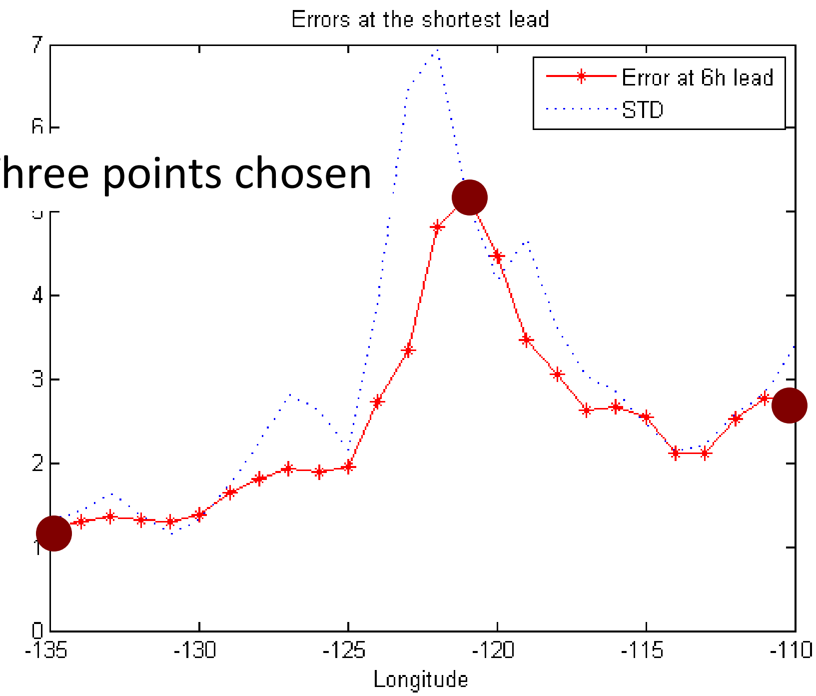
Points along 40° N. Total energy error variance data

Purpose: Asses whether the method captures the spatial characteristics of the variance reduction obtained by the observations

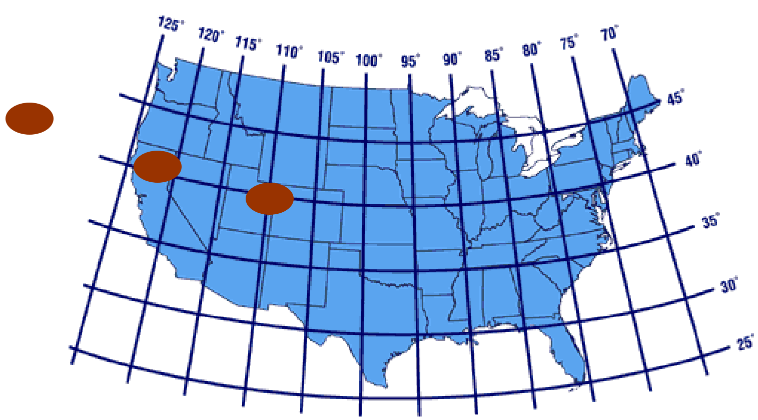
For all cases shown here:

- Assume exponential growth
- Minimization of cost function with L_∞ Norm
- First guess of the minimization procedure $u_0=[x_0\ \alpha_0\ \ \rho_0]$ varying $x_0=[1.5 \text{ to } 6.5]$ and $\rho_0=[0.05 \text{ to } 0.5]$, and $\alpha_0=0.15$



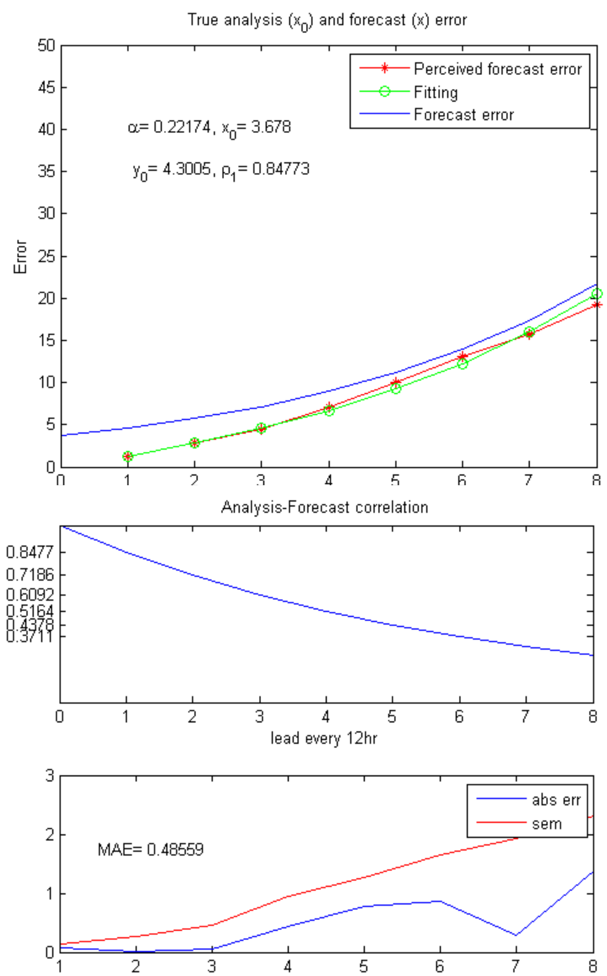


Three points chosen

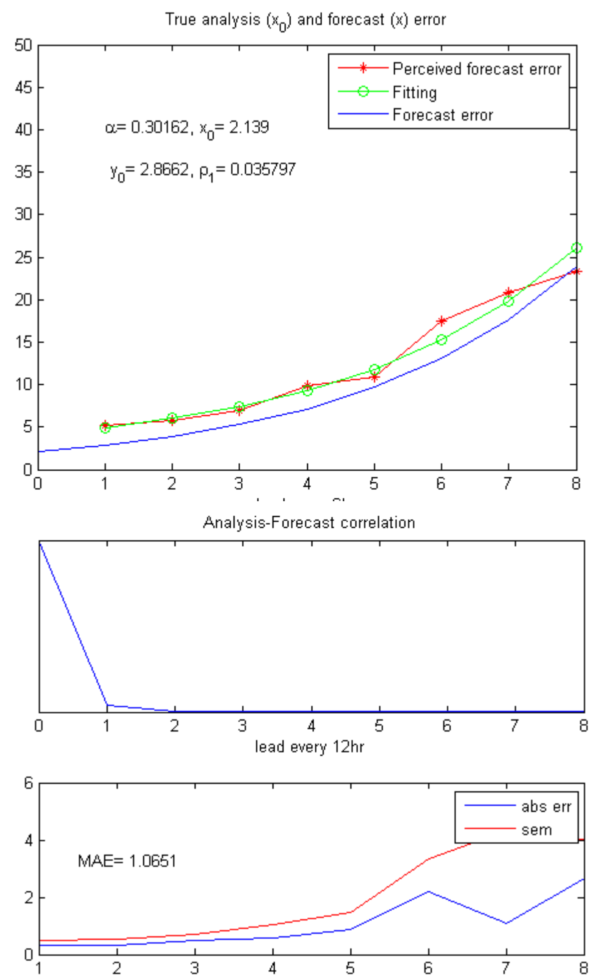


Fitting model: Exponential

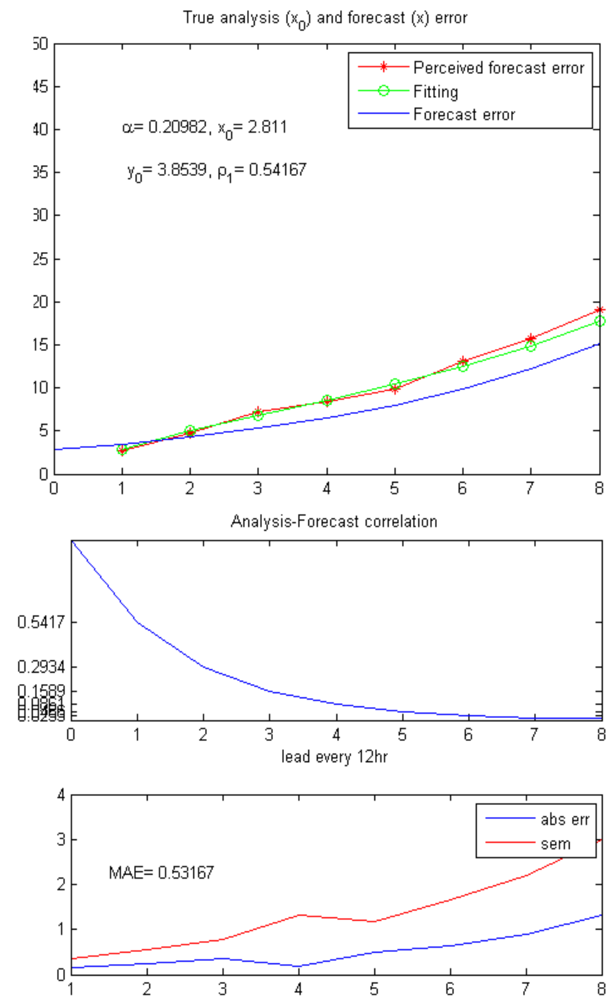
Point at 135W west (P. Ocean)



Point at 122.5W (W. coast)



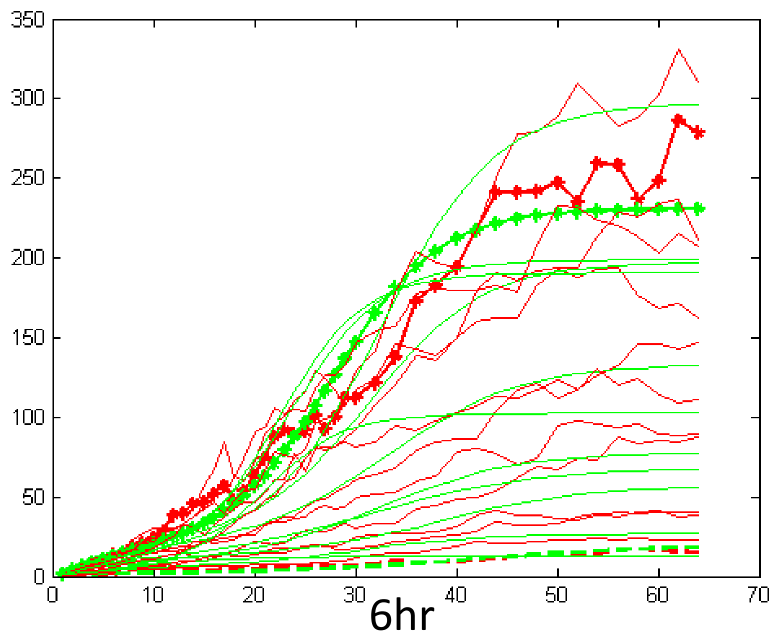
Point at 110W (R. Mountains)

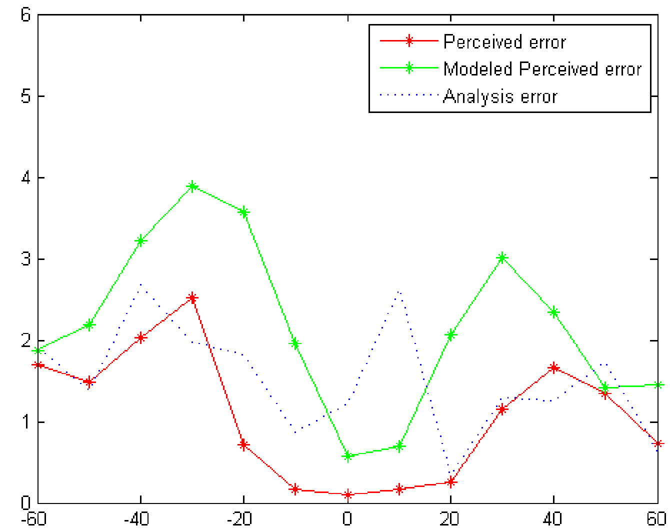


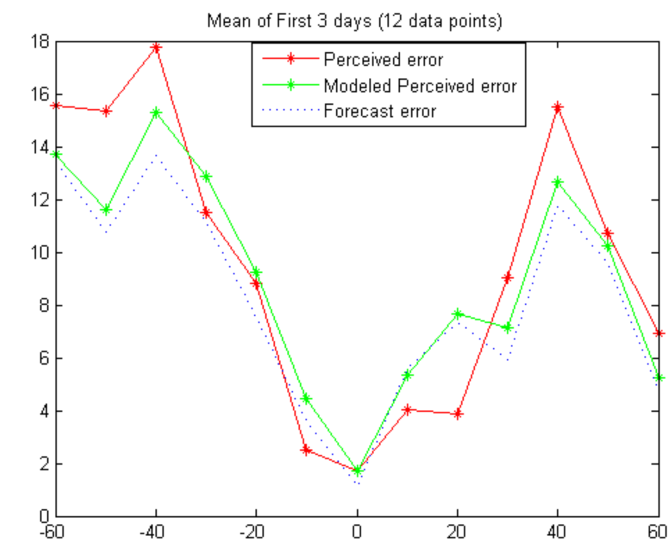
Fewer observations over ocean.
Forecast errors are underestimated.
Correlation is high.

Sufficient observations over land.
Forecast errors are uncorrelated to analysis errors

Points along 190W. Total energy error variance data

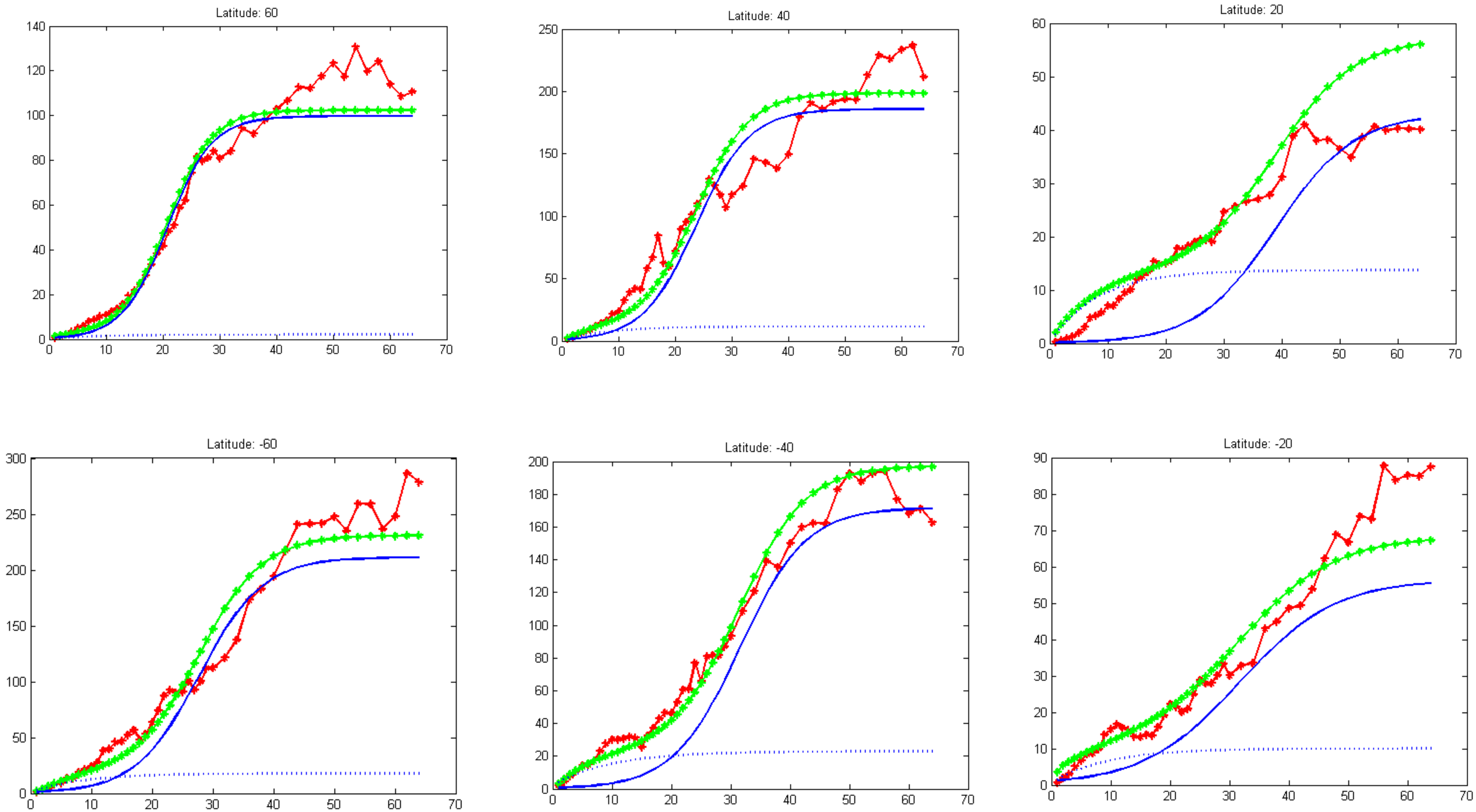






- Perceived errors for different latitudes from 60S to 60N
- 500hPa Total Energy errors
- Generally good fit
- Cost function computes parameters of all points at once. This allows some smoothing (not done yet)

7-parameter fitting curve: partitioning initial and model-related error



Concluding Remarks

- A method to estimate analysis and forecast error based on minimal assumptions is introduced.
    - The method assumes that small errors grow exponentially, that errors at short lead time are local and that correlation of errors decay following a power law
- The method was tested in the 3-variables Lorenz model in a perfect model scenario.
    - Results show accurate estimation of analysis errors and less accurate but still good estimate of short lead forecast errors
    - The method fails where assumptions are not met such that when the correlation of errors does not decay following a power law
- The method has been applied to gridpoints in the extratropics and in the tropics. For the 500hPa height field, the point in the extratropics is close to the perceived error; in the tropics the errors are underestimated.

Concluding Remarks

- With the method it is feasible to intercompare the performance of operational DA systems. Correlation of errors is a major diagnostic parameter to assess the performance of DA systems. It is shown that the ECMWF does have a superior DA system as the correlation is much smaller.
- More complex perceived error growth is addressed with a 7-parameter function. The fitting is excellent and allows partitioning of the two components of error growth (internal and drift).
- The method captures the geographical change in variance reduction due to observations (ocean-land) contrast

Backup slides

Optimization procedure (continuation):

- the weights w_i in (6) are introduced to make the fitting more accurate where the sampling standard error of the mean (SEM) is smaller, which is usually at short lead times.

$$w_i = \frac{SEM_i}{\sum_i SEM_i} \tag{7}$$

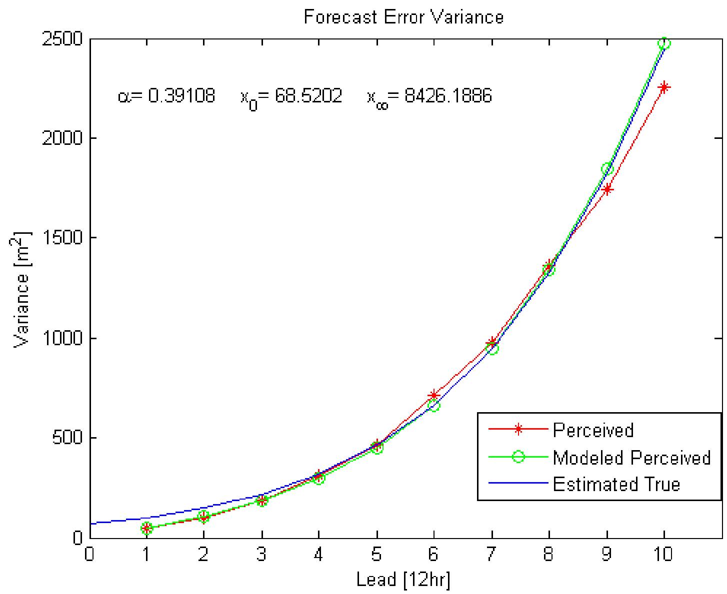
where $SEM_i = g \cdot \frac{s}{\sqrt{N}}$ and $g = \sqrt{\frac{1+(N-1)\cdot r}{1-r}}$

s = sample standard deviation, N = sample size and r = autocorrelation

- The minimum of (6) is found using the Nelder-Mead Simplex method available in Matlab (Reference: Lagarias, J.L., J. A. Reeds, M.H. Wrights and P.E. Wright (1998): Convergence properties of the Nelder-Mead Simplex Method in Low dimensions, *SIAM J. Optim.,* **9,** 112-147)

Forecast performance intercomparison

GFS



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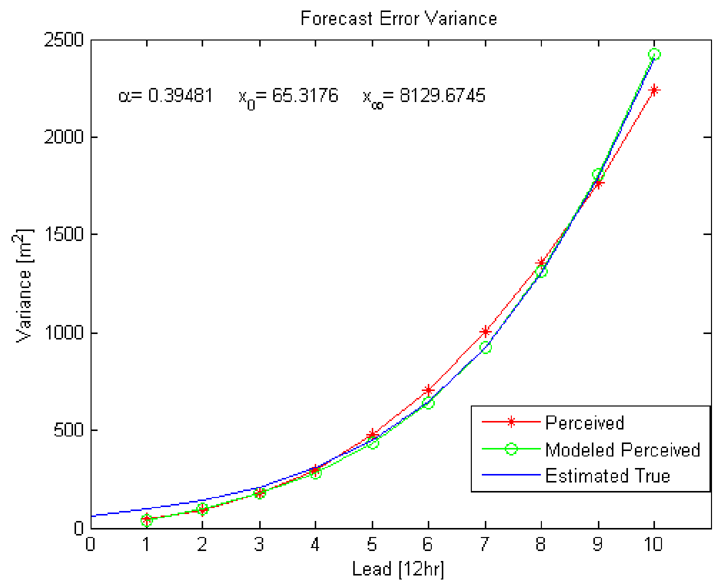
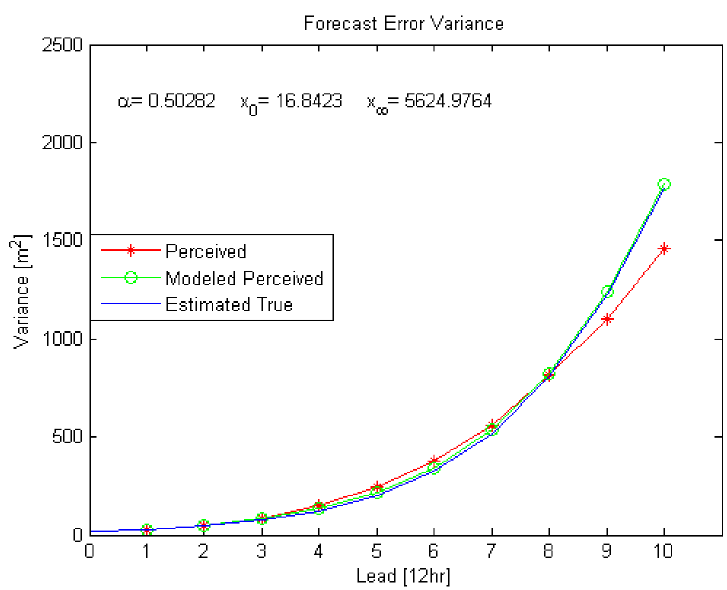


ECMWF



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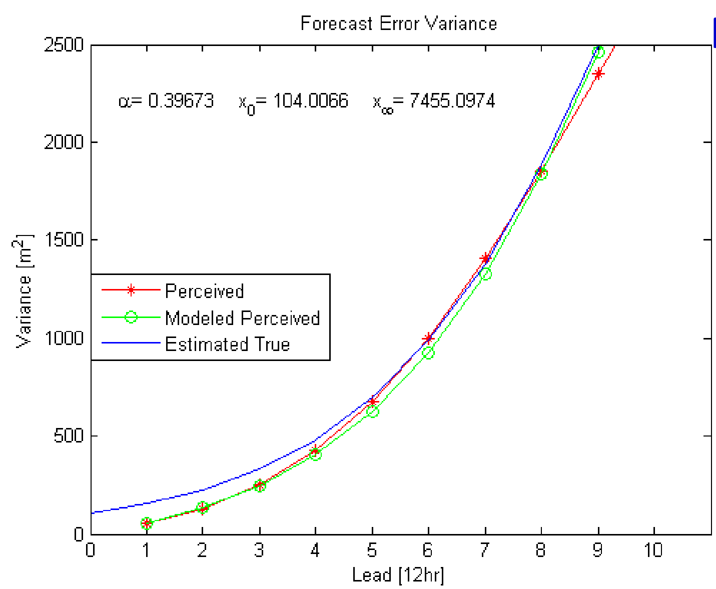
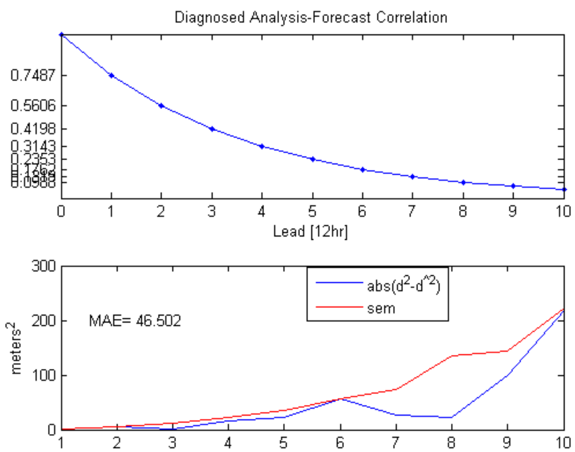


Forecast performance intercomparison

N.Hemisphere 500 hPa height field

GFS



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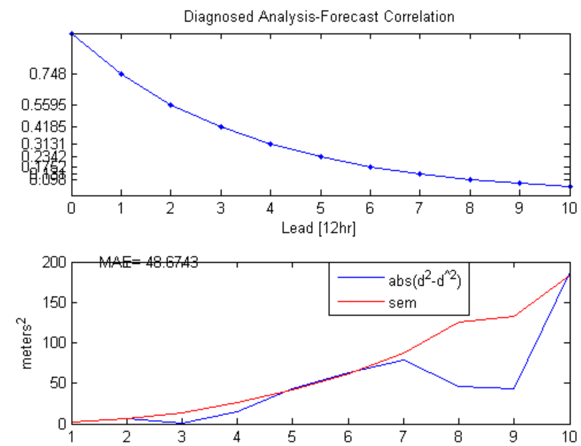
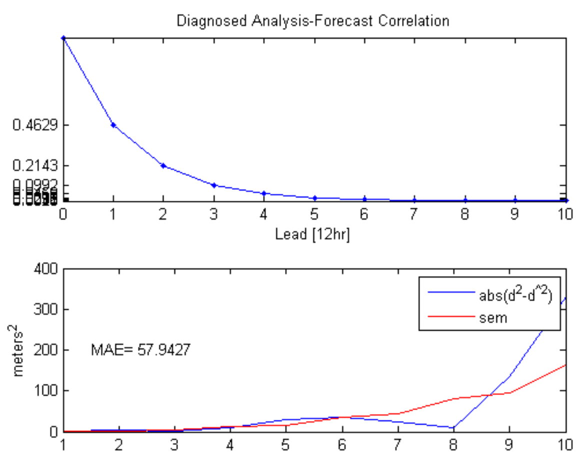


ECMWF



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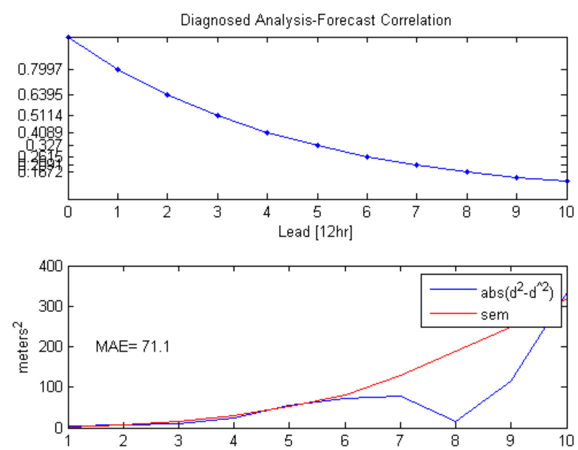

GFS 500 hPa height data. N.H average

- Four months of data, averaged over the N. Hemisphere provide a reliable estimate of the expected error variances.